Part V
Large Scale Structure

https://doi.org/10.1017/S0074180900216215 Published online by Cambridge University Press

New Cosmological Data and the Values of the Fundamental Parameters
IAU Symposium, Vol. 201, 2005
Anthony Lasenby and Althea Wilkinson, eds.

The Matter Power Spectrum

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Abstract. The main tools describing the formation of large-scale structure are reviewed. Detailed emphasis is given to the Cold Dark Matter model, since this appears to be a close match to observation. A consistent normalization that satisfies both CMB constraints and the cluster abundance requires $\Omega_m \simeq 0.35$ for a flat universe and scale-invariant fluctuations. Discrepancies between observed galaxy power spectra and CDM predictions are discussed; a heuristic model for galaxy bias is proposed that potentially allows such scale-dependent bias to be understood.

1. Structure formation in the CDM model

The origin and formation of large-scale structure in cosmology is a key problem that has generated much work over the years. Out of all the models that have been proposed, this talk concentrates on the simplest: gravitational instability of small initial density fluctuations. Furthermore, it is assumed that the mass density is dominated by a collisionless component, so that we are left with a theory very like the Cold Dark Matter model. This does not mean that CDM is an untestable religion, to which cosmologists cling in the face of all evidence. However, it is the simplest model for structure formation, and thus should be tested thoroughly before we move on to more complex alternatives.

Suppose there existed some primordial power-law spectrum, written dimensionlessly as the logarithmic contribution to the fractional density variance, σ^2:

$$\Delta^2(k) = \frac{d\sigma^2}{d\ln k} \propto k^{3+n}.$$

This undergoes linear growth

$$\delta_k(a) = \delta_k(a_0) \left[\frac{D(a)}{D(a_0)}\right] T_k,$$

where the linear growth law is $D(a) = a\,g[\Omega(a)]$ in the matter era, and the growth suppression for low Ω is

$$\begin{aligned} g(\Omega) &\simeq \Omega^{0.65} \text{ (open)} \\ &\simeq \Omega^{0.23} \text{ (flat)} \end{aligned}$$

The transfer function T_k depends on the dark-matter content as shown in figure 1.

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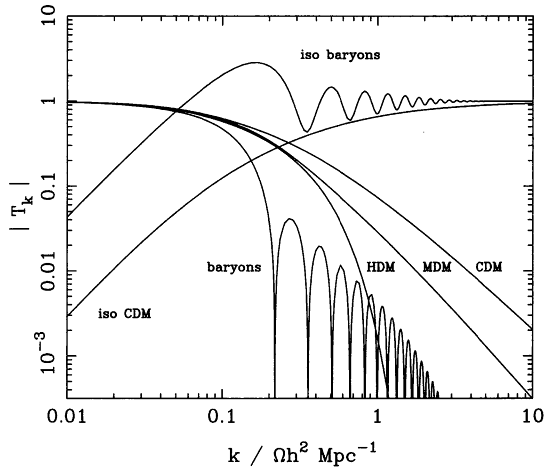

Figure 1. Transfer functions for various dark-matter models. The scaling with Ωh^2 is exact only for the zero-baryon models; the baryon results are scaled from the particular case $\Omega_{\rm B} = 1$, $h = 1/2$.

The state of the linear-theory spectrum after these modifications is illustrated in figure 2. The primordial power-law spectrum is reduced at large k, by an amount that depends on both the quantity of dark matter and its nature. Generally the bend in the spectrum occurs near $1/k$ of order the horizon size at matter-radiation equality, $\propto (\Omega h^2)^{-1}$. For a pure CDM universe, with scale-invariant initial fluctuations ($n = 1$), the observed spectrum depends only on two parameters. One is the shape $\Gamma = \Omega h$, and the other is a normalization. This can be set at a number of points. The COBE normalization comes from large angle CMB anisotropies, and is sensitive to the power spectrum at $k \simeq 10^{-3}\, h\, \mathrm{Mpc}^{-1}$. The alternative is to set the normalization near the quasi-linear scale, using the abundance of rich clusters. Many authors have tried this calculation, and there is good agreement on the answer:

$$\sigma_8 \simeq 0.5\, \Omega_m^{-0.6}.$$

(White, Efstathiou & Frenk 1993; Eke et al. 1996; Viana & Liddle 1996; Pierpaoli, Scott & White 2000; Wu 2000). In many ways, this is the most sensible normalization to use for LSS studies, since it does not rely on an extrapolation from larger scales. Within the CDM model, it is always possible to satisfy both these normalization constraints, by appropriate choice of Γ and n. This is illustrated in figure 3. Note that vacuum energy affects the answer; for reasonable values of h and reasonable baryon content, flat models require $\Omega_m \simeq 0.3$, whereas open models require $\Omega_m \simeq 0.5$.

