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CHANGES IN THE BEHAVIOUR OF THE UNTERAARGLETSCHER IN THE LAST 125 YEARS*

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Abstract. All the measurements involved concern the glacier tongue between its end and 2 600 m a.s.l. The total loss of volume of the Unteraargletscher since its last maximum advance (1871) is estimated to be 2.4 km³, which corresponds to a mean surface lowering of 0.67 m/year (referred to a total glacierized area of *c.* 40 km² on average). The considerable slowing down of the glacier flow velocity over the 125 years is primarily attributable to the marked decrease in the sliding component, whereas the shear component has only changed slightly. This behaviour is connected with the fact that the decrease in ice thickness has been accompanied by an increase in surface slope, so that the two effects on the shear component partially compensate each other. The seasonal variations in surface velocity were measured simultaneously at two profiles by Agassiz and his team in 1845/46. These variations are due to the variable amount of melt water and the resulting variations in hydrostatic pressure in the contact zone between ice and bedrock, in which the plastic contraction of the water channels plays a decisive role. This leads to the problem of water circulation in the interior of a glacier and its importance in the sliding process. Finally a simple method for the approximate calculation of the longitudinal profile of the surface of a glacier tongue in a steady state and with constant ablation is indicated.

Résumé. *Changements dans le comportement du glacier d'Unteraar durant les dernières 125 années.* Les mesures se rapportent à la langue du glacier entre son extrêmité et l'altitude de 2 600 m. La perte de masse totale depuis la dernière crue (1871) est estimée à 2,4 km³, correspondant à une diminution moyenne d'épaisseur de 0,67 m/année pour une surface de 40 km². Le ralentissement significatif du mouvement en 125 ans est caractérisé par la forte décroissance de la vitesse de glissement, la composante de cisaillement n'ayant que peu changé. Ce comportement est lié a une diminution d'épaisseur accompagnée d'une augmentation de la pente superficielle, les influences sur la composante de cisaillement se compensant partiellement. Les fluctuations annuelles de la vitesse superficielle ont été mesurées en 1845/46 simultanément dans deux profils par Agassiz et son équipe. Ces fluctuations sont attribuées à la quantité variable d'eau de fonte et à la variation de la pression hydrostatique dans la zone de contact entre le glace et le fond, le rétrécissement plastique des voies d'eau jouant un rôle décisif. Cela mène au problème de la circulation de l'eau au sein du glacier et à sa signification pour le glissement. Finalement une méthode simple est indiquée, permettant de calculer approximativement le profil en long de la surface d'une langue glaciaire stationnaire avec ablation constante.

Zusammenfassung. *Änderungen im Verhalten des Unteraargletschers in den letzten 125 Jahren.* Alle vorliegenden Messungen beziehen sich auf die Gletscherzunge zwischen deren Ende und 2 600 m ü.M. Der gesamte Massenverlust des Unteraargletschers seit seinem letzten Hochstand (1871) wird auf 2,4 km³ geschätzt, was einem mittleren Höhenverlust von 0,67 m/Jahr entspricht (bezogen auf totale vergletscherte Fläche von ca. 40 km² im Mittel). Die bedeutende Verlangsamung der Gletscherbewegung in 125 Jahren ist in erster Linie durch die starke Abnahme der Gleitgeschwindigkeit gekennzeichnet, während die Scherkomponente nur wenig änderte. Dieses Verhalten hängt damit zusammen, dass der Abnahme der Eismächtigkeit eine Zunahme des Oberflächengefälles gegenübersteht, wobei sich die beiden Einflüsse auf die Scherkomponente teilweise kompensieren. Die jahreszeitlichen Schwankungen der Oberflächengeschwindigkeiten wurden 1845/46 von Agassiz und seinem Team in 2 Profilen synchron gemessen. Diese Schwankungen werden auf die variable Schmelzwassermenge und die dadurch bedingten Änderungen des hydrostatischen Druckes in der Kontaktzone zwischen Eis und Untergrund zurückgeführt, wobei die plastische Verengung der Wasserwege eine massgebende Rolle spielt. Dies führt zu den Problemen der Wasserzirkulation im Innern des Gletschers und deren Bedeutung für den Gleitvorgang. Anschliessend wird auf eine einfache Methode zur angenäherten Berechnung des Längenprofils durch die Gletscheroberfläche einer stationären Gletscherzunge mit konstanter Ablation hingewiesen.

1. Introduction

The following studies have supplied information on the changes in the Unteraargletscher since 1842 as regards ice thickness, ice thinning and characteristics of movement:

1. The works of Hugi, which first drew the attention of glaciologists to the Unteraargletscher about 1827 (Hugi, 1830; Rütimeyer, 1881).
2. The pioneering investigations, measurements and cartographic surveys of L. Agassiz and his collaborators in 1841–46 (Wild's 1 : 10 000 survey in 1842 and that of Stengel in 1846) (Agassiz, 1847; Portmann, 1962).

* Paper presented at the general assembly of the International Union of Geodesy and Geophysics, Berne, Switzerland, 1967.

3. The systematic glacier measurements that have been carried out year by year since 1924 for the Kraftwerke Oberhasli, mainly by the Flotrons (father and son, both engineers), at the initial instigation of Dr A. Kaech (Flotron, unpublished; Jost, 1953) published regularly in *Die Alpen* from 1930 by the Gletscherkommission der Schweiz. Naturforschenden Gesellschaft.
4. The seismic exploration of the Unteraargletscher by the Gletscherkommission der Schweiz. Naturforschenden Gesellschaft (1936–50) and by A. Süsstrunk (for the Kraftwerke Oberhasli) (Renaud and Mercanton, [1950]; Kreis and others, 1952).
5. The various cartographic surveys of the Eidgenössische Vermessungsdirektion (1851, 1879-80, 1915 and 1932) and the photogrammetric air survey of the whole Unteraargletscher (including the firn region) to a scale of 1 : 10 000 for Eidgenössische Vermessungsdirektion in 1961, by A. Schlund.
6. Glaciological studies on the variations in the Unteraargletscher between 1840 and 1965, by R. Haefeli (report to Kraftwerke Oberhasli in 1967, 1969).

Fig. 1. Oberaargletscher (left) and Unteraargletscher (right). Aerial view 1934 by Mittelholzer. ○ Hôtel des Neuchâtelois on the top of the moraine between Lauteraargletscher (right) and Finsteraargletscher (left). Finsteraarhorn in the middle of the sky line.

Most of the changes observed in the Unteraargletscher in the course of the last 125 years may no doubt be regarded as representative of the behaviour of flat Alpine glacier tongues during the recession period that has now lasted for about 100 years (with a short interruption around 1920). An attempt will therefore be made here to investigate the possible causes of a few very typical features of the behaviour of the Unteraargletscher (ablation area). These investigations are closely bound up with the problem of thinning and of glacier movement as such.

A general view of the site is given in Figure 1, an aerial photograph of the Unteraargletscher with the Finsteraarhorn in the background. The circle marks the former position of the Hôtel des Neuchâtelois. In Figure 2 the confluence of the Unteraargletscher (right) and the Finsteraargletscher (left) at the so-called "Abschwung" is visible in the middle of the foreground.

Fig. 2. Junction (Abschwung) between Unteraargletscher and Finsteraargletscher. Jungfrau at highest point of the sky line.

