CORRESPONDENCE.

ON A TABLE FOR FACILITATING THE VALUATION OF ABSOLUTE REVERSIONS.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The accompanying Table, suggested by Mr. Sprague's Table of the Value of Life Interests (contained in the 8th volume of your *Journal*) will, I think, be found useful. Its title is sufficient explanation of the purpose for which it is intended.

The values indicated are based upon the well known formula $v-(1-v)a$, in which v is the present value of £1 due a year hence, and a the price of a whole-life annuity of £1; and it is evident that, whilst dealing with the same rate of interest, the difference between the results for any two given annuity prices consists of the difference between such prices multiplied by $(1-v)$. If a series of annuity prices be taken in arithmetical progression, the second term of the formula will form a series in like progression, causing the results of the whole expression to diminish by a constant quantity. When, therefore, the values for two prices are known, the value for any intermediate price can be found by the most simple method of interpolation.

If a be successively increased by one shilling, the series of values, starting from that corresponding to an annuity costing x pounds, will be—

	£	s.	d.	
for	x	0	0	$v-(1-v)x$
„	x	1	0	$v-(1-v)x-(1-v)\frac{1}{20}$
„	x	2	0	$v-(1-v)x-(1-v)\frac{2}{20}$
„	x	3	0	$v-(1-v)x-(1-v)\frac{3}{20}$
	. . . .			. . . .

And similarly, if the successive increase be one penny, we shall have

	£	s.	d.	
for	x	0	0	$v-(1-v)x$
,,	x	0	1	$v-(1-v)x-(1-v)\frac{1}{240}$
,,	x	0	2	$v-(1-v)x-(1-v)\frac{2}{240}$
,,	x	0	3	$v-(1-v)x-(1-v)\frac{3}{240}$
	....			....

from which it is seen that y shillings added to the annuity price diminish the value sought by $(1-v)\frac{y}{20}$, and that z pence diminish it by $(1-v)\frac{z}{240}$. Consequently the value corresponding with $\overset{£}{x}\ \overset{s.}{y}\ \overset{d.}{z}$ is

$$v-(1-v)x-(1-v)\frac{y}{20}-(1-v)\frac{z}{240}, \text{ or}$$

$$v-(1-v)\left(x+\frac{y}{20}+\frac{z}{240}\right)$$

In the Table are given, the value for each whole number of pounds which occurs in practice, and the quantity to be deducted for each possible number of shillings, and of pence; so that the means are afforded for readily obtaining the value arising from a combination. To prevent any misunderstanding in the matter, it may be well to give an example. Suppose that an absolute reversion to £1,000 cash has to be dealt with; that the Life Tenant is a male, presently aged 65; and that we desire to know what sum will just secure a purchaser 5 per cent upon the total sum which he must invest. Taking the government price of an annuity, £8. 17*s.* 10*d.*, we have

for the £8 . . .	571·4286
for the 17*s.*, to *deduct*,	40·4762
	530·9524
for the 10*d.*, to *deduct*,	1·9841
	528·9683=£528. 19*s.* 4*d.*

I need hardly remark that, in some cases, the Table is applicable where two or more persons are enjoying the proceeds of a Trust Fund.

I am, Sir,

Your obedient servant,

HENRY MOUNTCASTLE.

No. 1, *Moorgate Street, London,*
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Table for ascertaining the Value of an Absolute Reversion; so as to allow the Purchaser a given rate of interest upon his outlay; according to the price of an Annuity payable during the life of the person at whose death the Reversion will fall in.

Sum required for the purchase of an Annuity of £1 at the present Age of the Life Tenant.	Value of a Reversion of £1, assuming Interest at			
	4 per Cent.	4½ per Cent.	5 per Cent.	5½ per Cent.
£4	·807692308	·784688995	·761904762	·739336493
5	·769230769	·741626794	·714285714	·687203791
6	·730769231	·698564593	·666666667	·635071090
7	·692307692	·655502392	·619047619	·582938389
8	·653846154	·612440191	·571428571	·530805687
9	·615384615	·569377990	·523809524	·478672986
10	·576923077	·526315789	·476190476	·426540284
11	·538461538	·483253589	·428571429	·374407583
12	·500000000	·440191388	·380952381	·322274882
13	·461538462	·397129187	·333333333	·270142180
14	·423076923	·354066986	·285714286	·218009479
15	·384615385	·311004785	·238095238	·165876777
16	·346153846	·267942584	·190476190	·113744076
17	·307692308	·224880383	·142857143	·061611374
18	·269230769	·181818182	·095238095	·009478673
19	·230769231	·138755981	·047619048	
20	·192307692	·095693780		
21	·153846154	·052631579		
22	·115384615	·009569378		

Quantities to be *deducted* on account of *shillings* occurring in the price of the annuity.

