The Effect of Binary Motion on Period Changes in RR Lyrae Stars

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The present investigation is to determine what apparent period changes if any, would result from binary motions. In particular it is important to know whether or not such apparent period changes could be mistaken for intrinsic period changes caused by evolution.

Period changes in variable stars are investigated by studies of their light curves over a long interval of time. At any time t, the phase is computed with respect to a reference time t and is given by

$$\Phi = \frac{1}{2\pi}\int_{t_0}^{t} \omega \, dt,$$

$$\text{where } \omega = \frac{2\pi}{P}$$

and P is the period of the light variation.

Thus at any time t the phase shift due to changes in the period is

$$\Delta \Phi = -\int \frac{\Delta P}{P_0^2} dt.$$

If a variable star has a period P_0 and radial velocity V, the observed period will differ from the true period P_0 by an amount $\frac{P_0 V}{c}$. If the radial velocity changes, then the period of light variation will also appear to change. Thus, if the star belongs to a binary system, the period of its light variation will appear to vary because of the orbital motion. The apparent period at a given time t will be a function of the radial component of the orbital velocity which depends on the true anomaly Θ

$$P = P(\Theta) = P_0\left(1 + \frac{V_r(\Theta)}{c}\right)$$

$$V_r\,(\text{orbital}) = K\cos(\Theta + \omega) + Ke\cos\omega$$

$$\text{and } K = \frac{2\pi}{P_B}\,\frac{a\sin i}{\sqrt{1-e^2}}$$

where P_0 is the period of light variation
P_B the orbital period of the binary
2K is the total range of radial velocity
e the eccentricity of the orbit
ω the argument of perihelion
a the semi-major axis
i the inclination of the orbit with respect to the plane of the sky.

$$P = P_0 \text{ when } V_r(\Theta) = 0$$

$$\Delta P = P(\Theta) - P_0 = \frac{K P_0}{c}(\cos(\Theta + \omega) + e\cos\omega)$$

The phase shift of the light curve of a variable which is a component of a binary system is therefore given by

$$\Delta \Phi = \frac{-K}{P_0 c}\int_{P_0}^{P(\Theta)} (\cos(\Theta + \omega) + e\cos\omega)\, dt$$

provided that the period of the variable is otherwise constant. Applying Kepler's second law gives

$$\Delta \Phi = \frac{-2\pi}{c P_0 P_B} \cdot \frac{a \sin i}{\sqrt{1-e^2}} \cdot \frac{P_B a^2 (1-e^2)}{2\pi a^2 \sqrt{1-e^2}} \int_{\Theta_0}^{\Theta} \frac{\cos(\Theta+\omega) + e\cos\omega}{(1+e\cos\Theta)^2}\, d\Theta$$

$$= \frac{-a(1-e^2)\sin i}{c P_0} \int_{\Theta_0}^{\Theta} \frac{\cos\Theta\cos\omega - \sin\Theta\sin\omega + e\cos\omega}{(1+e\cos\Theta)^2}\, d\Theta$$

$$\text{Thus } \Delta \Phi = \frac{-a(1-e^2)\sin i}{c P_0} \left[\frac{\cos\omega\sin\Theta}{1+\cos\Theta} - \frac{\sin\omega}{e(1+e\cos\Theta)}\right]_{\Theta_0}^{\Theta}$$

Differentiating $\Delta \Phi$ and equating it to zero shows that the phase shift $\Delta \Phi$ has its extreme values when $\cos(\Theta+\omega) = -e\cos\omega$ and it can therefore be shown that the range of $\Delta \Phi$ is given by $\frac{2 a \sin i}{c P_0} \sqrt{1-e^2\cos^2\omega}$. According to Kepler's third law, $a^3 \propto (\mathfrak{M}_1 + \mathfrak{M}_2)\, P_B^2$ and so $a \propto P_B^{2/3}$. The range of $\Delta \Phi$ (in days) is given by:

$$\text{Range } (\Delta \Phi) = 0.01\, P_B^{2/3} \sqrt{1-e^2\cos^2\omega}\, \sin i \qquad \text{where}$$

P_B is the binary period in years and $\mathfrak{M}_1 + \mathfrak{M}_2 \simeq 1.25\, \mathfrak{M}_\odot$. From observations, the minimum detectable range in $\Delta \Phi$ depends on the quality of the light curves, but in general, this is about 0.03 days. Thus the period must be at least about 8 years.