2. Ice thinning and glacier retreat

Between 1842 and 1871 (29 years) the glacier advanced 150 m, or at an average of 5.2 m/year. In the next ninety years (1871–1961) the glacier tongue withdrew a total of 1 370 m, at a mean rate of retreat of approximately 15.0 m/year (Fig. 3). After averaging only 10 m per year from 1871 to 1931, the rate of retreat was greatly accelerated for a time by the damming of the glacier tongue area, which took place in 1931. From 1933 to 1937 the ice front withdrew by a mean figure of 353 m, or about 88 m per year (Fig. 3). In the decade 1955–65, however, the average retreat was again about 10 m per year. The minimum value of only 1 m was recorded in 1955/56, or 3.7 m/year during the 4 year period 1955–1959. In the last 3 years (1965–68) the average value of retreat increased to 17.5 m/year.

It we work out the loss of volume due to thinning below the Grunerhorn and Wildläger profiles, at a height of about 2 600 metres, we find that a reduction of the ice volume of about 0.5 km³ occurred in the 34 year period 1929–63, corresponding to a mean thinning of 1.22 m/year. Figure 4 shows the mean annual thinning versus altitude above sea-level, for various periods; the figures are measured up to 2 600 m by Flotron (unpublished). For the 35 year period from 1928–63 the following relationship was established:

$$\Delta H = \Delta H_F + (H_F - H)/800 \qquad (1)$$

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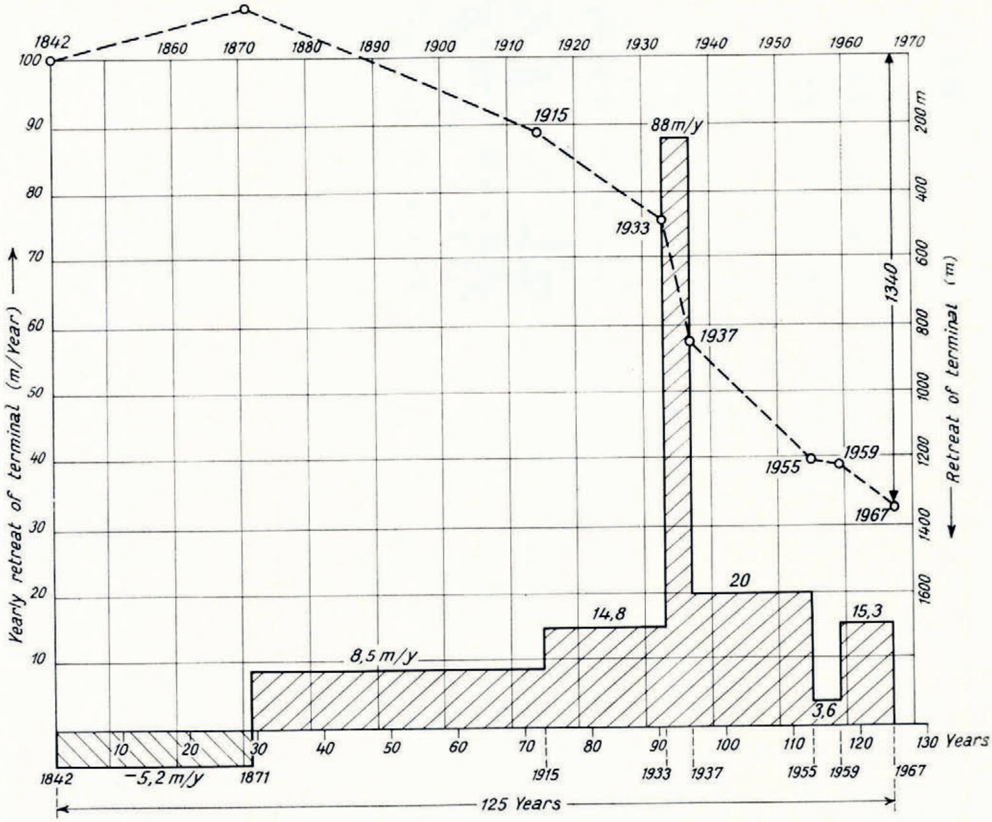

Fig. 3. Advance (1842–71) and retreat of the Unteraargletscher (terminal).

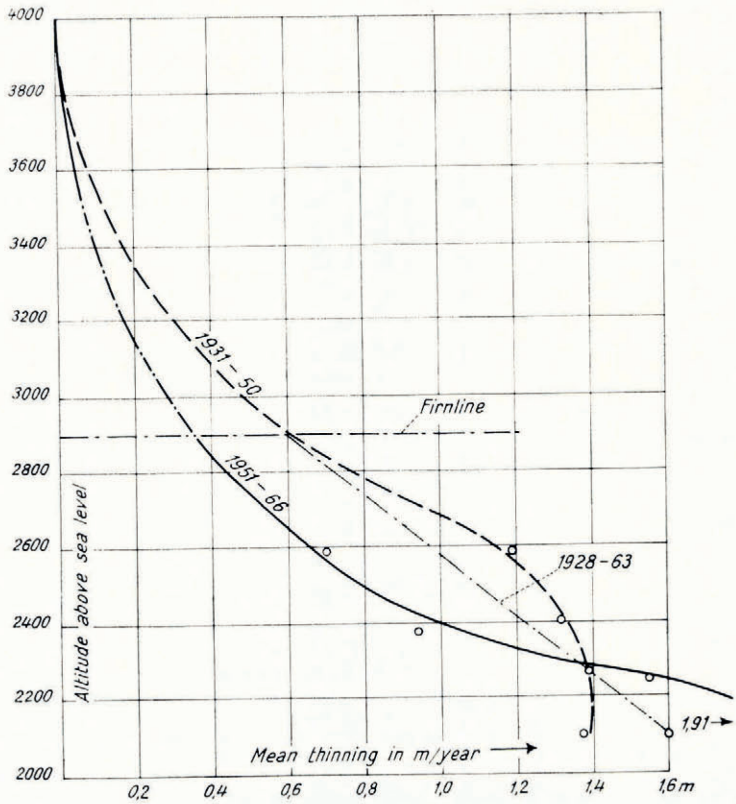

Fig. 4. Thinning of the Unteraargletscher in metres per year. Yearly measurements between 2 000–2 600 m. Estimation between 2 600–4 000 m.

where ΔH is the thinning per year at altitude H, ΔH_F is the thinning per year at the altitude of the firn line (= 0.6 m) and H_F is the height of the firn line (= 2 900 ± 100 m a.s.l.).

Figure 5 shows that very substantial scatter may occur in the 10 year means of the annual thinning.

In Figure 6 the distribution of the mean loss of ice volume over altitude increments of 200 m is shown for various periods. It is worthy of note that the annual thinning for the decade 1953–63 was only about one third that for the period 1941–53.

