4. Of course the n points of division required in AB could be got by first constructing $FB = \frac{1}{n}AB$ as in the first construction, and then laying off segments equal to it along AB. But, though this construction would be geometrographically simpler than the last, it would have the disadvantage of multiplying the initial error in FB by $n-1$, which might result in the later points of division in the process of dividing AB being affected by two great errors.

Query—What is the relation between the probable errors in the positions of E_r in the two cases?

5. Returning to the construction in 3, we may note that it gives a means of constructing a point F_r in AB such that $AF_r = r \cdot \frac{AB}{n}$.

There is nothing to prevent r from being greater than n in this construction, or less than zero. If r be greater than $n+1$, then D_r will be beyond D_{n+1}, and F_r will be in AB produced.

If r be negative, then D_r will be in DA produced, while E_{n-r} will be in $B\gamma$, so that F_r will lie in BA produced.

Thus we can by ruler and dividers alone divide a segment of a line internally or externally so that the parts are in any commensurable ratio to one another.

6. A modification of the construction of Art. 3 would be got by interchanging the rôles of the lines AD_{n+1} and AB. But this, though requiring fewer points of division, has the disadvantage of failing to give *immediately* the last dividing point required.

R. F. MUIRHEAD.

Elementary proof of the formula for $\alpha^n + \beta^n$ in terms of $\alpha + \beta$ and $\alpha\beta$.*—The proof given by Chrystal uses infinite series, but by a simple modification it may be put in an elementary form.

Write
$$\alpha + \beta = s,$$
$$\alpha\beta = p.$$
$$\alpha^n + \beta^n = s_n.$$
$$sx - px^2 = u.$$

* Chrystal's Algebra, 1st Edition, Chap. 27, Art. 7.

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Then $\dfrac{1}{1-\alpha x}+\dfrac{1}{1-\beta x}=\dfrac{2-sx}{1-u}$:

But $\dfrac{1}{1-\alpha x}=1+\alpha x+\alpha^2x^2+\ldots.+\alpha^nx^n+\dfrac{\alpha^{n+1}x^{n+1}}{1-\alpha x}$,

$$\frac{1}{1-\beta x}=1+\beta x+\beta^2x^2+\ldots+\beta^nx^n+\frac{\beta^{n+1}x^{n+}}{1-\beta x},$$

$$\frac{1}{1-u}=1+u+u^2+\ldots+u^n+\frac{u^{n+1}}{1-u}.$$

Thus $2+sx+s_2x^2+\ldots+s_nx^n+x^{n+1}\left(\dfrac{\alpha^{n+1}}{1-\alpha x}+\dfrac{\beta^{n+1}}{1-\beta x}\right)$

$$=(2-sx)(1+u+u^2+\ldots+u^n)+(2-sx)\frac{u^{n+1}}{1-u},$$

or $2+sx+s_2x^2+\ldots+s_nx^n-(2-sx)(1+u+\ldots+u^n)$

$$=-x^{n+1}\left(\frac{\alpha^{n+1}}{1-\alpha x}+\frac{\beta^{n+1}}{1-\beta x}\right)+(2-sx)\frac{u^{n+1}}{1-u}.$$

Since the left hand member here is a rational integral function of x, so also is the right hand member, and the latter obviously involves x^{n+1} as a factor.

Hence the coefficient of x^n on the left vanishes; that is, s_n is equal to the coefficient of x^n in

$$(2-sx)(1+u+\ldots+u^n),$$

which is easily found by the Binomial Theorem, as in Chrystal.

A similar proof of Newton's formulae connecting the sums of the powers of the roots of an equation :

Let the equation be

$$x^n+p_1x^{n-1}+p_2x^{n-2}+\ldots+p_n=0,$$

or, say, $$f(x)=0.$$

Let the roots be $\alpha_1, \alpha_2, \ldots, \alpha_n$.

Then, as is well known,

$$\frac{f(x)}{x-\alpha_1}+\frac{f(x)}{x-\alpha_2}+\ldots+\frac{f(x)}{x-\alpha_n}$$

$$=nx^{n-1}+(n-1)p_1x^{n-2}+\ldots+p_{n-1},$$

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or, writing $x = \frac{1}{z}$, and multiplying by z^{n-1},

$$(1 + p_1 z + \ldots + p_n z^n)\left(\frac{1}{1-\alpha_1 z} + \ldots + \frac{1}{1-\alpha_n z}\right)$$
$$= n + (n-1)p_1 z + \ldots + p_{n-1} z^{n-1}.$$

The left hand member is

$$\left(1 + p_1 z + \ldots + p_n z^n\right)\left(n + s_1 z + \ldots + s_n z^n + \frac{\alpha_1{}^{n+1} z^{n+1}}{1-\alpha_1 z} + \ldots + \frac{\alpha_n{}^{n+1} z^{n+1}}{1-\alpha_n z}\right),$$

that is,

$$(1 + p_1 z + \ldots + p_n z^n)\,(n + s_1 z + \ldots + s_n z^n)$$

together with a rational integral function of z containing z^{n+1} as a factor.

Hence, for $r = 0, 1, 2, \ldots, n$

the coefficient of z^r is the same in

$$(1 + p_1 z + \ldots + p_n z^n)\,(n + s_1 z + \ldots + s_n z^n)$$

and in

$$n + (n-1)p_1 z + \ldots + p_{n-1} z^{n-1},$$

from which Newton's formulae follow at once.

JOHN DOUGALL.

Arithmetical Solution of the Ages Problem.—The following solution of an old problem may be new to some readers.

"The ages of A and B are as 3 to 1, and in 15 years they will be as 2 to 1. Find their ages."

$$\frac{3}{1}\;,\quad \frac{2}{1}$$

$$\frac{3}{1}\;,\quad \frac{4}{2}$$

$$\frac{45}{15}\;,\quad \frac{60}{30}$$

In the first line the original and final ratios are written down as fractions. In the second line, without altering the value of the fractions, we make the difference of numerator and denominator

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