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WITT KERNELS OF BI-QUADRATIC EXTENSIONS IN CHARACTERISTIC 2

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Let k be a field of characteristic 2. The author's previous results (*Arch. Math.* (1994)) are used to prove the excellence of quadratic extensions of k. This in turn is used to determine the Witt kernel of a quadratic extension up to Witt equivalence. An example is given to show that Witt equivalence cannot be strengthened to isometry.

INTRODUCTION

An extension of fields L/k induces a homomorphism $\iota^* : W(L) \to W(k)$ of the Witt groups of (equivalence classes of) quadratic forms. The kernel of (ι^*) is precisely the (classes of) anisotropic k-forms that become hyperbolic over L. For fields of characteristic $\neq 2$, the Witt kernels of quadratic and bi-quadratic extensions are known (for example see [**5**, Theorem 3.2 p. 200] and [**4**, 2.12 p. 457]). In characteristic 2, the results for separable quadratic and separable bi-quadratic extensions are analogous to the characteristic $\neq 2$ case. Baeza in [**3**, 4.3] determined the Witt kernel of inseparable quadratic extensions up to Witt equivalence. In [**1**, 2.8], the Witt kernel of such extensions was determined up to isometry. The purpose of this note is to characterise the Witt kernel of bi-quadratic extensions in characteristic 2 (Theorems 10 and 11). As in the characteristic $\neq 2$ case the excellence of quadratic extensions plays an important role. Based on [**1**, 2.5] we establish the excellence of inseparable quadratic extension (Proposition 3). In the end we give an example to show that the characterisation (which is up to Witt equivalence) cannot be improved to isometry.

TERMINOLOGY AND NOTATION. Throughout, the field k will always have characteristic 2. We follow the standard notation as in [**2**]. In particular, a quadratic k-form (or simply a form) q is a map from a finite dimensional k-vector space V to k satisfying: (i) For every $a \in k$ and $\mathbf{x} \in V$, $q(a\mathbf{x}) = a^2 q(\mathbf{x})$, and (ii) $B_q(\mathbf{x},\mathbf{y}) := q(\mathbf{x}+\mathbf{y}) - q(\mathbf{x}) - q(\mathbf{y})$ is a bilinear map. A form q is called anisotropic if $\big(q(\mathbf{x}) = 0$ implies $\mathbf{x} = \mathbf{0}\big)$; and q is called nonsingular if the subspace $V^\perp := \{\mathbf{x} \in V \mid B_q(\mathbf{x},\mathbf{y}) = 0 \ \textit{for all}\ \mathbf{y} \in V\} = 0$. The isometry of forms is denoted by $\cong$, and the orthogonal sum of forms is denoted by $\perp$. The (non-singular) two dimensional quadratic space (V, q) with a basis $\{\mathbf{e}, \mathbf{f}\}$ such that

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$q(\mathbf{e}) = a$, $q(\mathbf{f}) = b$ and $B_q(\mathbf{e}, \mathbf{f}) = 1$ will be denoted by $[a, b]$. The hyperbolic plane $[0, 0]$ will be denoted by $\mathbb{H}$. The orthogonal sum $m \cdot \mathbb{H} := \mathbb{H} \perp \ldots \perp \mathbb{H}$ (m-summands) is called a hyperbolic form. It is known ([2]) that any non-singular k-form q decomposes into

$$q \cong m \cdot \mathbb{H} \perp [a_1, b_1] \perp \ldots \perp [a_r, b_r],$$

with $a_i, b_i \in k$ such that the form $q' = [a_1, b_1] \perp \ldots \perp [a_r, b_r]$ is anisotropic. The form q' and m are uniquely determined by q and are, respectively, called the anisotropic part and the Witt index of q. Two forms are Witt equivalent if their anisotropic parts are isometric. The set of Witt equivalence classes of non-singular quadratic k-forms with the operation $\perp$ define a group $W(k)$, the Witt group of k. Let L/k be a field extension. Then any k-form q can also be viewed as an L-form (denoted by q_L).