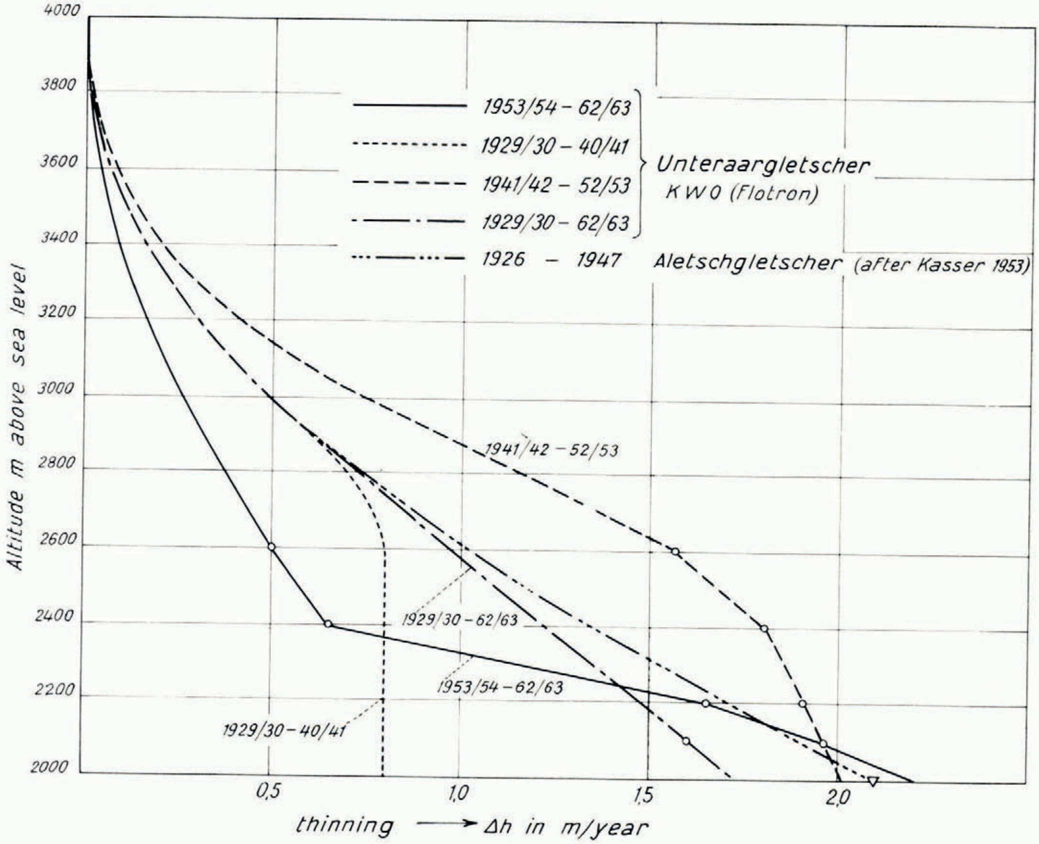

Fig. 5. Thinning of the Unteraargletscher during different decades in metres per year.

An estimate of the loss of ice volume of the whole Unteraargletscher for the 34 year period from 1930 to 1963 yielded a figure of at least 1 km³. This figure is not far from the average loss of ice per year of all glaciers in the Swiss Alps around the middle of this century (englaciated area ≈ 1 500 km²). A rough estimate of the total loss of mass of the Unteraargletscher from its last maximum in 1871 until 1961 came to about 2.4 km³. If we divide this figure by the mean englaciated area (about 40 km²) and the number of years in the period considered (90), we obtain a long-term mean for the annual thinning of:

$$\Delta H = 0.67 \text{ m/year} \qquad (1871\text{–}1961)$$

The change of the longitudinal profile of the Unteraargletscher is visible in Figure 7 (top). Table I shows the thinning which has been observed.

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3. Change in characteristics of movement

In the year 1967, it was exactly 125 years since L. Agassiz and A. Escher von der Linth carried out the first exact velocity measurements on the tongue of the Unteraargletscher (Agassiz, 1847). Since in 1845–46 the monthly velocity variations were measured as well as the annual velocities in several transverse profiles it has been possible to include both measurements in the following comparison.

Table I. Thinning of Unteraargletscher

Profile	*Altitude of the glacier surface (medial moraine) above sea-level in 1961*	*Thinning since 1842*
	m	m
Mieselenegg	2 386	88
Pavillon Dollfuss	2 252	119
Obere Brandlamm	2 080	113
Untere Brandlamm	1 920	207

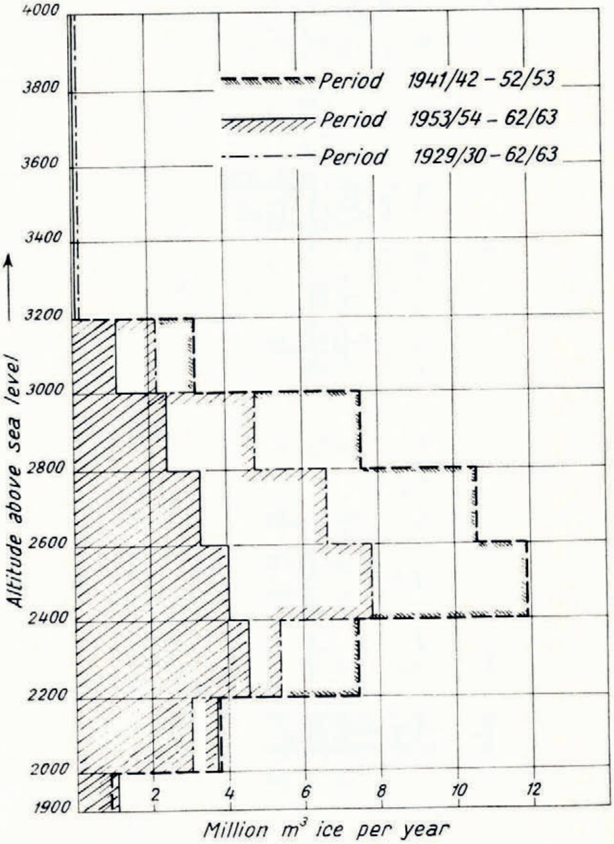

Fig. 6. Loss of ice of the Unteraargletscher in million cubic metres per year during different periods.

(1) *Mean annual velocities in the thalweg*

An attempt has been made in Figure 8 to fit the profile survey of Agassiz for the centrally situated transverse profile at Pavillon Dollfuss into those of the present day. While the glacier surface sank by about 120 m from 1841 to 1961, i.e. to 72% of the total ice thickness of 1842, or 1.0 m per year, the maximum surface velocity was reduced to 37% of its original value.

