

ASYMPTOTIC PROPERTIES OF LOW DEGREE GRAVITY MODES

Gabrielle Berthomieu and Janine Provost
Nice Observatory and CNRS (UA128)
B.P. 139
06003 Nice Cedex
France

ABSTRACT. Asymptotic properties of low degree gravity modes and their relation to the stratification of the model through the Brunt-Väissälä frequency are discussed for a solar type model and for a $10 M_{\odot}$ model. For the solar model, taking into account the quasi-adiabaticity of the solar convection zone, it is shown that two global constraints on the deep interior of the solar model can be derived from a set of g-modes periods in the observed range. However modes with large periods, i.e. larger than those up to now observed, are required to obtain informations on the stratification just below the convection zone. For the $10 M_{\odot}$ model, the preliminary results show that the existence of a layer with a large mean molecular weight gradient destroys the equidistance of the periods of gravity modes of a given degree and that some informations on the properties of this layer can be obtained from the analysis of the g-modes periods.

1. G-MODES OF A SOLAR TYPE MODEL

The results obtained concerning the asymptotic properties of solar g-modes are given in a paper to be published in *Astronomy and Astrophysics* (Provost *et al.*, 1986).

2. G-MODES OF A $10 M_{\odot}$ STAR

For stars with masses larger than $1.5 M_{\odot}$, the ZAMS model presents a convective core. As the age of the star increases, the size of the convective core decreases and a layer with a large mean molecular weight gradient $\nabla\mu$ develops. In this layer, which width increases with time, the Brunt-Väissälä frequency N is large and displays a very steep decrease at the boundaries as shown in Figure 1. This particular feature of N has a consequence on the g-modes periods. Here the theoretical asymptotic properties of low degree g-modes of such stars are considered. The g-modes periods of an evolved $10 M_{\odot}$ model have been computed in the Cowling's approximation and their properties compared to the theoretical predictions.

2.1. Asymptotic expression of periods of g modes

The asymptotic expression of the periods of g modes can be obtained following the methods developed by Tassoul (1980). In the convective core where the Brunt-Väissälä frequency

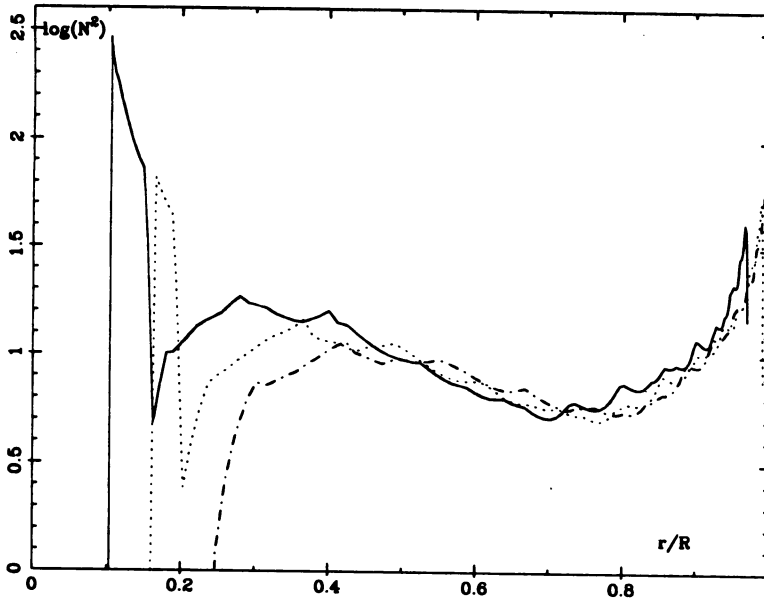


Figure 1: Variation of the logarithm of the square of Brunt-Väissälä frequency relatively to the radius for three models of $10M_{\odot}$ at different stage of evolution: - . - . - .ZAMS, $t = 0.98 \cdot 10^7 yr$, - - - - - $t = 0.17 \cdot 10^8 yr$.

