The Equality and Inequality of Fractions.—In a recent issue of *The Mathematical Gazette* (May 1910), Mr C. S. Jackson points out that certain algebraic theorems appear more difficult to some beginners than would have been anticipated, and he instances as a case in point the proof of the proposition that if $\frac{a}{b}=\frac{c}{d}$, each ratio is equal to $\frac{a+c}{b+d}$. He almost apologises (I think quite unnecessarily) for making the suggestion that some concrete illustrations of the proposition may render its simplicity more manifest, and thus prepare the way for an enunciation of the result in its most general form. The illustration Mr Jackson takes of the above proposition is:

> "A regiment of 1000 men contains 130 Yorkshiremen; another of 1200 men contains 156.
>
> The proportion of Yorkshiremen in each regiment is the same, for
>
> $$\frac{130}{1000}=\frac{156}{1200}.$$
>
> The proportion of Yorkshiremen in the total force is
>
> $$\frac{130+156}{1000+1200},$$
>
> and this is obviously the same as the proportion in either regiment."

Another simple type of illustration, capable of extension, is as follows:

Consider an alloy of two parts of copper to three parts of zinc.

(i) One lump of alloy might contain 2 lbs. of copper and 3 lbs. of zinc; another lump 6 lbs. of copper and 9 lbs. of zinc. If these two are made into one lump, it would contain $(2+6)$ lbs. of copper and $(3+9)$ lbs. of zinc. But the ratio of copper to zinc is unaltered. Hence

$$\frac{2}{3}=\frac{6}{9}=\frac{2+6}{3+9}.$$

Generally

$$\frac{a}{b}=\frac{c}{d}=\frac{a+c}{b+d}.$$

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(ii) If a lump of the alloy containing say 2 lbs. of copper and 3 lbs. of zinc be fused with other 4 lbs of copper and other 4 lbs. of zinc into another lump, then the second lump is "more coppery" than the first. Hence

$$\frac{2}{3} < \frac{2+4}{3+4},$$

or

$$\frac{a}{b} < \frac{a+c}{b+c} \text{ if } a < b.$$

Similarly

$$\frac{a}{b} > \frac{a+c}{b+c} \text{ if } a > b.$$

Also we may illustrate the inequality between

$$\frac{a}{c} \text{ and } \frac{a-c}{b-c}.$$

(iii) If a lump of an alloy of copper and zinc containing a parts of copper and b parts of zinc be fused with a lump of a second alloy of copper and zinc containing c parts of copper and d parts of zinc, then the lump so formed will contain $(a+c)$ parts of copper and $(b+d)$ parts of zinc. If $a/b \neq c/d$, then the new alloy is "less coppery" than the one and "more coppery" than the other. That is $\dfrac{a+c}{b+d}$ lies between $\dfrac{a}{b}$ and $\dfrac{c}{d}$. Similarly, if we suppose n alloys to be fused into one, we see that

$$(a_1 + a_2 + \ldots + a_n)/(b_1 + b_2 + \ldots + b_n)$$

lies between the least and greatest of the fractions a_1/b_1, a_2/b_2, etc.

Graphical illustrations of these propositions are also instructive.

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Proof of a Property of Simson's Line.—The following is a slightly simplified version of a well-known proof of this theorem :—

The Simson's Line of P, with respect to $\triangle$ABC, bisects PO, where O is the orthocentre of the triangle.