Figure 1 shows that rather large oscillatory features would be expected if the universe was baryon dominated. The lack of observational evidence for such features is one reason for believing that the universe might be dominated by collisionless nonbaryonic matter (consistent with primordial nucleosynthesis if $\Omega_m \gtrsim 0.1$). Nevertheless, baryonic fluctuations in the spectrum can become

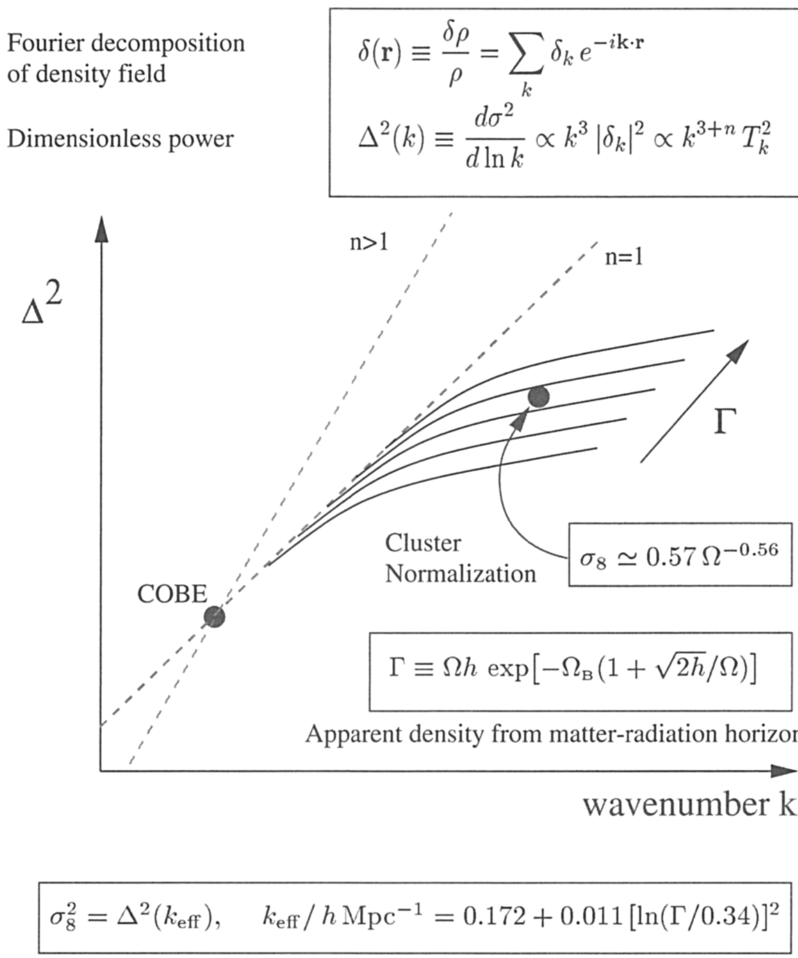

Figure 2. This figure illustrates how the primordial power spectrum is modified as a function of density in a CDM model. For a given tilt, it is always possible to choose a density that satisfies both the COBE and cluster normalizations.

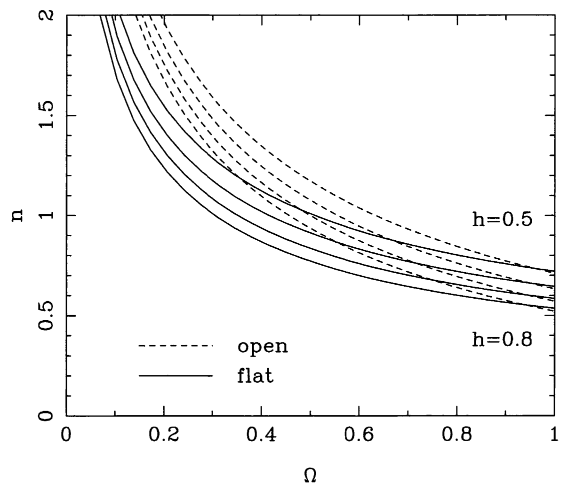

Figure 3. For 15% baryons, the value of n needed to reconcile COBE and the cluster normalization in CDM models. If $h = 0.7$, then $\Omega_m \simeq 0.35$ is required in a flat universe containing scale-invariant fluctuations.