N is zero, the method does not apply and a numerical integration must be performed. Solutions at the boundary of the convective core $x = x_0$ and near the surface are obtained and their asymptotic developpement at the first order in the frequency σ are fitted at the boundary of the large mean molecular weight gradient layer x_{μ} . At this point the steep decrease of N is treated as a discontinuity and the fitting of the solutions gives an implicit dispersion equation for the frequency of the mode:

$$\begin{aligned} \cos\left(\frac{\sqrt{l(l+1)}}{\sigma} \int_{x_0}^1 \frac{N}{x} dx - \frac{n_e \pi}{2} - \frac{\pi}{4} + \varphi\right) = \\ \frac{1-\alpha}{1+\alpha} \cos\left(\frac{\sqrt{l(l+1)}}{\sigma} \left(\int_{x_{\mu}}^1 \frac{N}{x} dx - \int_{x_0}^{x_{\mu}} \frac{N}{x} dx\right) - \frac{n_e \pi}{2} - \frac{\pi}{4} + \varphi\right) \end{aligned} \tag{1}$$

where σ is the frequency, n_e the polytropic index of the surface of the star, $\alpha = N(x_{\mu}^-)/N(x_{\mu}^+)$ is a measure of the discontinuity of N . φ is a phase introduced by the boundary condition $x = x_0$, and tends to zero with σ . Assuming $\varphi = 0$ and using the computed values of the integrals for the evolved $10 M_{\odot}$ model, it results from formula (1) that the periods of g modes of a given degree are not regularly spaced (with a period related to the equidistance period $P_0 = 2\pi^2 / (\int_{x_0}^1 (N/x) dx)$) as if N had a smooth variation ($\alpha = 1$).

The departure from the equidistance period P_0 is a function of the ratio α and has a maximum value equal to $P_0 \arcsin((1-\alpha)/(1+\alpha))/\pi$. If it is assumed that $(1-\alpha)/(1+\alpha)$ is small, the dispersion of the periods around the equidistance period P_0 can be expressed

by:

$$\frac{\overline{P}_k}{P_0} \equiv \sqrt{l(l+1)} \frac{(P(k+1) - P(k))}{P_0} \sim 1 + \frac{(-1)^k (1 - \alpha)}{\pi (1 + \alpha)} \tag{2}$$

$$[\cos(\overline{\varphi} + (\overline{\varphi} + k)P_0\Phi)(1 + \cos P_0\Phi) - \sin P_0\Phi \sin(\overline{\varphi} + (\overline{\varphi} + k)P_0\Phi)]$$

where $\Phi = \frac{1}{2\pi} \left[\int_{x_0}^1 \frac{N}{x} dx - 2 \int_{x_0}^{x_\mu} \frac{N}{x} dx \right]$, $\overline{\varphi} = n_e/2 + 3/4 - \varphi$ and $P(k) = 2\pi/\sigma$.

With $\varphi = 0$ and $n_e = 3/2$, the points corresponding to $(\overline{P}_k - P_0)/P_0$ for radial order k from 0 to 30 are plotted (o) and linked by a dotted line in Figure 3. They are distributed along two symmetric sinusoids with an amplitude related to the discontinuity α of the Brunt-Väissälä frequency at x_μ and a period related to $P_0\Phi$.

2.2. Comparison with computed periods

A set of g modes periods for $l = 1, 2, 3, 4$ and k from 2 to 30 has been computed for the most evolved model of $10M_\odot$ and plotted in Figure 2. The points are not distributed along a straight line with slope equal to P_0 , which would be the case if $\alpha = 1$, but are situated within the two dotted lines (with slope P_0) corresponding to the maximum theoretical departure from equidistance.