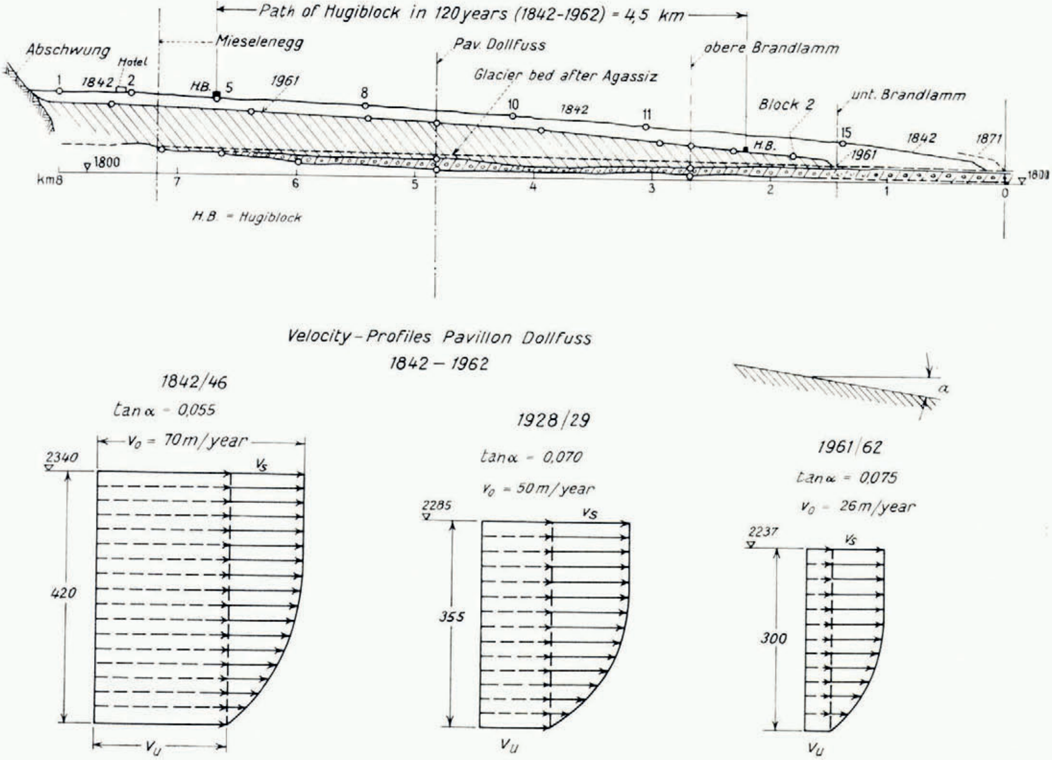

Fig. 7. Top: Longitudinal profile through the Unteraargletscher 1842 and 1961. Compare the ice thickness predicted by Agassiz with the result of seismic sounding. Bottom: Change of velocity in the thalweg at Pavillon Dollfuss 1842–1961. The big decrease of the measured surface velocity is mainly due to the decrease of the slip component.

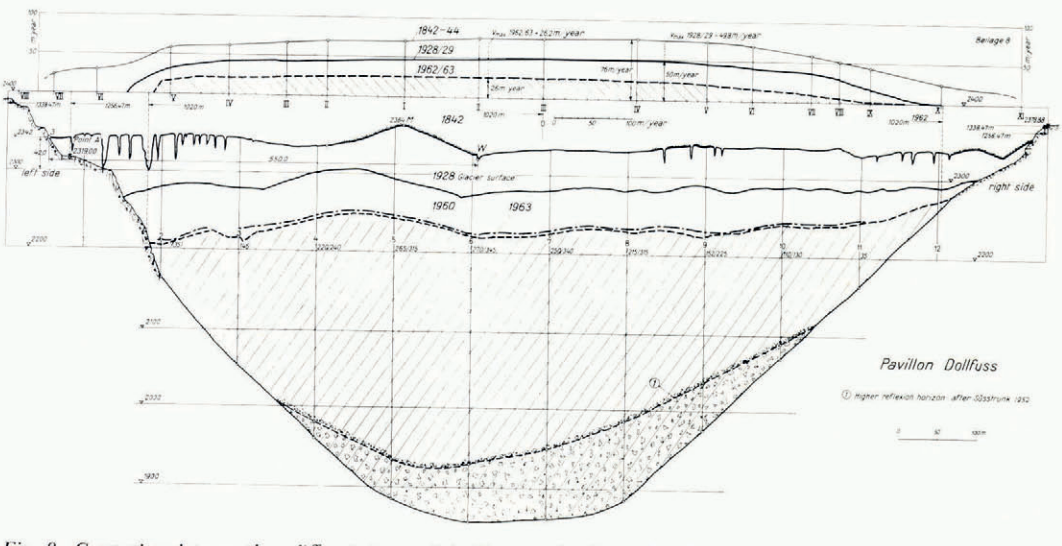

Fig. 8. Comparison between three different stages of the Unteraargletscher at Pavillon Dollfuss with velocity profiles on the top.

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If the maximum surface velocity is considered as consisting of two components, a shear and a slip component, we would like to know what percentage of the total movement is due to each of these components in different periods.

At first sight it is astonishing that the calculation of the percentage due to shear on the basis of the flow law for ice (for $n = 3$), the measured ice thickness, and the gradient of the surface, reveals no great velocity changes between 1842 and 1961. This is explained by the fact that the influence of the decreasing ice thickness is in part compensated by the gradual steepening of the surface gradient. We also have to take into account that in our calculation the fourth power of the ice thickness appears as against the cube of the gradient (see Equation (1)). The considerable reduction of the surface velocity observed during the period 1842–1961 at the Pavillon Dollfuss transverse profile can therefore only be accounted for by the pronounced diminution of the slip component v_u (Fig. 7).

The method used to separate the shear component from the slip component, was a semi-empirical one based on the equation (Haefeli, 1961):

$$v_s = k\rho^n \tan^n\alpha h^{n+1}(1-h/b)/(n+1) \tag{1}$$

where v_s is the shear component of movement, k is the temperature coefficient of the flow law of ice (Glen, 1952), n $(=3)$ is the exponent of the flow law of ice, ρ is the density of the ice, α is the slope of the glacier surface, h is the thickness of the ice and b is the width of the glacier.

Table II. Attempt to separate the slip component v_u for the profile at Pavillon Dollfuss for three different periods

		Period 1	*Period 2*	*Period 3*
		1842–46	1928–29	1961–62
h	m	405	360	300
$\tan\alpha$		0.0548	0.0625	0,070
b	m	1 200	1 100	1 000
$1-h/b$		0.66	0.67	0.70
v_0 measured	m/year	70	50	25
v_s calculated and estimated	m/year	26	25	18
$v_u = v_0 - v_s$	m/year	44	25	7

v_0 = total velocity on the glacier surface (thalweg),
v_s = shear component, v_u = slip component.

The shear component (18 m/year) of the third period has been controlled by measurement. Then Equation (1) was applied in order to calculate the parameter k for this period. The calculation of the other two shear components is based on the assumption that k is constant.

Although the accuracy of the calculation is less than might be desirable, it is quite clear from the comparison of the three velocity profiles in Figure 7 and Table II, that the conditions have in part been reversed since 1842. Whereas the slip component was then predominant, it is the shear component that is most in evidence today. The velocity reduction has thus been due chiefly to a diminishing slip component.

This reversal of the mechanism of motion of the glacier tongue comes out particularly clearly in Figure 9, where the measured velocities along the longitudinal axis are shown under the longitudinal profile of the glacier.

Whereas in 1842–46 the velocity measured just above the snout pointed to a slip component of about 25 m/year, the velocity diagram for the period 1959–63 drops to zero at the terminus (Fig. 9). So Block 2, which lies about 100 m above the snout, underwent a horizontal movement of only 0.5 m/year with a vertical displacement of −0.8 m (1965–66) according to the measurements of Flotron (unpublished). The "Hugiblock", which is at a distance of about 400 m from the terminus, was displaced horizontally by about 2 m in the same period (1965–66).