1*s*.	·001923077	·002153110	·002380952	·002606635
2	·003846154	·004306220	·004761905	·005213270
3	·005769231	·006459330	·007142857	·007819905
4	·007692308	·008612440	·009523810	·010426540
5	·009615385	·010765550	·011904762	·013033175
6	·011538462	·012918660	·014285714	·015639810
7	·013461538	·015071770	·016666667	·018246445
8	·015384615	·017224880	·019047619	·020853081
9	·017307692	·019377990	·021428571	·023459716
10	·019230769	·021531100	·023809524	·026066351
11	·021153846	·023684211	·026190476	·028672986
12	·023076923	·025837321	·028571429	·031279621
13	·025000000	·027990431	·030952381	·033886256
14	·026923077	·030143541	·033333333	·036492891
15	·028846154	·032296651	·035714286	·039099526
16	·030769231	·034449761	·038095238	·041706161
17	·032692308	·036602871	·040476190	·044312796
18	·034615385	·038755981	·042857143	·046919431
19	·036538462	·040909091	·045238095	·049526066

Quantities to be *deducted* on account of *pence* occurring in the price of the annuity.

1*d.*	·000160256	·000179426	·000198413	·000217220
2	·000320513	·000358852	·000396825	·000434439
3	·000480769	·000538278	·000595238	·000651659
4	·000641026	·000717703	·000793651	·000868878
5	·000801282	·000897129	·000992063	·001086098
6	·000961538	·001076555	·001190476	·001303318
7	·001121795	·001255981	·001388889	·001520537
8	·001282051	·001435407	·001587302	·001737757
9	·001442308	·001614833	·001785714	·001954976
10	·001602564	·001794258	·001984127	·002172196
11	·001762821	·001973684	·002182540	·002389415

*** We print this modification of Orchard's tables in the form adopted by our correspondent; but we think no good purpose is served by giving more than five decimal places. In practice, it would probably be better to make the corrections *additive:* thus, taking the above example,

£8. 17*s.* 10*d.*=£9—2*s.* 2*d.*

Value for the £9=	523·809524
Add for the 2*s.*	4·761905
„ „ 2*d.*	·396825
Value, as above,	528·968254

Ed. *J. I. A.*

ON "TEN YEAR NON-FORFEITURE POLICIES."

To the Editor of the Journal of the Institute of Actuaries.

Sir,—If leisure had permitted I intended to have given in the last Number of the *Journal* a development of the suggestion contained in your foot note to my letter in the January Number, and to have looked at the American ten year non-forfeiture policies from the surrender point of view. I now propose to do this, and as all numerical results given in the present communication are based upon the Experience rate of mortality and three per cent interest, it will be advisable first to give the following recomputed values, on the same basis, of the numerical illustrations contained in my last letter.

Age at Entry.	Law of Surrender. $p_1=1,\ p_2=p_3=p_4\ \ldots\ =p_9(=p)$.				Law of Surrender. $p_1=1,\ p_2=p_3=p_4=p_5(=p),\ p_6=p_7=p_8=p_9=1$.			
	$p=0$.	$p=\frac{1}{3}$.	$p=\frac{2}{3}$.	$p=1$.	$p=0$.	$p=\frac{1}{3}$.	$p=\frac{2}{3}$.	$p=1$.
30	4·674	4·690	4·686	4·691	4·674	4·689	4·683	4·691
40	5·636	5·633	5·637	5·630	5·636	5·632	5·635	5·630
50	7·002	7·091	7·080	7·088	7·002	7·088	7·056	7·088

Each of these results denotes the annual premium per cent.

If, now, we call V_n the true cash surrender value of a policy at the end of the nth year, just before the $(n+1)$th premium becomes due, and

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