Standard phase-shift diagrams have been plotted with the aid of tables giving the relationship between Θ, the true anomaly and time (SCHLESINGER and UDECK 1912). Figure 1 shows these diagrams for an eccentricity of 0.75 and four values of ω, the argument of perihelion. Curves have not been shown for values of ω, 180° or greater, because when ω is greater than 180°, the O—C curve is the same as that for $\omega - 180°$, but inverted.

These standard curves have been compared with O—C diagrams for RR Lyrae stars in the globular cluster M 3. SZEIDL's (1965) work is the most complete survey of period changes of variables in any globular cluster. He gives O—C diagrams for 112 RR Lyrae variables. A comparison of these diagrams with the derived standard curves indicates that about 76 of these stars could be binaries. Some of his diagrams are illustrated in Figures 2 and 3 and it can be seen that they resemble the curves of Figure 1. (If the range of $\Delta \Phi$ in the O—C diagram exceeds 0.01 $P_B^{2/3}$ days, then the diagram cannot be attributed to binary motion.) The curve for V 20 appears similar to that of $\omega = 0$, while V 10 appears to have $\omega = 270°$ and V 123, $\omega = 135°$. For variable 10, a parabola can also be fit to SZEIDL's O—C diagram, implying a uniform period change and not a periodic one. In fact for about half of these possible binaries in M 3, the O—C diagrams can be attributed to uniform period changes and the accuracy of the observations does not permit a distinction between the two interpretations. However, if all these stars are binaries, we should expect to see a random distribution of the phase of the O — C diagram observed for each star. Instead, we always see the same portion — beginning and ending with the maximum (or minimum) value of $\Delta \Phi$. This indicates that the diagrams are probably a result of uniform period changes — not apparent period changes from orbital motion. In Figure 3, we see that V 13 has an O — C diagram that is parabolic, with periodic fluctuations superimposed. There are other stars with diagrams like this in M 3 and they are probably binaries.

A more realistic value for the number of binaries among RR Lyrae stars in M 3 is about 20 when we assume that the observed phases of the O — C diagrams should be randomly distributed. The orbital periods for these probable binaries in M 3 range from about 8 to 80 years.

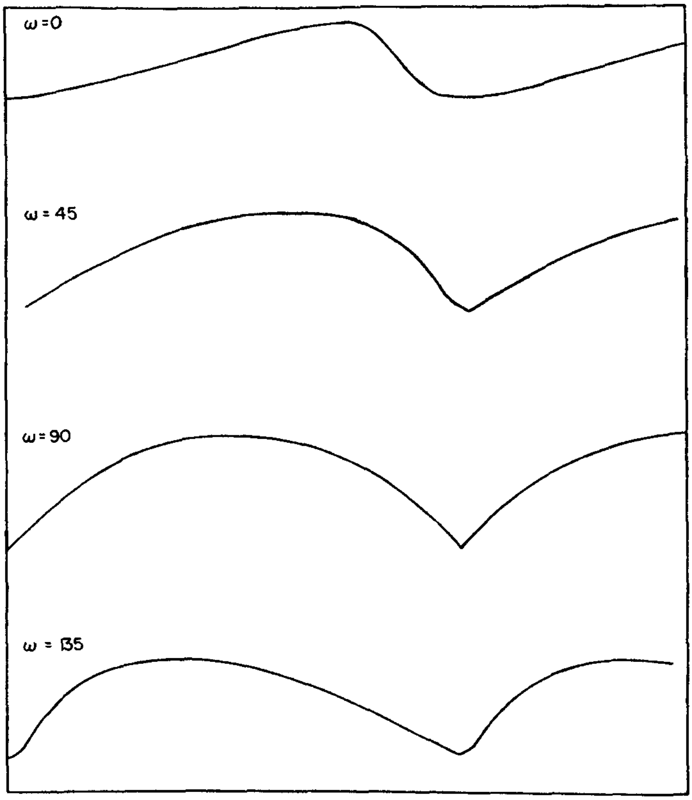

Fig. 1: Standard phase-shift (O—C) diagrams computed for an eccentricity 0.75 and four values of ω, the argument of perihelion.