As Figure 9 shows, the surface velocities (from 1.4 km to 4.8 km) are today directly proportional to the distance from the terminus. It follows that the terminus, which coincides with the zero point of the velocity diagram, is at present hardly involved in any sliding movement. In this special case the retreat of the glacier tongue is only due to ablation.

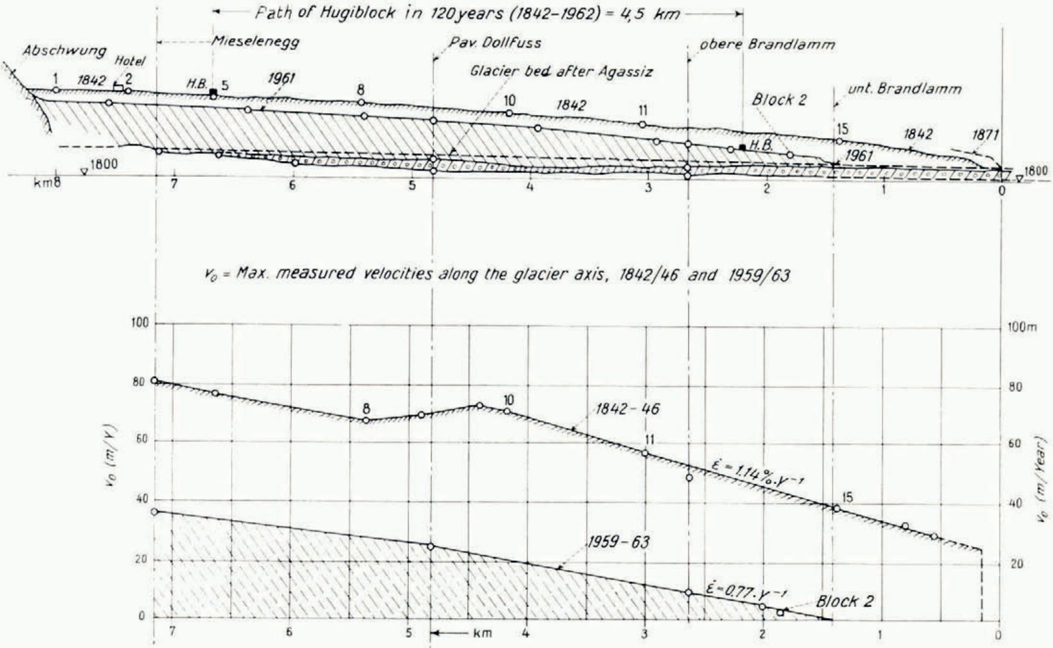

Fig. 9. Top: Longitudinal profile of the Unteraargletscher 1842 and 1961. Bottom: Measured velocities along the longitudinal profile. Comparison between velocity distribution in 1842 and 1961. HB = *Hugiblock.*

TABLE III. COMPARISON OF SURFACE VELOCITIES, 1842–46 AND 1956–63 (THALWEG)

Distance from terminus km	*1842–46* v_1 m/year	Δv_1 m/year	*1956–63* v_2 m/year	Δv_2 m/year	*Proportion* v_2/v_1 %
0	25		0		0
		11		8	
1.0	36		8		22
		11		7	
2.0	47		15		32
		12		7	
3.0	59		22		39
		12		6	
4.0	70		28		40

Table III shows velocities and velocity changes in the longitudinal profile taken from Figure 9 at equal distances from the terminus.

It also appears from Figure 9 that the strain-rate in the pressure zone of the glacier tongue (in the direction of flow) has decreased since 1842 from about 1.14% to about 0.77% per year. With transverse expansion horizontally inhibited, this means that the longitudinal stress has diminished from about 1.4 bar to about 0.8 bar, a fact which may have contributed to the observed diminution of the sliding process.

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For the profile at Pavillon Dollfuss, Figure 10 shows (1) the thinning process during the period 1924–60, (2) the change of yearly velocity of the glacier surface in the same period. The much higher scatter around the second curve (2) may be due to changes in the inter-glacial hydraulic system, interconnected with variable sliding conditions.

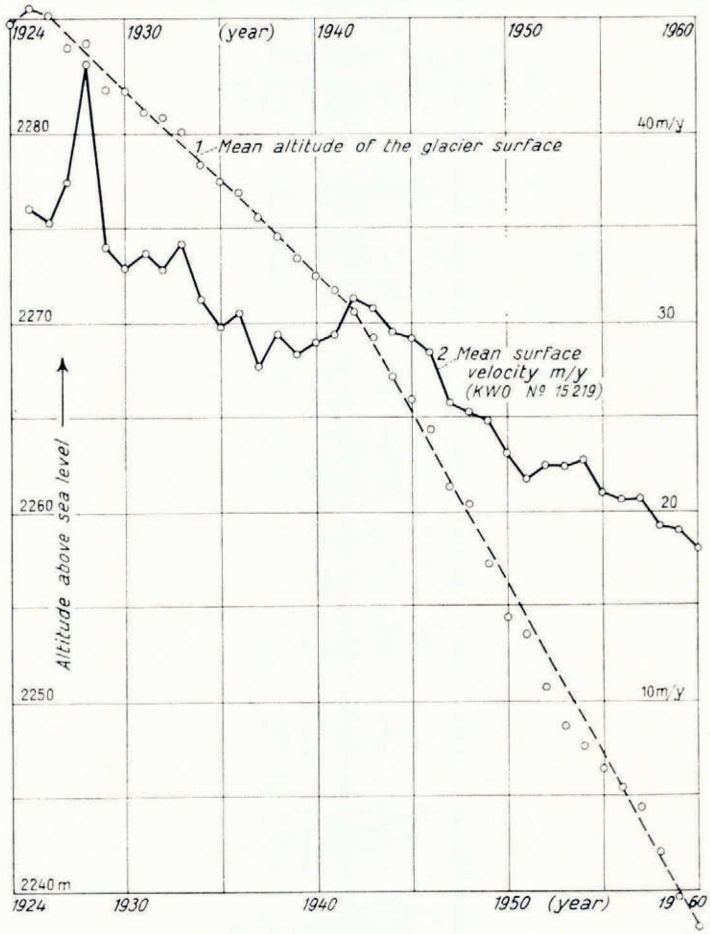

Fig. 10. Comparison between change of ice thickness (curve 1 = mean altitude of glacier surface) and mean surface velocity (curve 2) measured in the cross-section at Pavillon Dollfuss (1924–60).

(b) *Seasonal velocity fluctuations*

Despite the well known fact that access to the Unteraargletscher in winter is seriously endangered by avalanches, Agassiz and his co-workers succeeded in measuring the velocities of the glacier in two profiles every month for a whole year (Agassiz, 1847). The distance between the two profiles, of which the upper one (Hôtel des Neuchâtelois) is situated near the confluence already mentioned (Abschwung) and the lower one (Trift) about 300 m below the Pavillon Dollfuss, is roughly 2 900 m. The results of the monthly measurements as published by Agassiz are plotted in Figure 11, which shows the very marked and irregular fluctuations. The two velocity diagrams 1 and 3 have a similar shape and should be compared with the velocity diagram for the year 1965/66 measured by A. Flotron (curve 4). In all three diagrams the maximum velocities occur in the hot season and the minima in the cold season, however, the sudden increase of surface velocity in January 1846 cannot be explained.

