

# CORRIGENDUM

Are cascading flows stable? – CORRIGENDUM

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Regrettably, an error has been found in equation (A10), for the non-dimensional rate of energy loss per unit channel width in a hydraulic transition of a stratified shear flow. It should be

$$\begin{aligned}
 & E'/[\rho_0 g \Delta U_1 h_1^2] \\
 &= (U_1^2/2g\Delta h_1) \left\{ \int_0^1 F_1^3(x) dx - \int_0^1 F_2^3(x) dx \left[ \int_0^1 F_1(x) dx / \int_0^1 F_2(x) dx \right]^3 P^3/q^2 \right\} \\
 &+ 2 \left\{ \int_0^1 x f_1(x) F_1(x) dx + \int_0^1 F_1(x) \int_x^1 f_1(y) dy dx \right\} \\
 &- 2q \left\{ \int_0^1 f_1(x) F_1(x) dx / \int_0^1 f_2(x) F_2(x) dx \right\} \left\{ \int_0^1 x f_2(x) F_2(x) dx \right. \\
 &\left. + \int_0^1 F_2(x) \int_x^1 f_2(y) dy dx \right\}, \tag{1}
 \end{aligned}$$

where, as before, the velocity upstream and downstream of the transition is uniform in the horizontal  $x$ -direction and depends only on the vertical coordinate,  $z$ ; the profiles of velocity and density are given by

$$U_i(z) = U_i F_i(z/h_i) \text{ when } 0 \leq z \leq h_i, \tag{2}$$

with  $U_i(z) = 0$  when  $z > h_i$ , and

$$\rho_i(z) = \rho_0 [1 - \Delta + 2\Delta f_i(z/h_i)] \text{ when } 0 \leq z \leq h_i, \tag{3}$$

with  $\rho_i(z) = \rho_0(1 - \Delta)$  when  $z > h_i$ , subscripts 1 and 2 denoting the flow upstream and downstream of the transition, respectively, and  $q = h_2/h_1$ , and  $P$  is the entrainment factor.<sup>2</sup>

The error affects the estimates of the smallest ‘Froude number’,  $Fr_c = U_{max}^2/g\Delta h_1$ , at which a transition can occur with a (physically necessary) zero or positive energy loss. ( $U_{max}$  is the maximum velocity in the flow upstream of the transition). The value of  $Fr_c$  calculated in Sections 3.2.1 to 3.2.3 is reduced to one substantially less than the Froude number,  $Fr = 3.34$ , of an observed flow cascading down the side of Lake Geneva in cold winter weather. For example, a ‘shape-preserving’ transition of flows that have the same velocity and density profiles as those observed can occur if  $Fr \geq 0.529$ , substantially less than the measured Froude number. Consequently, it is

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<sup>2</sup>Unlike the equation given by Thorpe and Ozen, (1) reduces to the equation for the energy loss in a single-layer, free-surface flow as given, for example, by Lighthill (1978).

possible that the cascading flows in Lake Geneva can undergo hydraulic jumps and, contrary to the prediction made by Thorpe and Ozen (2007), the observed flow is not marginally unstable to hydraulic jumps: jumps of substantial amplitude may occur.

## REFERENCES

- LIGHTHILL, J. 1978 *Waves in Fluids*. Cambridge University Press, 504 pp.  
THORPE, S. A. & OZEN, B. 2007 Are cascading flows stable? *J. Fluid Mech.*, **589**, 411–432.