significant for high-precision measurements. Figure 4 shows that order 10% modulation of the power may be expected in realistic baryonic models (Eisenstein & Hu 1998; Goldberg & Strauss 1998). Most of these features are however removed by nonlinear evolution. The highest-k feature to survive is usually the second peak, which almost always lies near $k = 0.05\,\mathrm{Mpc}^{-1}$ (no h, for a change). This feature is relatively narrow, and can serve as a clear proof of the past existence of baryonic oscillations in forming the mass distribution (Meiksin, White & Peacock 1999).

2. The observed galaxy power spectrum

The real-space galaxy power spectrum has been estimated from angular clustering in the APM survey (Baugh & Efstathiou 1993, 1994; Maddox et al. 1996). The APM survey was generated from a catalogue of $\sim 10^6$ galaxies derived from UK Schmidt Telescope photographic plates scanned with the Cambridge Automatic Plate Measuring machine. The APM result has been investigated in detail by a number of authors (e.g. Gaztañaga & Baugh 1998; Eisenstein & Zaldarriaga 1999; Efstathiou & Moody 2000) and found to be robust, although there is some evidence that the true power uncertainty due cosmic variance may have been underestimated.

One advantage of the APM estimate of the power spectrum is that it is based on a deprojection of angular clustering, and is thus immune to the complicating effects of redshift-space distortions. These effects were first fully analyzed by Kaiser (1987), and distort the apparent density field in a 3D redshift survey in a number of ways. For a survey that subtends a small angle (i.e. in the distant-observer approximation), a good approximation to the anisotropic redshift-space Fourier spectrum is given by a systematic large-scale anisotropy, together with

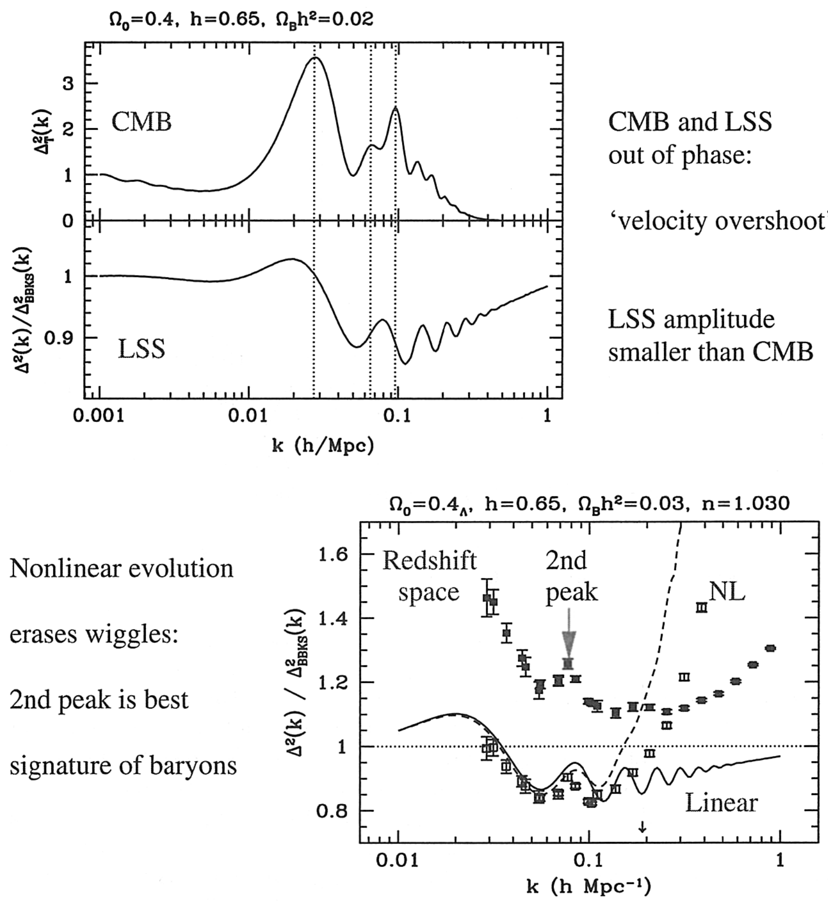

Figure 4. Baryonic fluctuations in the spectrum can become significant for high-precision measurements. Although such features are much less important in the density spectrum than in the CMB (first panel), the order 10% modulation of the power is potentially detectable. However, nonlinear evolution has the effect of damping all beyond the second peak. This second feature is relatively narrow, and can serve as a clear proof of the past existence of oscillations in the baryon-photon fluid (Meiksin, White & Peacock 1999).