In comparing the seasonal velocity fluctuations of the Hôtel des Neuchâtelois and Trift profiles (1845–46), one can differentiate between velocity changes that occur simultaneously in both profiles and those to which this does not apply. Velocity peaks that present themselves simultaneously in two or more profiles, such as that of September 1845, are to be ascribed to general climatic influences (e.g. high temperature, with a sudden increase of melt water or hydraulic pressure), while velocity fluctuations that appear in only one profile must be due to some local factor, the nature of which we shall revert to in the discussion.

The ratio of the maximum measured velocity to the mean annual velocity can be gathered from Table IV.

TABLE IV. MEAN AND EXTREME VALUES OF THE VELOCITIES MEASURED IN THREE PROFILES BY AGASSIZ

	Hôtel des Neuchâtelois		*Trift*		*Pavillon Dollfuss*	
	Period	*Velocity* m/year	*Period*	*Velocity* m/year	*Period*	*Velocity* m/year
v_0 max.	16.4–30.5.1846	137	25.5–6.6.1846	138	9–11.7.1966	40.1
v_0 mean	1845/46	83.0	1845/46	74.0	1965/66	25.6
$\frac{v_0 \text{ max.}}{v_0 \text{ mean}}$	1845/46	1.65	1845/46	1.87	1965/66	1.57
v_0 min.	19.12.1845–11.1.1846	48.5	24.10–20.11.1845	57.6	9.9.1965–19.1.1966	18.6
$\frac{v_0 \text{ max.}}{v_0 \text{ min.}}$	1845–46	2.81	1845/46	2.40	1965/66	2.16

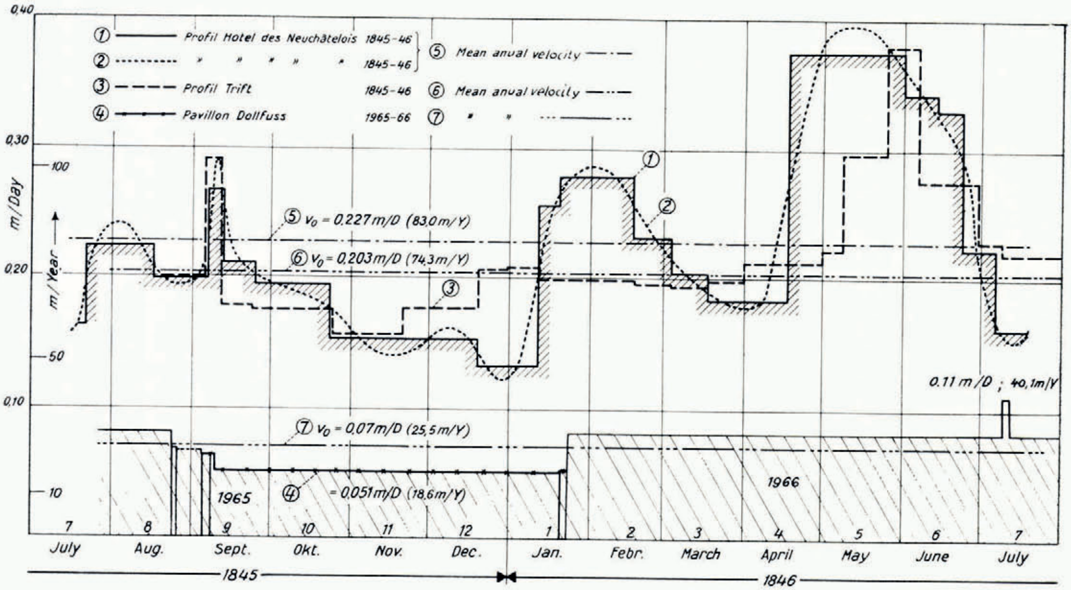

Fig. 11. Seasonal velocity fluctuations measured by Agassiz and his team in two different profiles of the Unteraargletscher in m/year and m/day. Below: Seasonal velocity measured by A. Flotron at Pavillon Dollfuss, 1965/66.

4. CHECKING THE CONTINUITY OF GLACIER MOVEMENT BY MEANS OF OGIVES

So-called ogives usually appear below steep ice falls; a bulge running transversally across the glacier is formed each year and is later distorted into an ogival shape. In the present case this process can be very clearly observed, especially on aerial photographs, below the ice fall of the Finsteraargletscher (see Fig. 12). Since there are heavy accumulations of snow in the depression between two ogives in winter, the wave crests lose their snow covering earlier and in summer striation of the glacier surface occurs. As can be seen below the ice fall on the aerial photograph, the wave crests appear dark against lighter depressions.

There are various theories about the mechanism of ogive formation, but we cannot review them here. It may only be mentioned that ogive formation is not limited to ice but is characteristic also for other viscous materials like milk (Streiff-Becker, 1951) or lavas (Thorarinsson, 1953). In our case the important point is that ogive formation produces yearly features on the surface of the glacier whose mean distance from each other, at least in stationary glaciers, corresponds to the yearly velocity of the point in question (Haefeli, 1966). If the glacier is not

Fig. 12. Air view on Finsteraargletscher and Unteraargletscher (11 September 1961). Copyright Eidg. Vermessungsdirektion, alle Rechte vorbehalten. G = *Grunerhorn profile,* M = *Mieselenegg profile,* D = *Cross-section Pavillon Dollfuss.*

stationary, the wavelength (distance between ogives) at any point below the ogive formation (ice fall) is only identical with the mean yearly velocity if the latter varies in the same proportion in all observed points of the glacier surface. This condition seems to be approximately satisfied in the present case, so that we can compare the measured velocities (annual means) directly with the mean wavelengths of the ogives.

To this end we consider the velocity conditions of the Finsteraargletscher between the Grunerhorn (G) and Mieselenegg (M) profiles marked in Figure 12, the distances between which in longitudinal profile is *c.* 2 900 m. In Figure 12 (an aerial photograph taken on 11 September 1961) a zebra-like striation of the glacier surface can be clearly distinguished and is remarkable for its great regularity. If we designate the distance between profiles G and

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M as L, while n is the number of ogives, the mean distance l_m between two ogives can be calculated as

$$l_m = L/n.$$

The number n of ogives was found in six counts to be an average of 65, so that we can write:

$$l_m = 2\,900/65 = 44.5 \text{ m/year}.$$

On the other hand in 1961–62 the following velocity values (annual means, thalweg) were measured in the two profiles:

Grunerhorn, $v_0 = 52.7$ m/year,
Mieselenegg, $v_0 = 36.4$ m/year,
mean velocity $= 44.5$ m/year.

The agreement of this mean velocity with that calculated above shows that in the present case the mean wavelength of the ogives is in fact representative for the mean annual velocity within the stretch of glacier under consideration, which should comprise at least five ogives to allow for annual scatter.

This relationship enables us to estimate the approximate annual velocity of the glacier surface with the aid of the wavelength of the ogives, i.e. on the basis of an aerial photograph, even for points lying outside the range of direct velocity measurements, e.g. at the foot of the ice fall. From the first five clearly visible waves below the ice fall we obtain from Figure 12 a wavelength of 80 m.

