NONLINEAR ALFVÉN WAVES WITH LARGE LARMOR RADIUS EFFECT

NAGENDRA KUMAR and KRISHNA M. SRIVASTAVA
Department of Mathematics
University of Roorkee
Roorkee 247667 INDIA

The nonlinear behaviour of Alfvén waves propagating parallel to a uniform magnetic field in a compressible fluid with large Larmor radius effect is investigated. We have considered an inviscid non-heat conducting gas of infinite electrical conductivity with scale-length of interest L such that $L << \rho_i$; where ρ_i is the ion gyroradius. Its motion is governed by basic MHD equations. A reductive perturbation analysis, including weak spatial variation in the transverse direction, based upon the coordinate stretchings $\xi=\epsilon(x-\lambda t), \eta=\epsilon^{3/2}y, \zeta=\epsilon^{3/2}z$, $\tau = \epsilon^2 t$ is performed. The transverse components of magnetic field and velocity, denoted by means of the complex quantities $\tilde{V}(= V_y+iV_z)$ and $\tilde{B}(= B_y+iB_z)$, have expansions of the form $\tilde{V} = \epsilon^{1/2}(\tilde{V}^{(1)} + \epsilon\tilde{V}^{(2)} + \ldots)$ and $\tilde{B} = \epsilon^{1/2}(\tilde{B}^{(1)}+\epsilon\tilde{B}^{(2)}+\ldots)$, while the other variables have expansions $B_x = B_0+\epsilon B_x^{(1)}+\ldots$, $V_x = \epsilon V_x^{(1)}+\epsilon^2 V_x^{(2)}+\ldots$, $\rho = \rho_0+\epsilon\rho^{(1)}+\ldots$, $p = p_0+\epsilon p^{(1)}+\ldots$. Using the above stretchings and expansions, we obtain the set of equations governing the evolution of the transverse $\tilde{B}$ and parallel B_x magnetic field perturbations. If the spatial variation in the transverse direction is negligible, then we get the "Derivative Nonlinear Schrödinger (DNLS) equation"

$$\frac{\partial\tilde{B}}{\partial\tau} + C_2\frac{\partial}{\partial\xi}(|\tilde{B}|^2\tilde{B})+iC_1\frac{\partial^2\tilde{B}}{\partial\xi^2} = 0; \quad C_2 = v_A^3/(4B_o^2(v_A^2-\alpha^2))$$

The solution of the DNLS equation and its properties have been discussed by many researchers. When C_1 $(= Mcv_A^2/2eB_o) = 0$, the dispersive term in the DNLS equation vanishes and higher-order dispersive effects are required to contain the nonlinear growth of the wave. It means that the LLR effect provides a dispersive term to balance the nonlinear growth of the Alfvén wave. It is conjectured that the present calculations may be applicable to the study of the structure of MHD waves in both laboratory and space plasmas e.g. plasma flow near small planetary bodies such as comets, plasma dynamics near collisionless shock fronts.

Reference:

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