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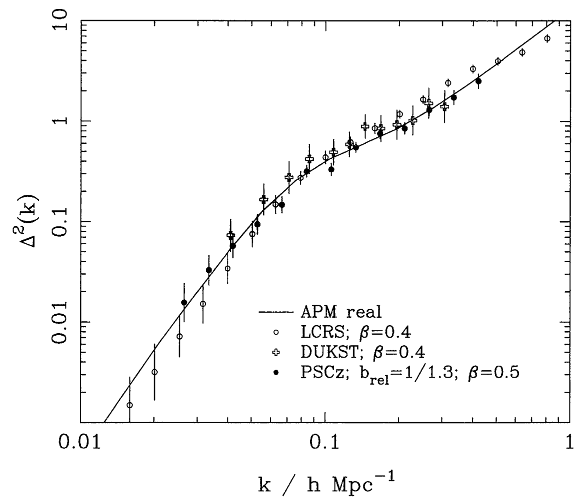

Figure 5. A recent compilation of measurement of the power spectrum of galaxy clustering (dimensionless power against wavenumber). The real-space APM result was deprojected from angular clustering. The other points show the results from various redshift surveys, with an approximate correction for redshift-space distortions (see text). For the IRAS-based PSCz survey, an additional relative bias factor of 1.3 has been applied.

a damping term from nonlinear effects:

$$\delta_k^s = \delta_k^r (1 + \beta\mu^2) D(k\sigma\mu).$$

Here, $\beta = \Omega_m^{0.6}/b$, b being the linear bias parameter of the galaxies under study, and $\mu = \hat{\mathbf{k}} \cdot \hat{\mathbf{r}}$. For an exponential distribution of relative small-scale peculiar velocities (as seen empirically), the damping function is $D(y) \simeq (1 + y^2/2)^{-1/2}$, and $\sigma \simeq 400\,\mathrm{km\,s^{-1}}$ is a reasonable estimate for the pairwise velocity dispersion of galaxies (e.g. Ballinger, Peacock & Heavens 1996). In principle, this distortion should be a robust way to determine Ω (or at least β); see the reviews by Strauss & Willick (1995) and Hamilton (1997). At this meeting, Colless presented preliminary results for β from the 2dF Galaxy Redshift Survey, which show the distortion effect clearly for the first time. For the present purpose, we are interested in the scale-dependent correction to the angle-averaged power, which may be deduced by integrating over μ in the above expression.

We can thus carry out an approximate comparison between the APM deprojection and the power measured in some of the largest recent redshift surveys, which contain of order 10^4 galaxy redshifts (LCRS: Shectman et al. 1996; DUKST: Hoyle et al. 1999; PSCz: Saunders et al. 2000) This comparison is performed in figure 5, and indicates that there is now good agreement on the form of the galaxy power spectrum for wavelengths up to several hundred h^{-1} Mpc. Indeed, this conclusion has been stable for a number of years (see Peacock & Dodds 1994 for an earlier compilation).

A number of authors have pointed out that the detailed spectral shape inferred from galaxy data appears to be inconsistent with that of nonlinear evolution from CDM initial conditions. (e.g. Efstathiou, Sutherland & Maddox 1990; Klypin, Primack & Holtzman 1996; Peacock 1997). Perhaps the most detailed work was carried out by the VIRGO consortium, who carried out $N = 256^3$ simulations of a number of CDM models (Jenkins et al. 1998). Their results are shown in figure 6, which gives the nonlinear power spectrum at various times (cluster normalization is chosen for $z = 0$) and contrasts this with the APM data. The lower small panels are the scale-dependent bias that would required if the model did in fact describe the real universe, defined as

$$b(k) \equiv \left(\frac{\Delta^2_{\rm gals}(k)}{\Delta^2_{\rm mass}} \right)^{1/2} .$$

In all cases, the required bias is non-monotonic; it rises at $k \gtrsim 5\,h^{-1}\,\mathrm{Mpc}$, but also displays a bump around $k \simeq 0.1\,h^{-1}\,\mathrm{Mpc}$.