In addition to these individual velocity figures, conclusions as to the continuity of glacier movement in the past can also be drawn from the ogive formation. Thus a regular ogive pattern indicates that there have been no sudden local changes and distortions in the glacier movement within a certain period of time. The perfectly regular ogive pattern between the ice fall and the Mieselenegg profile further shows that there has been a steady change in the velocities since the end of the last century. If the ogives could be counted as far as the Pavillon Dollfuss, we should be able to review a further period right back to the days of Agassiz. Here again the ogive pattern shows no irregularities which would point to a discontinuous change in the surface velocity of the glacier. In this way the continuity of glacier movement since 1800 can be demonstrated. Closer investigation would be needed to decide whether one is justified in concluding, on the basis of this continuity, that within a certain period no major kinematic waves have occurred.

5. Water circulation and sliding movement

It is well known that the intensity of the sliding process depends to a high degree on the hydrostatic pressure of the water present at the sliding surface (Weertman, 1957; Haefeli, 1957; Lliboutry, 1968). This pressure is in its turn determined by the hydrodynamic, thermodynamic and rheological conditions prevailing in the interior of the glacier. Changes in interglacial waterways are due on the one hand to the pressure, temperature (melting power) and quantity of the flowing water and on the other to their own deformation. Therefore these interglacial waterways are subject to constant changes of a continuous and discontinuous nature (plastic deformation and collapse).

To obtain some idea of how a water passage of this kind—for instance a circular shaft—can close up, it is an advantage to introduce the "half-life" T required for the diameter of an empty shaft to be reduced to one half. It will be seen from Figure 13 that the "half-life of a shaft" in temperate ice at a depth of about 100 m is about 75 d, and at 200 m only about 10 d, provided that the unknown critical depth ($>$400 m) at which closure takes place by collapse is not reached.

The following formula for the "half-life" T was deduced from Glen's flow law of ice (Glen, 1952, 1955) and the formula of Nye (1953) when the vertical cylindrical shaft (for example a pot-hole) is empty:

$$T = \frac{2 \ln 2}{k_1 (p/n\tau_1)^n} \tag{2}$$

where p is the overburden pressure, τ_1 is the 1 bar and k_1 is the temperature parameter for $\tau_1 = 1$ bar.

For $k_1 = 0.25$/year and $n = 3$, we can write with good approximation:

$$T = 150 p^{-3} \text{ years} = 55(p/10)^{-3} \text{ days} \tag{3}$$

if p is measured in bars. The ratio $r_0 : r$ is given by the equation

$$\ln (r_0/r) = \tfrac{1}{2} k_1 (p/n\tau_1)^n t = 1.27(p/10)^3 (t/100).$$

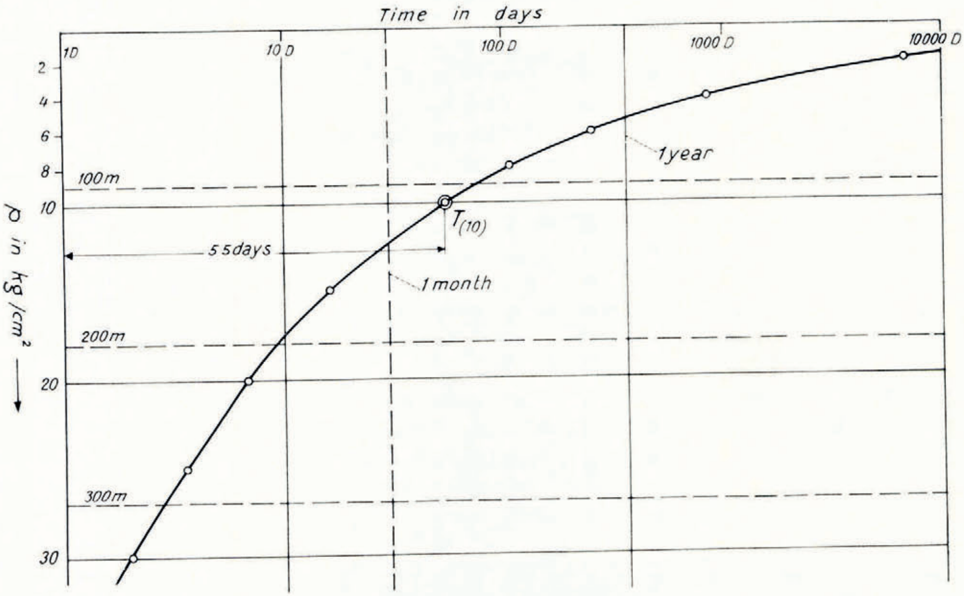


Fig. 13. Half-life curve of the closing process of a vertical cylindrical shaft, as a function of the overburden pressure or the depth, for temperate ice.

In this way it is easy to represent the deformation of a circular shaft for different moments as in Figure 14, which shows that vertical circular shafts become funnel-shaped when they close. If the shaft in a temperate glacier is empty during the cold season, a period of only 100 d is sufficient, at a depth of only 150 m, for such a space to close but for a small funnel-like opening. This fact is of great importance for the subglacial water circulation.

It may be concluded from this that hollows in the interior or on the bed of a large temperate glacier can only survive the cold season if they are partly or completely filled with water and if the hydrostatic pressure of the water is sufficient to prevent rapid plastic closing of the aperture. Otherwise they close except for a narrow passage or are completely blocked by a collapse (if the overburden pressure reaches the compressive strength of the ice). At great depths below the surface of the glacier the circulatory system must therefore (if it does not remain water-filled) be reconstituted each year. Old water passages are widened by pressure and melting processes and new ones are created. This explains why marginal glacier lakes such as the Gornersee or the former Märjelensee reform each year and then, in summer when the water level is high, suddenly break out into the interior or bed of the glacier.

In Figure 15 an attempt has been made to represent schematically some of the relationships involved with the aid of a simple model. Let it be supposed that the circulatory system has the form of a tube with a right-angle bend OCB, the vertical limb OC being a pot-hole and the horizontal limb CB a water passage along the bed of the glacier or an interglacial gallery (Streiff-Becker, 1951).

When water flows through this system (Fig. 15a), a definite pressure line A_0–B is formed for each water quantity Q and is given by the piezometer levels h_0, h_1, h_2, etc. If the cross-section of the tube is constant and the surface roughness uniform (case a), the piezometer line A_0–B is straight. If the water flow gets small, the piezometer line sinks, as in A′–B in Figure 15a, which would normally correspond to autumn conditions. When the system is partly empty during the winter, intensive closing of the cross-sections sets in and causes the piezometer line