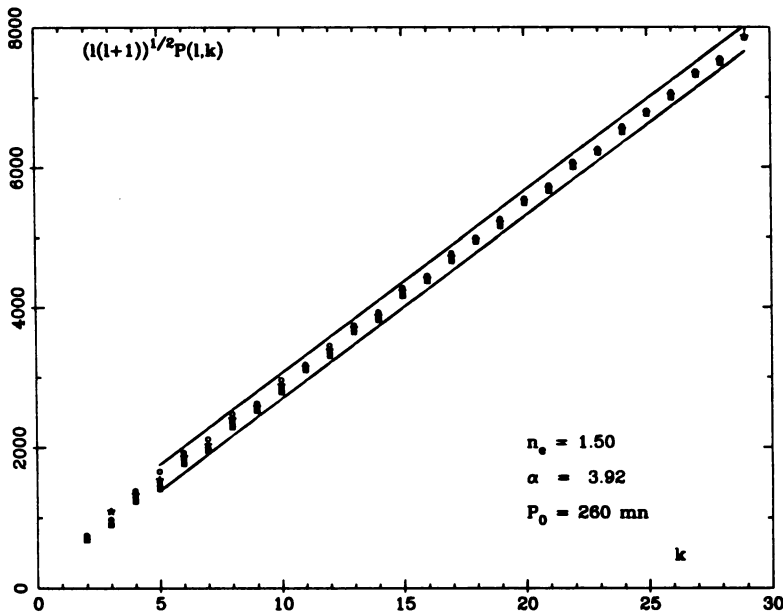


Figure 2: Periods P (multiplied by $\sqrt{l(l+1)}$) are plotted as a function of the radial order k .

The relative difference between the computed period interval \overline{P}_k separating two consecutive modes and P_0 is plotted in Figure 3. Except for low values of k where second order terms should be taken into account, the asymptotic predictions of equation (2) (dotted lines) agree qualitatively with the numerical values. Theoretical and computed amplitudes

are very close. The period of variation $P_0 \Phi / 2\pi$ is reasonably recovered, except for the existence of small shift depending on l and σ . More detailed analysis is needed to account for this shift. However these preliminary results show that informations on the properties of stratification of the model and on a possible discontinuity of the Brunt-Väissälä frequency due to the existence of a layer with large mean molecular weight gradient can be obtained from the analysis of g modes periods. In particular the quantities P_0 , Φ and α could be estimated. Further work is in progress.

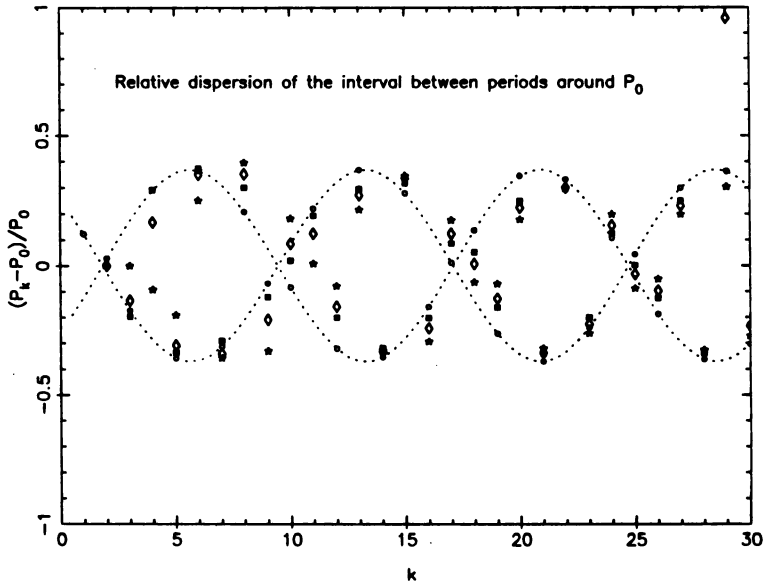


Figure 3: Relative difference between the computed period interval \overline{P}_k and P_0 as a function of radial order k for $l = 1, 2, 3$.

REFERENCES

- Provost, J., Berthomieu, G.: 1986, *Astron. Astrophys.*, in press.
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