3. Galaxy formation and biased clustering

The disagreement between galaxy clustering and the mass clustering expected in CDM universes might be resolved if the relation between galaxies and the overall matter distribution is sufficiently complicated. Indeed, the formation of galaxies must be a non-local process to some extent, and the modern paradigm was introduced by White & Rees (1978): galaxies form through the cooling of baryonic material in virialized haloes of dark matter. The virial radii of these systems are in excess of 0.1 Mpc, so there is the potential for large differences in the correlation properties of galaxies and dark matter on these scales.

A number of studies have indicated that the observed galaxy correlations may indeed be reproduced by CDM models. The most direct approach is a numerical simulation that includes gas, and relevant dissipative processes. This is challenging, but just starting to be feasible with current computing power (Pearce et al. 1999). The alternative is 'semianalytic' modelling, in which the merging history of dark-matter haloes is treated via the extended Press-Schechter theory (Bond et al. 1991), and the location of galaxies within haloes is estimated using dynamical-friction arguments (e.g. Kauffmann et al. 1993, 1999; Cole et al. 1994; Somerville & Primack 1999; van Kampen, Jimenez & Peacock 1999; Benson et al. 2000a,b). Both these approaches have yielded similar conclusions, and shown how CDM models can match the galaxy data: specifically, the low-density flat ΛCDM model that is favoured on other grounds can yield a correlation function that is close to a single power law over $1000 \gtrsim \xi \gtrsim 1$, even though the mass correlations show a marked curvature over this range (Pearce et al. 1999; Benson et al. 2000a; see figure 7). These results are impressive, yet it is frustrating to have a result of such fundamental importance emerge from a complicated calculational apparatus. There is thus some motivation for constructing a simpler heuristic model that captures the main processes at work in the full semianalytic models. This section describes an approach of this sort (Peacock & Smith 2000; see also Seljak 2000).

An early model for galaxy clustering was suggested by Neyman, Scott & Shane (1953), in which the nonlinear density field was taken to be a superposition of randomly-placed clumps. With our present knowledge about the evolution of