Such binaries also appear to exist in other clusters. In his study of M 53, MARGONI (1967) has drawn attention to the periodic nature of some of the O—C diagrams. In M 5, which has been studied by COUTTS and SAWYER HOGG (1969), there are also RR Lyrae stars which could belong to binary systems.

Now we must consider whether such long period binaries can exist in clusters or if their orbits would be disrupted by encounters with other stars. For a binary with a period of 80 years, the mean separation of the components is about 20 astronomical units. According to ALLEN (1963), M 3 has a mass of 21×10^4 $\mathfrak{M}_\odot$ and a linear diameter of 13 pc. This means that there are about 10^6 stars in M 3 and that they occupy a volume of about 10^{19} cubic astronomical units. Thus the mean separation of the stars in M 3 is about 10^4 A. U. which is much larger than the mean separation of binary components with an orbital period of 80 years. It therefore seems probable that binaries can exist in globular clusters. According to SAWYER's catalogue (1955), there are eclipsing binaries observed in globular clusters. In particular, variable 78, which appears to be a member of ω Centauri has been studied by GEYER (1967) and SISTERO (1968). Spectroscopic binaries are difficult to detect in globular clusters, owing to the faintness of the stars.

The method of detecting binaries from O—C diagrams favours the discovery of binaries with long periods (at least 8 years) because the longer the period, the greater the range in

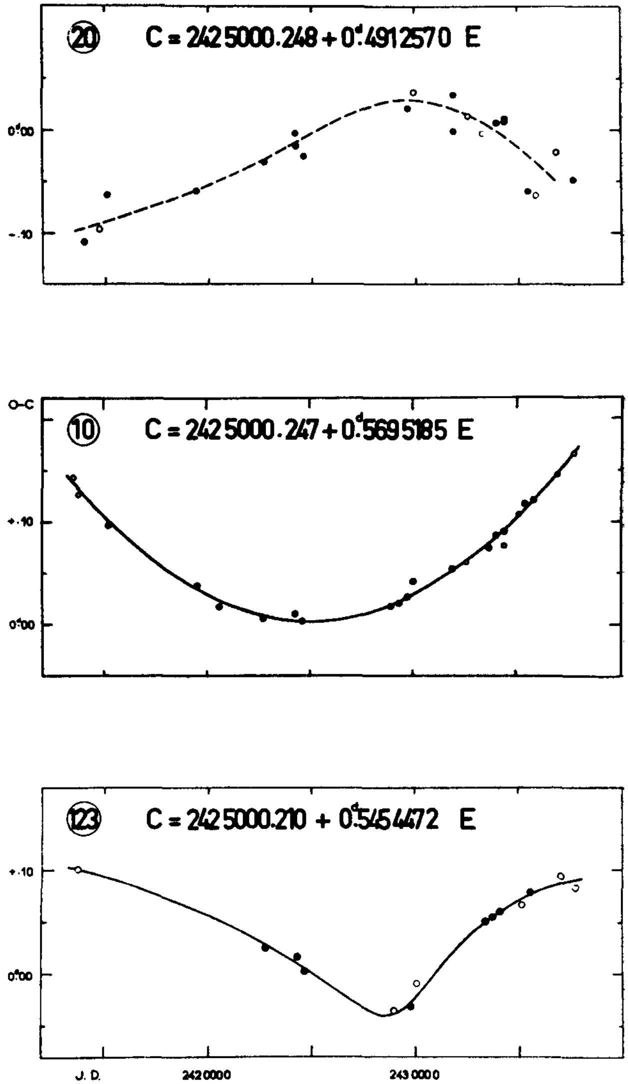

Fig. 2: O—C Diagrams for Variables in M 3 from SZEIDL (1965).