1. Excellence of quadratic extensions

Let L/k be a field extension. An L-form φ is said to be defined over k if there exists a k- form γ such that $\varphi \cong \gamma_L$. The extension L/k is called excellent if the anisotropic part over L of any non-singular k-form is defined over k.

It is known that separable quadratic extension are excellent. This is an immediate consequence of the following proposition.

Proposition 1.1. [2, Theorem 4.2 p. 121] *Let k be a field of characteristic 2 and let $K = k(\beta)/k$ where $\beta \notin k$ and $\beta^2 - \beta = b \in k$. If a non-singular anisotropic k-form q has Witt index s over K, then $q \cong c_1[1, b] \perp \ldots \perp c_s[1, b] \perp q_0$ for some $c_1, \ldots, c_s \in k$ and a k-form q_0. In particular, if q becomes hyperbolic over K, then $q \cong c_1[1, b] \perp \ldots \perp c_r[1, b]$ for some $c_1, \ldots, c_r \in k$*

Before showing that inseparable quadratic extensions are excellent, we make the following.

Remark 1.2. Let $K = k(\sqrt{d})$.

(i) By [1, 2.5], any non-singular anisotropic k-form q can be written as

$$q \cong q_1 \perp (q_0 \perp dq_0) \perp \big([a_1, b_1] \perp d[a_1, c_1]\big) \perp \ldots \perp \big([a_r, b_r] \perp d[a_r, c_r]\big),$$

where q_0 and q_1 are k-forms and $a_i, b_i, c_i \in k$, $(i = 1, \ldots, r)$ such that the Witt index of q over $K = k(\sqrt{d})$ equals $\dim(q_0) + r$. In particular, q_1 remains anisotropic over K, and $[a_i, b_i] \perp d[a_i, c_i]$ is not hyperbolic over K.

(ii) Let $a, b, c \in k$. Then, over K, we have

$$[a, b] \perp d[a, c] \cong [a, b] \perp [a, c] \cong [0, b] \perp [a, b + c] \cong \mathbb{H} \perp [a, b + c]$$

The first isometry follows because $d \in K^2$. To see the second isometry, take $\mathbf{u}_1, \mathbf{v}_1, \mathbf{u}_2, \mathbf{v}_2$ to be a basis associated with the presentation $[a, b] \perp [a, c]$,

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and rewrite the form with respect to the new basis $\mathbf{u}_1 + \mathbf{u}_2, \mathbf{v}_1, \mathbf{u}_2, \mathbf{v}_2 + \mathbf{v}_1$ to get $[0, b] \perp [a, b + c]$.

PROPOSITION 1.3. *Let k be a field of characteristic 2. Let q be a non-singular anisotropic k-form. Then the anisotropic part of q over $K = k(\sqrt{d})$ is defined over k; i. e. K/k is excellent.*

PROOF: Replacing q by its anisotropic part, we may assume that q is anisotropic over k. Write

$$q \cong q_1 \perp (q_0 \perp dq_0) \perp \left([a_1, b_1] \perp d[a_1, c_1]\right) \perp \ldots \perp \left([a_r, b_r] \perp d[a_r, c_r]\right),$$

as in (i) of Remark 1.2. By (ii) of 1.2, $[a_i, b_i] \perp d[a_i, c_i] \cong \mathbb{H} \perp [a_i, b_i + c_i]$, and therefore

$$q_K \cong (\dim q_0 + r)\mathbb{H} \perp q_1 \perp [a_1, b_1 + c_1] \perp \ldots \perp [a_r, b_r + c_r].$$

Since the Witt index of q_K equals $\dim q_0 + r$ (Remark 1.2.(i)), the anisotropic part of q over K is $q_1 \perp [a_1, b_1 + c_1] \perp \ldots \perp [a_r, b_r + c_r]$, which is defined over k. ∎

As in characteristic $\neq 2$, the excellence property implies (see [**3**, Lemma 2.1])

COROLLARY 1.4. *Let $K = k(\sqrt{d})$ be a quadratic extension over k. Let σ and δ be non- singular k-forms and let γ be a non-singular K-form. If $\sigma_K \cong \delta_K \perp \gamma$, then γ is defined over k.*

To determine the Witt kernel of bi-quadratic extensions, one needs a "characteristic 2" analogue of [**3**, Proposition 2.11.(a)]. For separable quadratic extensions we have the following statement whose proof is identical to that of [**3**, Proposition 2.11.(a)].

