# 60. SMALL AMPLITUDE DENSITY WAVES ON A FLAT GALAXY

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**Abstract.** By numerical methods we have found an unstable two-armed density wave on a flat galactic model. We present the results in a form of four plots, and briefly discuss the observational implications as well as the uncertainties involved in the models and the calculations.

## 1. Brief Description of the Calculations

We have investigated the stability of a flat model galaxy derived from the rotation curve of M31 (Van de Hulst *et al.*, 1957). From 4 kpc outward sufficiently high random motions were incorporated in the model to make it stable against local axisymmetric collapse. To stabilize the inner part rather large orbital eccentricities are required. Rather than extrapolate our epicyclic orbits to eccentricities larger than 0.2, we reduced the response by supposing that only a fraction of the stars participated in the collective modes. Such a procedure appears to be qualitatively correct and is consistent with the results reported below. The perturbations or modes are assumed to be small in amplitude and are found by solving the Poisson and linearized Vlasov equations. The solutions of these equations can be written in the form $\mathrm{Re}[A(r)\exp i(m\theta+\omega t)]$ and are density waves which rotate around the axis of the galaxy with a pattern speed $=\mathrm{Re}(-\omega/m)$. The amplitude $A(r)$ and the frequency $\omega$ are obtained by solving an integral equation (Kalnajs, 1965).

The two armed or $m=2$ modes are special for the reasons pointed out by Lindblad (1958). We have obtained numerically the largest discrete $m=2$ mode. By largest we mean that the gravitational interactions associated with it are strongest as measured by the shift of the pattern speed ($=30$ km s$^{-1}$ kpc$^{-1}$) from the kinematical value ($\Omega-\kappa/2\approx 10$ km s$^{-1}$ kpc$^{-1}$) which would obtain if the gravitational effects of the perturbation were neglected (Lindblad, 1958). The calculation itself involved the replacement of the kernel of the integral equation by a $60\times 60$ complex matrix. The eigenvector solution was numerically stable.

After finding a self-consistent mode it is possible to calculate the response of any subsystem of the galaxy to it. Since the spiral structure is seen most prominently in the gas and objects with the lowest velocity dispersion, we have calculated the density response and velocity fields of zero velocity dispersion objects. This is a reasonable approximation since the pressure effects in the gas are negligible on the scale of kiloparsecs.

The results are graphically summarized in Figures 1–4. The rotation of the galaxy as well as of the pattern is counterclockwise. The amplitude of the latter grows at the rate of two powers of $e$ in $10^9$ yr. For the sake of clarity only the positive values of the perturbed quantities have been printed. The numbers in the figures must be multiplied by the indicated scale factors to obtain a consistent set of values. The

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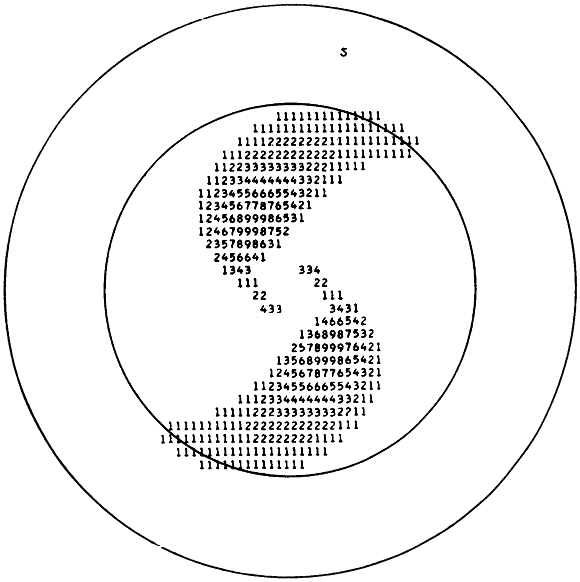


Fig. 1. Excess mass density. Scale factor is 6.17.

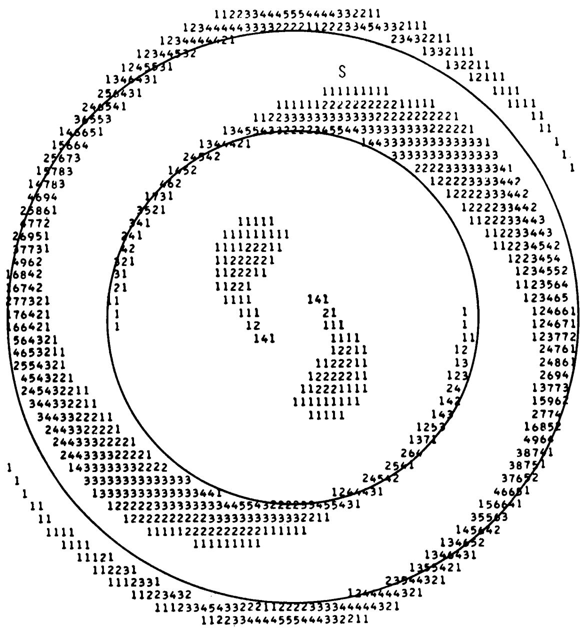


Fig. 2. Excess gas density. Scale factor is 0.0557.