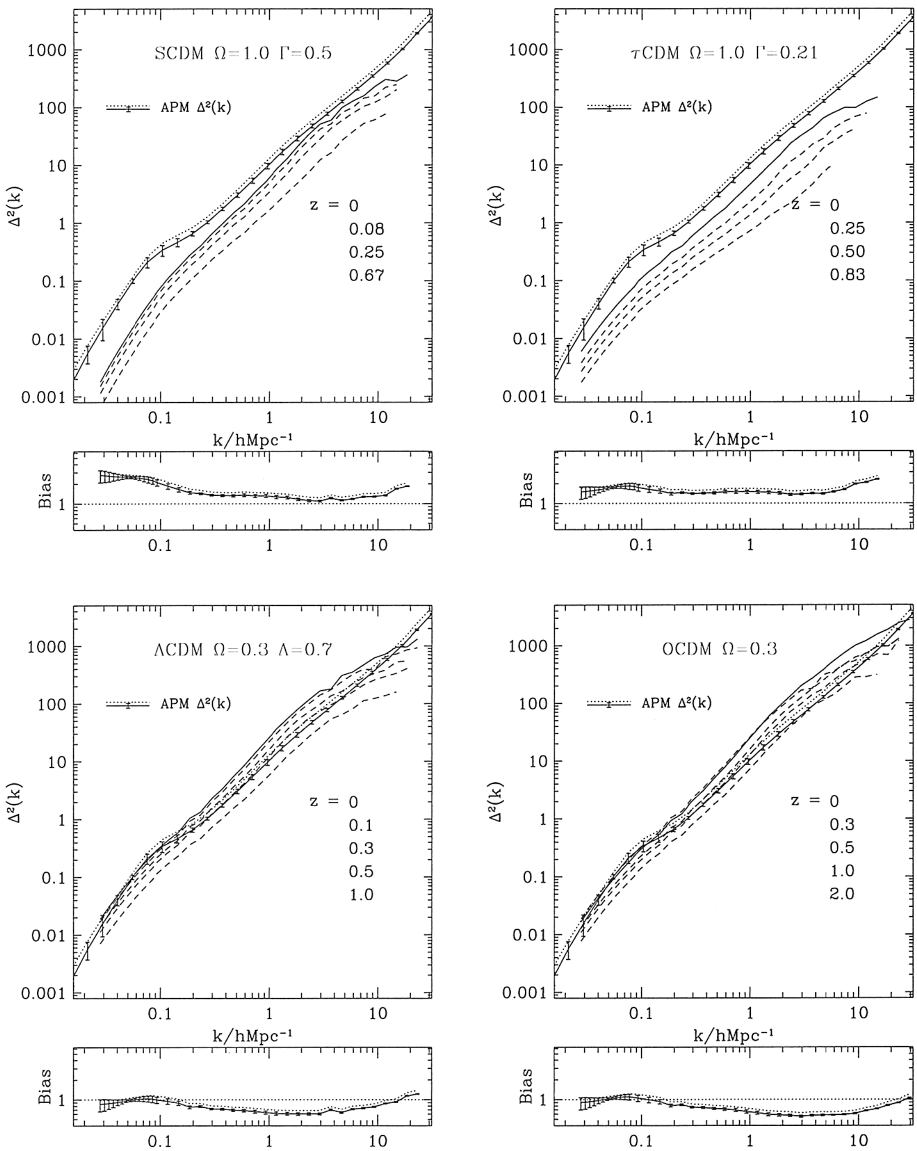

Figure 6. The nonlinear evolution of various CDM power spectra, as determined by the Virgo consortium (Jenkins et al. 1998). The dashed lines show the evolving spectra for the mass, which at no time match the shape of the APM data. This is expressed in the lower small panels as a scale-dependent bias at $z = 0$: $b^2(k) = P_{\rm APM}/P_{\rm mass}$.

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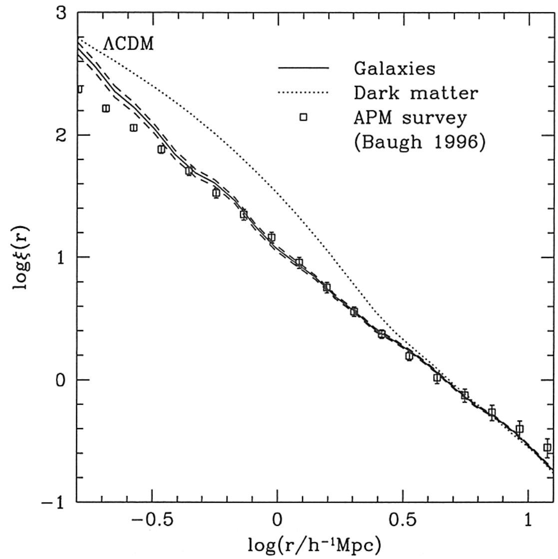

Figure 7. The correlation function of galaxies in the semianalytical simulation of an LCDM universe by Benson et al. (2000a). The predicted galaxy correlations are very close to a single power law, especially around $1\,h^{-1}$ Mpc, where the predicted mass correlations rise above the APM data.

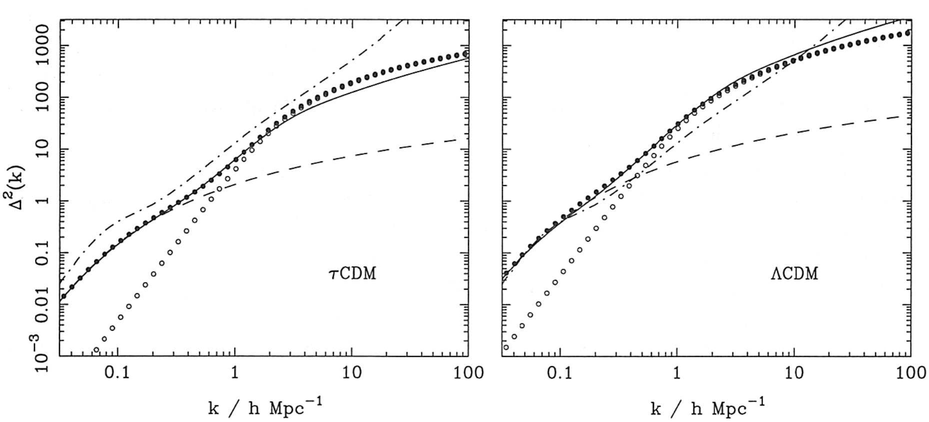

Figure 8. The power spectrum for the τCDM and ΛCDM models. The solid lines contrast the linear spectrum with the nonlinear spectrum, calculated according to the approximation of PD96. The spectrum according to randomly-placed haloes is denoted by open circles; if the linear power spectrum is added, the main features of the nonlinear spectrum are well reproduced. In neither case is the resulting nonlinear spectrum at all the same shape as the APM observations (shown asa dot-dashed line).