THEOREM 1.5. *Let k be a field of characteristic 2. Let K/k be an excellent extension of k. Let $q \cong e_1[1, b] \perp \ldots \perp e_r[1, b]$ where $b \in k^*$ and $e_1, \ldots, e_r \in K$. If q is defined over k, then there exist $c_1, \ldots, c_r \in k$ such that $q \cong c_1[1, b] \perp \ldots \perp c_r[1, b]$.*

In the case of inseparable quadratic extensions we have

THEOREM 1.6. *Let K be an excellent extension of k, and let $d \in k^*$. Let γ be a non-singular K-form such that the form $\gamma \perp d\gamma$ is defined over k. Then there exists a k-form δ such that $\gamma \perp d\gamma \cong (\delta \perp d\delta)_K$.*

The proof will be broken into two lemmas.

LEMMA 1.7. *Let $a, b, c, d \in k$ such that the form $\delta \cong [a, b] \perp d[a, b]$ represents c. Then $\delta \cong [c, b'] \perp d[c, b']$ for some $b' \in k$.*

PROOF: Note that $\delta \cong a\alpha$ where α is the Pfister form $[1, ab] \perp d[1, ab]$. Since α represents ac, we have

$$\delta \cong a\alpha \cong a^2 c\alpha \cong c\alpha \cong c[1, ab] \perp dc[1, ab]$$

But $c[1, ab] \cong [c, b']$ for some $b' \in k$. ∎

Lemma 1.8. *Let K be a field extension of k, and let $d \in k^{\bullet}$. Let γ be a non-singular K-form such that the form $\gamma \perp d\gamma$ represents an element of k. Then there exist $a \in k$, $b \in K$, and a K-form γ_1 such that*

$$\gamma \perp d\gamma \cong \big([a,b] \perp d[a,b]\big) \perp (\gamma_1 \perp d\gamma_1)$$

Proof: Let (V,γ) be a non-singular K-quadratic space. If d is a square in K, then $\gamma \perp d\gamma$ is a hyperbolic form of dimension $2(\dim\gamma)$ which is divisible by 4. Hence in this case we take $a = b = 0$ and γ_1 to be the hyperbolic form of dimension $(\dim\gamma - 1)$.

If γ is isotropic, then $\gamma \cong [0,0] \perp \gamma_1$ and we then let $a = b = 0$. So, we may assume that γ is anisotropic and $\gamma \perp d\gamma$ represents an element $c \in k^{\bullet}$. So, there exist $\mathbf{v}_1, \mathbf{v}_2 \in V$ not both zero such that

$$c = \gamma(\mathbf{v}_1) + d\gamma(\mathbf{v}_2).$$

If $\mathbf{v}_1$ (respectively $\mathbf{v}_2$) is the zero vector, then $\gamma(\mathbf{v}_1)$ (respectively $\gamma(\mathbf{v}_2)$) equals c (respectively c/d). So, γ represents an element a of $k^{\bullet}$ where $a = c$ or $a = c/d$. Therefore $\gamma \cong [a,b] \perp \gamma_1$ for some $b \in K$ and a K-form γ_1, and the conclusion follows.

Therefore we may assume that $\mathbf{v}_1$ and $\mathbf{v}_2$ are both non-zero. First assume that $\mathbf{v}_1$ and $\mathbf{v}_2$ are not orthogonal or $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly dependent. Since γ is non-singular, in either case the vectors $\mathbf{v}_1, \mathbf{v}_2$ are contained in a non-singular two dimensional quadratic subspace V_0 of V. Then

$$(V,\gamma) \cong (V_0, \gamma|_{V_0}) \perp \gamma_1 \tag{1}$$

for some K-form γ_1. Let $f = \gamma(\mathbf{v}_1)$ and $g = \gamma(\mathbf{v}_2)$. Then by [**1**, 2.2], we have

$$(V_0, \gamma|_{V_0}) \cong [f, f'] \quad \text{and} \quad (V_0, \gamma|_{V_0}) \cong [g, g'], \tag{2}$$