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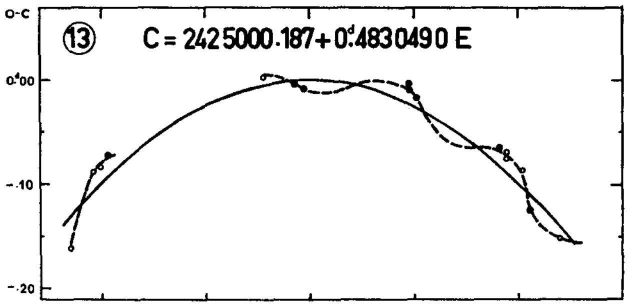

Fig. 3: O—C Diagram for Variable 13 in M 3 from SZEIDL (1965).

$\Delta\Phi$. It also favours the discovery of orbits with values of e and $\cos\omega$ less than 0.75. If the period of the orbit is longer than the interval of observations, then detection again becomes difficult because we might not observe the total range of $\Delta\Phi$.

Not many binaries are known among RR Lyrae stars. However, EPPS (1971) has pointed out that from the work of JONES (1970), it appears that the frequency of visual companions to RR Lyrae stars does not differ from other stars when distances and magnitude differences are taken into account. WISNIEWSKI (1971) has discussed an RR Lyrae variable in an eclipsing system. Now we have a new class of binaries among RR Lyrae variables: those which are detected from effects in the phase-shift diagrams determined from light curves.

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Discussion to the paper of COUTTS

F. B. WOOD: How strong is the evidence that the eclipsing variables are really in the cluster and not foreground objects?

COUTTS: Variable 78 in ω Centauri has been studied more than the others and seems to be a member of the cluster but this has not been established definitely.

DETRE: Your nice idea for the interpretation of O—C diagramms must be proved by observation of radial velocities, because it is very dangerous to draw any conclusion from O—C diagrams alone. (e. g. IRWIN's method for eclipsing binaries lead to wrong results in most cases).

COUTTS: Yes, but unfortunately, it will be difficult to make such observations in globular clusters. This makes it difficult to prove or disprove my hypothesis.

HERCZEG: Let me add to Dr. DETRE's remark that we at Hamburg recently investigated 15 well studied cases of possible light-time effect among eclipsing binaries. None of them turned out definitely as a member of a triple system, although a few of them are still to be kept on the list of possible candidates. In many cases, indeed, we have a nice „periodicity“ of the O—C values for several years or decades, which later turns out to be spurious.

Concerning membership of eclipsing binaries in globular clusters I should like to emphasize that the three possible members have as a counterpart some 1500 RR Lyrae stars discovered in clusters. There are virtually tens of thousands of stars investigated and the small number of eclipsing binaries found indicates clearly how extremely infrequent these stars are in the magnitude range considered. I think the membership of V 78 in ω Centauri is still a question not yet settled.

WALTER: Are there cases where the variations of periods repeated several times as one should expect that they arise by the binary character?

COUTTS: In Variable 13 in M 3, the cycle is repeated.

PERCY: It seemed that the amplitude of the phase variation was consistent with the approximate periods. Is this so?

COUTTS: In many cases.

BAKOS: Have you looked at the variations of O—C among non-cluster RR Lyrae variables?

COUTTS: No, this investigation has been limited to one cluster only.

Simultaneous Photometric and Image-Tube Spectroscopic Observations of Short Period Binary Systems and Rapid Intrinsic Variables *

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An extremely important new frontier in variable star research lies in the application of image tube techniques to the study of the spectroscopic phenomena in extremely rapid intrinsic variables and in short-period binary systems. Within limits, the use of image tubes for the spectroscopic observation of constant or slowly varying stars can be regarded as a convenience — as a means of increasing the efficiency of use of time at the telescope; the same results could, in principle, be obtained by observing for a longer time by conventional means. However, when one wishes to observe rapid changes, there is no substitute for the higher information collection rate provided by the image tube. Electronographic image tubes are especially well suited to this type of investigation owing to their high quantum efficiency, high resolution, large storage capacity, and linearity of response. Typically, image tubes of this type may have quantum efficiencies of 10% to 20%, an information-rate gain of 10 to 20 over the fastest photographic plate (baked Kodak IIaO at 4000 A), and resolutions between 80 and 100 lp/mm. With proper choice of photographic emulsion to record the electronic images, a linear relationship exists between photographic density and intensity of the incident light up to densities of about six, making electronographic image tubes especially valuable for spectrophotometric observations. Despite the great potential of this technique,

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