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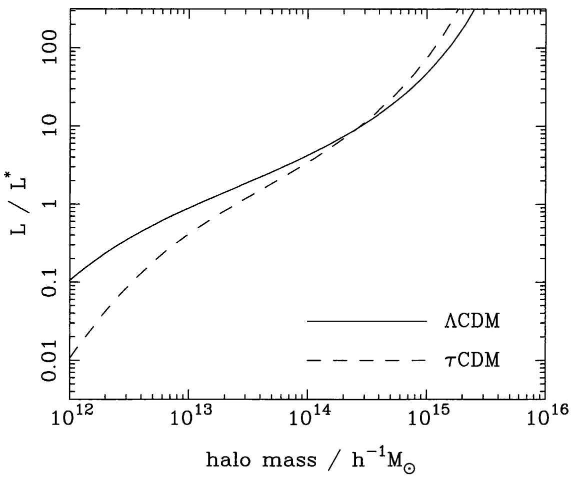

Figure 9. The empirical luminosity–mass relation required to reconcile the observed AGS luminosity function with two variants of CDM. L^* is the characteristic luminosity in the AGS luminosity function ($L^* = 7.6 \times 10^{10} h^{-2} L_{\odot}$). Note the rather flat slope around $M = 10^{13}$ to $10^{14} h^{-1} M_{\odot}$, especially for ΛCDM.

CDM universes, we can make this idealised model considerably more realistic: hierarchical models are expected to contain a distribution of masses of clumps, which have density profiles that are more complicated than isothermal spheres. These issues are well studied in N-body simulations, and highly accurate fitting formulae exist, both for the mass function and for the density profiles. Briefly, we use the mass function of Sheth & Tormen (1999; ST) and the halo profiles of Moore et al. (1999; M99). Using this model, it is then possible to calculate the correlations of the nonlinear density field, neglecting only the large-scale correlations in halo positions. The power spectrum determined in this way is shown in figure 8, and turns out to agree very well with the exact nonlinear result on small and intermediate scales. The lesson here is that a good deal of the nonlinear correlations of the dark matter field can be understood as a distribution of random clumps, provided these are given the correct distribution of masses and mass-dependent density profiles.

How can we extend this model to understand how the clustering of galaxies can differ from that of the mass? There are two distinct ways in which a degree of bias is inevitable:

(1) Halo occupation numbers. For low-mass haloes, the probability of obtaining an L^* galaxy must fall to zero. For haloes with mass above this lower limit, the number of galaxies will in general not scale with halo mass.

(2) Nonlocality. Galaxies can orbit within their host haloes, so the probability of forming a galaxy depends on the overall halo properties, not just the density at a point. Also, the galaxies will end up at special places within the haloes: for a halo containing only one galaxy, the galaxy will

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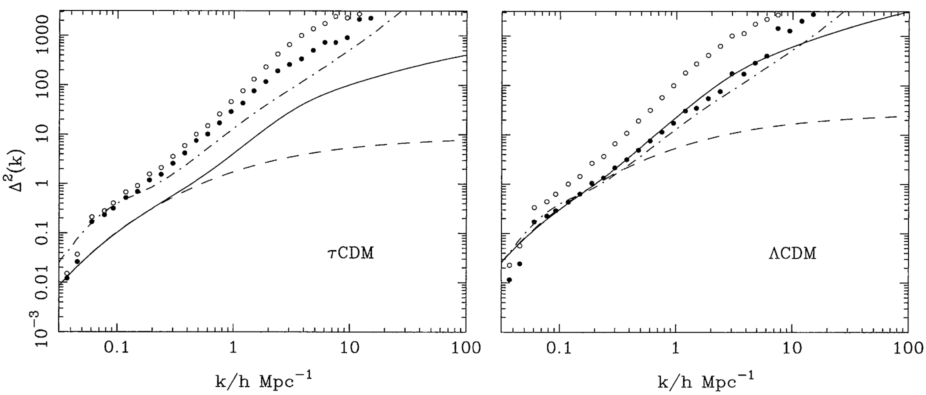

Figure 10. The power spectrum for galaxy catalogues constructed from the τCDM and ΛCDM models. A reasonable agreement with the APM data (dot-dashed line) is achieved by simple empirical adjustment of the occupation number of galaxies as a function of halo mass, plus a scheme for placing the haloes non-randomly within the haloes. In contrast, the galaxy power spectrum differs significantly in shape from that of the dark matter (linear and nonlinear theory shown as in figure 8).

clearly mark the halo centre. In general, we expect one central galaxy and a number of satellites.