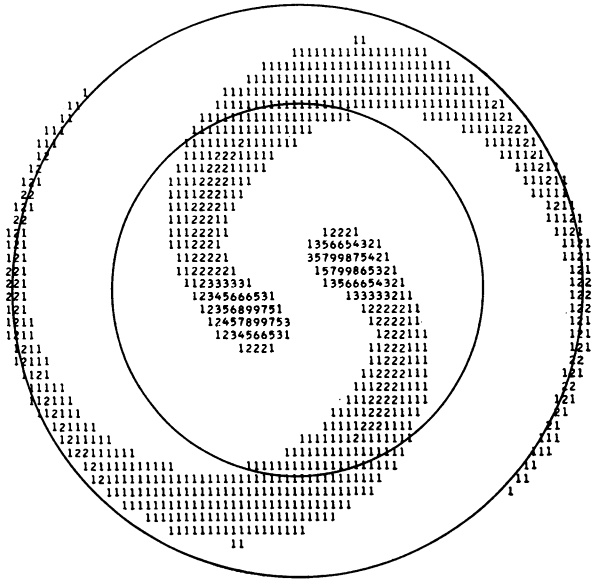

Fig. 3. Tangential velocity. Scale factor is 2.32.

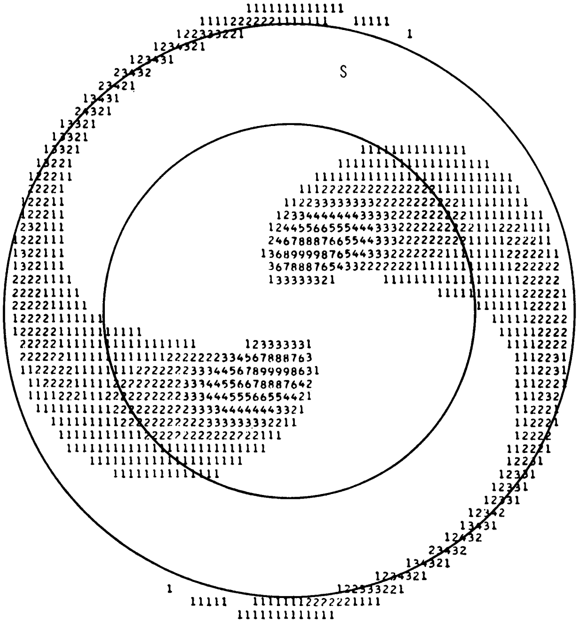


Fig. 4. Radial velocity. Scale factor is 2.42.

surface mass density is in solar masses $pc^{-2}$, the velocities are in km $s^{-1}$, and the surface gas density is normalized by its equilibrium value. In the calculation the latter was assumed to be constant, however a smooth variation with radius will not change the pattern significantly.

The high frequency ($\omega = -60-2i$) gives rise to two resonances. At the particle resonance (inner circle, $r=9$ kpc) the stars travel with the wave, whereas at the outer resonance (outer circle, $r=14$ kpc) the stars see the force field varying at their epicyclic frequency. Energy and angular momentum are conserved over the whole disk but they are redistributed outward: the stars inside the particle resonances, which move faster than the wave, lose both to the stars on the outside of the resonance. An angular momentum transfer is already apparent from the trailing nature of the pattern.

## 2. Discussion

The interesting result of these calculations is the strong dependence of the perturbed densities on the velocity dispersion of the subsystem. The density wave is essentially a bar-like distortion of the central region of the galaxy which drives the gas. The tightly wound pattern and large density contrast in the latter are due to the presence of the resonances. The position of the resonance is determined by the inner part of the model, whereas the growth rate of the pattern depends on the mass density chiefly at the outer resonance. A decrease of the latter means a slower growth rate which tends to break up the gas pattern into nearly circular arcs around the resonance radii.

Insofar as the model is entirely determined by the rotation curve, the above results should apply to our galaxy since the rotation curves are similar (at least from 4 kpc outward). The sun's radial position would fall half-way between the two resonances. If we take the observed north-south asymmetry in the rotation curve (Kerr, 1964) as evidence for the presence of such a mode, then we can explain it if the sun is placed in the position *s*, and the local standard of rest (LSR) has a radial motion which is one-half that of the gas, or $-4$ km $s^{-1}$. The asymmetry also determines the amplitude of the mode which is twice as large as that quoted in the figures. If we further assume that the LSR moves with the gas in the tangential direction (at 6.5 km $s^{-1}$) we can fill in most of the dip in the rotation curve which occurs at 6.7 kpc. There are other features such as the 50 km $s^{-1}$ maximum in the radial velocity field in the direction of the galactic center which would appear as an outward moving 'arm', the general asymmetry with respect to the center, and the fact that the mode is unstable, which encourage further investigation.

There are also many deficiencies, notably those in the equilibrium model and nonlinearities in the gas distribution, which have to be corrected. Small corrections in the model arise from the fact that the presence of the mode increases the Oort constants of the gas, $A$, $B$, $C$, and $K$ by 2.7, 0.7, $-1.7$, and $-3.5$ km $s^{-1}$ $kpc^{-1}$ respectively. A larger uncertainty comes from the central region. The model we used has no central condensation which would lead to an inner resonance.

The interpretation that leads to the 1965 Schmidt model, which is the commonly accepted one, tends to overestimate the central rotation rates. We have repeated the above calculation using this model and find that the mode is not very much affected in the outer regions ($r > 5$ kpc), but is significantly modified in the vicinity of the inner resonance point at 2.3 kpc. If an amplitude is chosen to match the outer parts, we find non-circular velocities at 2.3 kpc in the range of 100–150 km $s^{-1}$. Such amplitudes severely strain the linear theory. The credibility of the result is further lessened by our orbit approximations which in the presence of a resonance do become suspect. However, viewed as an order of magnitude calculation, the result suggests that a bar-like mode of the disk can produce large non-circular motions if a central condensation exists, and that the nature of the motions will have to be understood in order to obtain a quantitative description of the center. It is conceivable that the wide wings on the line profiles near the center are associated with an inner Lindblad resonance, which would imply a smaller central concentration than suggested by the Schmidt model.

Another interesting feature of the inner resonance is the coincidence of the maxima in gas densities with the maxima in the inward radial velocities.

A more detailed description of these calculations will be published later.

## Acknowledgement

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