if l/m be one of the roots of (β).

Hence $$l/(y-\beta) = -m/(x-\alpha).$$

If then for l and m we substitute in (β) $y-\beta$ and $-(x-\alpha)$ respectively, we shall obtain the equation to the four normals which can be drawn from (α, β) to the conic S.

4. It may be worth considering what the condition (α) becomes for the circle—

$$x^2+y^2+2gx+2fy+c=0.$$

Here
$$a=b=1\ ;\ h=0$$
$$\Delta = c-f^2-g^2,$$
$$A=c-f^2, B=c-g^2, C=1,$$
$$F=-f,\quad G=-g,\quad H=fg.$$

Hence in this case (α) becomes

$$(l^2+m^2)[l^2(c-f^2)+m^2(c-g^2)+n^2-2fmn-2gnl+2fglm]$$
$$=(l^2+m^2)^2(c-f^2-g^2).$$

Dividing by l^2+m^2, we obtain

$$c(l^2+m^2)-f^2l^2-g^2m^2+n^2-2fmn-2gnl+2fglm$$
$$=c(l^2+m^2)-f^2l^2-f^2m^2-g^2l^2-g^2m^2,$$

or $$g^2l^2+f^2m^2+2fglm-2n(gl+fm)+n^2=0,$$

or $$(gl+fm-n)^2=0.$$

Hence the condition that $lx+my+n=0$ should be a normal to the circle is $$gl+fm-n=0.$$

But this is the tangential equation to the point $(-g, -f)$, the centre of the circle.

Third Meeting, 10th January 1890.

George A. Gibson, Esq., M.A., Ex-President, in the Chair.

Note on a curious operational Theorem.

By Professor Tait.

The idea in the following note is evidently capable of very wide development, but it can be made clear by a very simple example.

Whatever be the vectors α, β, γ, δ, we have always

$$V.V\alpha\beta V\gamma\delta = \alpha S.\beta\gamma\delta - \beta S.\alpha\gamma\delta.$$

But vector operators are to be treated in all respects like vectors, provided each be always kept *before* its subject.

Let $$\sigma = i\xi + j\eta + k\zeta,$$
where ξ, η, ζ are functions of x, y, z; and let
$$\nabla = i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz},$$
as usual. Also let σ_1, ∇_1 be their values when x_1, y_1, z_1 are put for x, y, z.

Then by the first equation, attending to the rule for the place of an operator,
$$\mathrm{V}.\mathrm{V}\nabla\sigma\mathrm{V}\nabla_1\sigma_1 = \nabla\mathrm{S}.\sigma\nabla_1\sigma_1 - \mathrm{S}(\nabla_1\sigma_1\nabla)\sigma.$$

If we suppose the operations to be completed, and *then* make $x_1 = x$, $y_1 = y$, $z_1 = z$, the left-hand member must obviously vanish. So therefore must the right.

That is :— $$\nabla\mathrm{S}.\sigma\nabla_1\sigma_1 = \mathrm{S}(\nabla_1\sigma_1\nabla)\sigma\,;$$
if when the operations are complete, we put $\sigma_1 = \sigma, \nabla_1 = \nabla$.

In Cartesian co-ordinates this is equivalent to three equations, of the same type. I write only one, viz. :—
$$\frac{d}{dx}\begin{vmatrix} \xi & \eta & \zeta \\ \frac{d}{dx_1} & \frac{d}{dy_1} & \frac{d}{dz_1} \\ \xi_1 & \eta_1 & \zeta_1 \end{vmatrix} = \begin{vmatrix} \frac{d}{dx_1} & \frac{d}{dy_1} & \frac{d}{dz_1} \\ \xi_1 & \eta_1 & \zeta_1 \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \end{vmatrix}\xi,$$
if, *after* operating, we put $x_1 = x$, $\xi_1 = \xi$, &c., &c.

On a property of odd and even polygons.

By R. E. Allardice, M.A.

The property referred to comes to light on consideration of the problem, "To inscribe in a given n-gon the n-gon of minimum perimeter."

Triangle.

Let us consider first the case of the triangle (fig. 29). If ABC is the given triangle and DEF the inscribed triangle of minimum perimeter, it is obvious that we must have ∠ FDB = ∠ EDC($=\alpha$, say), ∠ DEC = ∠ FEA($=\beta$), ∠ EFA = ∠ DFB($=\gamma$). This condition is satisfied if D,E,F, are the feet of the perpendiculars from the oppo-

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