The numbers of galaxies that form in a halo of a given mass is the prime quantity that numerical models of galaxy formation aim to calculate. However, for a given assumed background cosmology, the answer may be determined empirically. Galaxy redshift surveys have been analyzed via grouping algorithms similar to the 'friends-of-friends' method widely employed to find virialized clumps in N-body simulations. With an appropriate correction for the survey limiting magnitude, the observed number of galaxies in a group can be converted to an estimate of the total stellar luminosity in a group. This allows a determination of the All Galaxy System (AGS) luminosity function: the distribution of virialized clumps of galaxies as a function of their total luminosity, from small systems like the Local Group to rich Abell clusters.

The AGS function for the CfA survey was investigated by Moore, Frenk & White (1993), who found that the result in blue light was well described by

$$d\phi = \phi^* \left[(L/L^*)^\beta + (L/L^*)^\gamma\right]^{-1} dL/L^*,$$

where $\phi^* = 0.00126h^3\text{Mpc}^{-3}$, $\beta = 1.34$, $\gamma = 2.89$; the characteristic luminosity is $M^* = -21.42 + 5\log_{10} h$ in Zwicky magnitudes, corresponding to $M_B^* = -21.71 + 5\log_{10} h$, or $L^* = 7.6 \times 10^{10} h^{-2} L_\odot$, assuming $M_B^\odot = 5.48$. One notable feature of this function is that it is rather flat at low luminosities, in contrast to the mass function of dark-matter haloes (see Sheth & Tormen 1999). It is therefore clear that any fictitious galaxy catalogue generated by randomly sampling the mass is unlikely to be a good match to observation. The simplest cure for this deficiency is to assume that the stellar luminosity per virialized halo is a

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monotonic, but nonlinear, function of halo mass. The required luminosity–mass relation is then easily deduced by finding the luminosity at which the integrated AGS density $\Phi(> L)$ matches the integrated number density of haloes with mass $> M$. The result is shown in figure 9.

We can now return to the halo-based galaxy power spectrum and use the correct occupation number, N, as a function of mass. This is needs a little care at small numbers, however, since the number of haloes with occupation number unity affects the correlation properties strongly. These haloes contribute no correlated pairs, so they simply dilute the signal from the haloes with $N \geq 2$. The existence of antibias on intermediate scales can probably be traced to the fact that a large fraction of galaxy groups contain only one $> L_*$ galaxy. Finally, we need to put the galaxies in the correct location, as discussed above. If one galaxy always occupies the halo centre, with others acting as satellites, the small-scale correlations automatically follow the slope of the halo density profile, which keeps them steep. The results of this exercise are shown in figure 10, and are encouragingly similar to the scale-dependent bias found in the detailed calculations of Benson et al. (2000a), shown in figure 7.

4. Conclusions

Although this talk has presented only a partial selection of the evidence, there is a good case for convergence in our understanding of the basis of large-scale structure. The power spectrum of the matter density that results from gravitational instability is understood in detail, including small corrections from nonlinear effects and fine signatures of the baryon content. On the observational side, the next generation of galaxy surveys will yield sufficiently precise results that we may be able to detect these features.

The great barrier to using large-scale structure as a tool for quantitative cosmology has long been the uncertainties associated with galaxy bias, and this problem is not yet beaten. Nevertheless, both the detailed semianalytic results and their incarnation in the generic 'halo model' give grounds for optimism that we may be starting to attain a physical understanding of the origin of galaxy bias. If this provisional understanding holds up, we should be able to use observations of the galaxy power spectrum with confidence in order to measure the properties of the matter density. It is not yet clear whether the result of this process will be a familiar standard model for structure formation, or a complete revolution. However, there appears to be no fundamental obstacle that will prevent a decision being reached.

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