for some $f', g' \in K$. Since $f + d_2 g = \gamma(\mathbf{v}_1) + d\gamma(\mathbf{v}_2) = c$, the form $[f,f'] \perp d[g,g']$ represents $c \in k$. By the previous lemma, there exists $b \in K$ such that

$$[f,f'] \perp d[g,g'] \cong [c,b] \perp d[c,b] \tag{3}$$

From the equations (1)–(3) we have

$$\begin{aligned} \gamma \perp d\gamma &\cong \big([f,f'] \perp \gamma_1\big) \perp d\big([g,g'] \perp \gamma_1\big) \\ &\cong \big([f,f'] \perp d[g,g']\big) \perp (\gamma_1 \perp d\gamma_1) \\ &\cong \big([c,b] \perp d[c,b]\big) \perp (\gamma_1 \perp d\gamma_1) \end{aligned}$$

as desired.

Finally, assume that $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal and linearly independent. Then by [**1**, 2.2],

$$\gamma \cong [e_1, f_1] \perp [e_2, f_2] \perp \gamma_0 \tag{4}$$

where $e_i = \gamma(\mathbf{v}_i)$, $i = 1, 2$, $f_1, f_2 \in K$ and γ_0 is a K-form. Since $e_1 + de_2 = \gamma(\mathbf{v}_1) + d\gamma(\mathbf{v}_2) = c$, the form $[e_1, f_1] \perp d[e_2, f_2]$ represents $c \in k$. By [**1**, 2.2] again, there exists $b, r, s \in K$ such that

$$[e_1, f_1] \perp d[e_2, f_2] \cong [c, b] \perp [r, s] \tag{5}$$

From the equations (4) and (5) we have

$$\begin{aligned}\gamma \perp d\gamma &\cong \big([e_1, f_1] \perp [e_2, f_2] \perp \gamma_0\big) \perp d\big([e_1, f_1] \perp [e_2, f_2] \perp \gamma_0\big) \\ &\cong \big([e_1, f_1] \perp d[e_2, f_2] \perp \gamma_0\big) \perp d\big([e_1, f_1] \perp d[e_2, f_2] \perp \gamma_0\big) \\ &\cong \big([c, b] \perp [r, s] \perp \gamma_0\big) \perp d\big([c, b] \perp [r, s] \perp \gamma_0\big) \\ &\cong \big([c, b] \perp d[c, b]\big) \perp (\gamma_1 \perp d\gamma_1)\end{aligned}$$

where $\gamma_1 := \big([r, s] \perp \gamma_0\big)$. This completes the proof of the lemma. □

REMARK 1.9.

(i) The forms $[a, b]$, $a[1, ab]$ and $a[1, a^2b^2]$ are isometric because they are two dimensional forms representing a common element a and have the same Arf invariant (see [**6**, Lemma 4.4.(i), p. 341]).

(ii) Let $K = k(\sqrt{d})$, $a \in k$ and $b \in K$. By (i), $[a, b] \cong a[1, a^2b^2]$; hence is defined over k because $a^2b^2 \in k$. So in the conclusion of Lemma 1.8, we may assume that both a and b are in k.

To complete the proof of Theorem 1.6, note that if $\gamma \perp d\gamma$ is defined over k, then it represents an element of k. By 1.8 and 1.9

$$\gamma \perp d\gamma \cong \big([a, b] \perp d[a, b]\big) \perp (\gamma_1 \perp d\gamma_1)$$

where $a, b \in k$ and γ_1 is a K-form. If $\dim(\gamma) = 2$, then we are done. If $\dim(\gamma) > 2$, then corollary 4 implies that $\gamma_1 \perp d_2\gamma_1$ is defined over k. The assertion of the theorem follows by induction.

2. WITT KERNELS OF BI-QUADRATIC EXTENSIONS

We start with the inseparable case first. One distinguishes between two types of inseparable bi-quadratic extensions: the purely inseparable case where $L = k(\sqrt{d_1}, \sqrt{d_2})$ with $d_1, d_2 \in k$; and the case L/k contains an intermediate separable extension. In the latter case, $L = k(\beta, \sqrt{d})$ for some non-square element $d \in k$ and $\beta \notin k$ such that $\beta^2 - \beta = b \in k$.