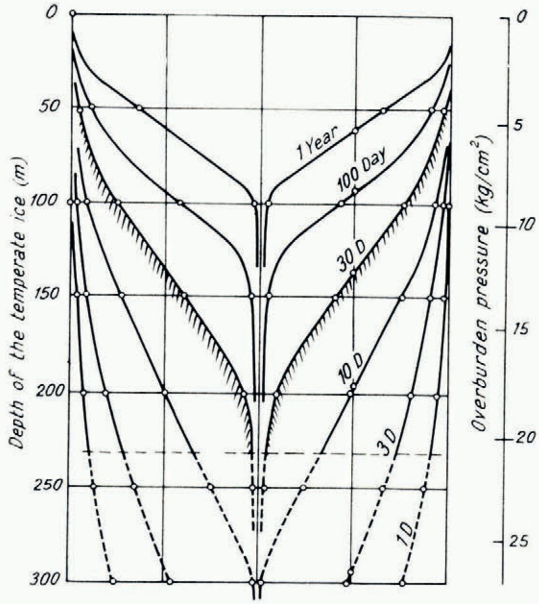
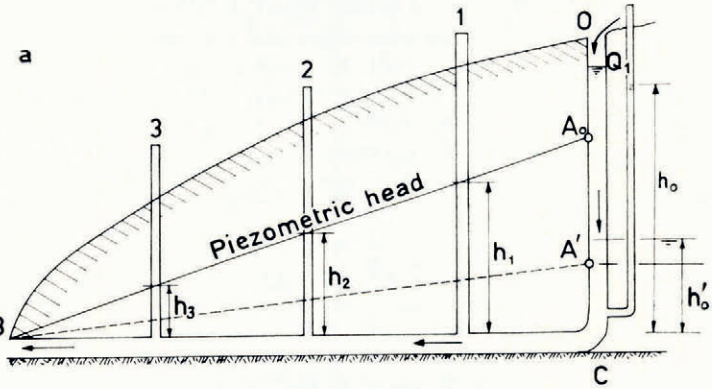


Fig. 14. Different stages of the closing process of a vertical cylindrical shaft in temperate ice.

to become curved, as in B–A_1 (Fig. 15b). The rate of closure depends chiefly on the difference between the overlying pressure of the ice and the water pressure. If this difference exceeds the critical figure a collapse occurs. In spring, when the narrowing of the water cross-sections has reached its maximum, a comparatively small water quantity Q_2 is sufficient to raise the piezometer level (for instance from A_1 to A_2) and thus to activate the sliding movement, provided that the circulatory system is in communication with the sliding surface (on the bed). Water quantity, narrowing of cross-section, melting processes and piezometer level are interconnected by very complex relationships and to some extent mutually affect each other. As the sliding speed largely depends on the hydrostatic pressure in the contact layer between ice and bed, the apparently arbitrary velocity fluctuations measured by Agassiz, as shown in Figure 11, are basically quite susceptible of explanation.

Many new factors would have to be taken into account in proceeding from a simple model to a study of the glacier itself. Several circulatory systems may exist without intercommunication. Therefore different ground-water stories may be established. If it were possible to take piezometer readings at numerous points on the glacier bed, as was done on the Gornergletscher, the totality of the readings would present an extremely complicated picture, viz. a surface with an irregular relief containing discontinuities and in a continuous state of change. Slip velocities on the glacier bed and their fluctuations depend to a large extent on this surface, which represents a sort of living "ground-water table" of the glacier bed.



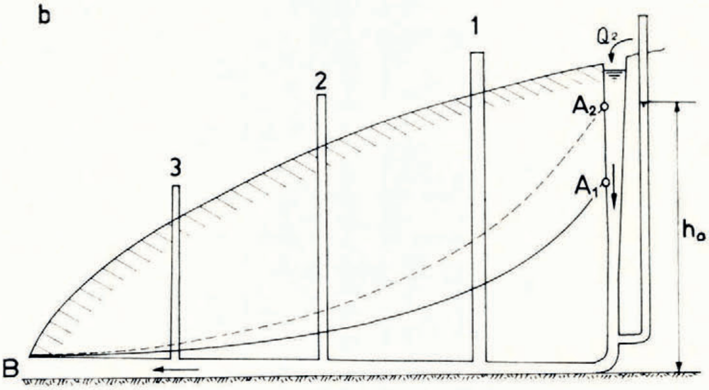


Fig. 15. Simplified hydraulic model (schematic) for water circulation in a glacier snout (temperate ice).

6. Comparison between measured and calculated long profile of the glacier snout

After the early work of Koechlin (1944) a first comparison between the theoretical and the measured long profile has been given by Nye (1952), based on the assumption that the shear stress over the bed and also the inclination of the bed are constant. The same author (Nye, 1967) improved Orowan's parabolic profile, which is based on a constant yield stress between bed and ice (Orowan, 1949).

The following calculation, which is based on constant ablation a, contains no assumption about the shear stress on the bed, whose inclination is neglected.

In Figure 16, a given cross-section ($x = 0$) is taken as starting point for the calculation. The amount of ice flowing through this cross-section per metre width and year under steady conditions is designated as q_0. The following expression is valid, on the basis of the mass balance, for the mean velocity along a vertical line at a distance x from the starting cross-section:

$$v_{x\mathrm{m}} = (q_0 - ax)/y \tag{4}$$

where y is the thickness of the ice at cross-section x and a is the constant annual ablation.

On the other hand, the following relationships based on Glen's flow law of ice hold good when the breadth of the ice flow is limited (Haefeli, 1961):

$$v_{x\mathrm{m}} = C_3(-\mathrm{d}y/\mathrm{d}x)^n y^{n+1}\lambda, \qquad C_3 = k\rho^n/(n+2), \tag{5}$$

where λ is an empirical reduction factor resulting from the limited breadth of the glacier tongue.

With the aid of Equations (4) and (5), the equation for the longitudinal profile is obtained by integration. It reads:

$$y = y_0(q_0 - ax)^{\frac{1}{2}}/q_0^{\frac{1}{2}} = h_0(1 - ax/q_0)^{\frac{1}{2}}, \tag{6}$$

$$\bar{y} = y/y_0 = (1 - ax/q_0)^{\frac{1}{2}} = (a\bar{x}/q_0)^{\frac{1}{2}}, \qquad \bar{x} = L - x.$$

We thus obtain for the longitudinal profile a parabola, similar to the solution of Orowan.

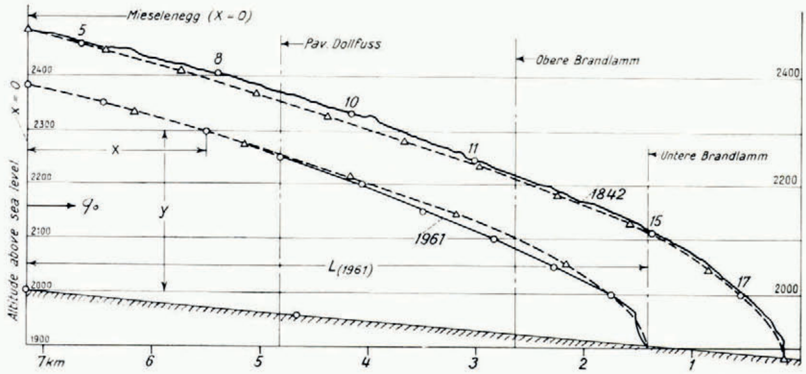


Fig. 16. Comparison between measured and calculated longitudinal profile through the Unteraargletscher for 1842 and 1961. △ *calculated,* ○ *measured.*

Starting from the Mieselenegg profile, the longitudinal profile through the tongue of the Unteraargletscher was calculated on the basis of Equation (6), first for the almost steady state of 1842 and secondly for the present (i.e. 1961). The results are set forth in Figure 16. The longitudinal profile plotted by Wild under the direction of Agassiz (Fig. 16, top) runs along the ridge of the medial moraine. If this profile line is reduced by the estimated height of the moraine, very good agreement of measurement and calculation results. A satisfactory agreement also exists between the measured and calculated long profile in 1961.

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