THEOREM 2.1. *Let L/k be an inseparable bi-quadratic extension over k. Let q be an anisotropic non-singular k-form such that q is hyperbolic over L.*

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(i) *If $L = k(\sqrt{d_1}, \sqrt{d_2})$ with $d_1, d_2 \in k$, then q is Witt equivalent to a form of the shape*

$$(q_1 \perp d_1 q_1) \perp (q_2 \perp d_2 q_2)$$

for some k-forms q_1 and q_2.

(ii) *If $L = k(\beta, \sqrt{d})$ where $d \in k - k^2$, $\beta \notin k$ and $\beta^2 - \beta = b \in k$, then q is Witt equivalent to a form of the shape*

$$\big(c_1[1,b] \perp \ldots \perp c_r[1,b]\big) \perp (q_0 \perp dq_0)$$

for some $c_i \in k$ $(i = 1, \ldots, r)$ and a k-form q_0.

PROOF: For (i), let $K = k(\sqrt{d_1})$. If q is hyperbolic over K, the theorem follows immediately from [**1**, 2.8]. So, assume q_K is not hyperbolic. Let φ denote the anisotropic part of q over K. By proposition, φ is defined over k. Since q is hyperbolic over $L = K(\sqrt{d_2})$, φ_L is hyperbolic; hence there exists a K- form q_2 such that $\varphi \cong q_2 \perp d_2 q_2$. By Theorem 1.6, we may assume that q_2 is a k-form. Consider the k-form $\alpha := q \perp -(q_2 \perp d_2 q_2)$. Over K, the form α is hyperbolic because (in $W(K)$) $[\alpha_K] = \big[q \perp -(q_2 \perp d_2 q_2)_K\big] = [\varphi \perp -\varphi] = 0$. So, by [1, 2.8], α is Witt equivalent (over k) to $q_1 \perp d_1 q_1$ for some k-form q_1. Therefore in the Witt ring of k we have

$$\big[q \perp -(q_2 \perp d_2 q_2)\big] = [q_1 \perp d_1 q_1];$$

or equivalently,

$$[q] = \big[(q_2 \perp d_2 q_2) \perp (q_1 \perp d_1 q_1)\big]$$

as desired.

For (ii), we let $K = k(\beta)$. If q_K is hyperbolic, then we are done by 1.1. So, assume that q_K is not hyperbolic and let φ be its anisotropic part. As in part (i), it follows that $\varphi \cong q_0 \perp d_2 q_0$ for some k-form q_0 and the k-form $\alpha := q \perp q_0 \perp d_2 q_0$ is hyperbolic over K. Proposition 1.1 implies that α is Witt equivalent (over k) to $c_1[1,b] \perp \ldots \perp c_r[1,b]$ for some $c_i \in k$ $(i = 1, \ldots, r)$. Therefore q is Witt equivalent to $\big(c_1[1,b] \perp \ldots \perp c_r[1,b]\big) \perp (q_0 \perp dq_0)$ ∎

Using an argument similar to that in the proof of Theorem 2.1 (or similar to [**3**, Proposition 2.12] together with Theorem 1.5) we get

THEOREM 2.2. *Let $L = k(\alpha, \beta)$ be a (separable) bi-quadratic extension over k with $\alpha^2 - \alpha = a \in k$ and $\beta^2 - \beta = b \in k$. Let q be an anisotropic non-singular k-form. If q is hyperbolic over L, then q is Witt equivalent to a form of the shape*

$$\big(e_1[1,a] \perp \ldots \perp e_r[1,a]\big) \perp \big(f_1[1,b] \perp \ldots \perp f_s[1,b]\big)$$

for some $e_i, f_j \in k$ $(i = 1, \ldots, r; j = 1, \ldots, s)$.

We conclude this note by an example which shows that the Witt equivalence in the conclusions of Theorems 2.1 and 2.2 above cannot be strengthened to isometry.

EXAMPLE. Let k_0 be a fixed field of characteristic two. Let $k = k_0(r, s, t, u)$ where r, s, t, u are algebraically independent elements over k_0 and set

$$q \cong [1, r] \perp t[1, s] \perp u[1, r + s].$$

Let α, β (in the algebraic closure of k) be such that $\alpha^2 - \alpha = r$ and $\beta^2 - \beta = r + s$. Then

(i) The form q is anisotropic over k because r, s, t, u are algebraically independent elements over k_0 (see [5, ex. 1 p. 273]).

(ii) Over the fields $K_1 = k(\sqrt{t})$, $K_2 = k(\sqrt{u})$, $K_3 = k(\alpha)$ and $K_4 = k(\beta)$, the form q is isotropic and have Witt index 1. We see this as follows: First over K_1, $t \in K_1^2$ and $[1, r] \perp t[1, s] \cong [1, r] \perp [1, s] \cong \mathbb{H} \perp [1, r + s]$ (see (ii) of Remark 1.2). Therefore

$$q_{K_1} \cong \mathbb{H} \perp [1, r + s] \perp u[1, r + s]. \tag{6}$$

Since $r+s$ and u are algebraically independent over $k_0(\sqrt{t})$, the form $[1, r+s] \perp u[1, r+s]$ is anisotropic over K_1, and therefore q_{K_1} has Witt index 1. Similarly, we can show that q_{K_2} also has Witt index 1.

Now over $K_3 = k(\alpha)$, the form $[1, r]$ is isotropic and $[1, r + s] \cong_{K_3} [1, s]$ (for they have the same Arf invariant over K_3 and represent 1). Therefore, over K_3,

$$q_{K_3} \cong \mathbb{H} \perp t[1, r + s] \perp u[1, r + s] \tag{7}$$

and $t[1, s] \perp u[1, s]$ is anisotropic over K_3 because s, t and u are algebraically independent over $k_0(\alpha)$. Therefore, q_{K_3} has Witt index 1. Likewise, q_{K_4} has Witt index 1.

(iii) The form q is hyperbolic over the fields $L_1 = k(\sqrt{t}, \sqrt{u})$, $L_2 = k(\sqrt{t}, \beta)$ and $L_3 = k(\alpha, \beta)$:

Note that $u \in L_1^2$ and therefore the form $[1, r + s] \perp u[1, r + s] \cong [1, r + s] \perp [1, r + s] \cong 2\mathbb{H}$. Since $K_1 \subset L_1$, we have from equation (6) above that

$$q_{L_1} \cong \mathbb{H} \perp [1, r + s] \perp u[1, r + s] \cong 3\mathbb{H}$$

That is, q_{L_1} is hyperbolic.

Since β belongs to L_2 and L_3, $[1, r+s] \cong \mathbb{H}$ over L_2 and L_3 because $\beta^2+\beta+(r+s) = 0$. Therefore the form $[1, r+s] \perp u[1, r+s]$ (respectively, $t[1, r+s] \perp u[1, r+s]$) is hyperbolic over L_2 (respectively, L_3). Therefore equation (6) (respectively, equation (7)) implies that q_{L_2} (respectively, q_{L_3}) is hyperbolic.

(iv) Theorems 2.1 and 2.2 imply that over k the form q is Witt equivalent to forms of the shape

(a) $(q_1 \perp tq_1) \perp (q_2 \perp uq_2)$.

(b) $(q_1 \perp tq_1) \perp (c_1[1, r + s] \perp \ldots \perp c_n[1, r + s])$.

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(c) $\big(b_1[1,r] \perp \ldots \perp b_m[1,r]\big) \perp \big(c_1[1,r+s] \perp \ldots \perp c_n[1,r+s]\big)$.

where $b_j, c_i \in k$ and q_1 and q_2 are non-singular k-forms. This Witt equivalence cannot be improved to isometry. For if q is isometric to (a), (b) or (c), then by comparing dimensions we have either $\dim q_1 \geqslant 2$, $\dim q_2 \geqslant 2$, $m \geqslant 2$ or $n \geqslant 2$. This respectively imply that the Witt index over K_1, K_2, K_3 or K_4 is $\geqslant 2$; contradicting part (ii) of this example.

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