

# ESTIMATION OF GALACTIC MODEL PARAMETERS BY INTERVAL METHOD

S. A. KUTUZOV

*Saint Petersburg State University, Faculty PM-PU,  
Stary Peterhof, 198904, Saint Petersburg, Russia*

The interval method of estimating model parameters (MPs) for the Galaxy was suggested earlier (Kutuzov 1988). Intervals are proposed to be used both for observational estimates of galactic parameters (GPs) and for the values of MPs. In this work we consider a model as a tool for studying mutual interaction of GPs. Two-component model is considered (Kutuzov, Ossipkov 1989). We have to estimate the array  $\mathbf{P}$  of eight MPs.

We use the following array of seven GPs:  $\mathbf{Y} = (R_{\odot}, V, U, A, C, m, M)$ . Here  $R_{\odot}$  is the distance of the Sun from the centre of the Galaxy,  $V$  is the circular velocity,  $U$  is the velocity of escape,  $A$  is the Oort parameter,  $C$  is the Kuzmin parameter,  $m$  is the mass of the central parsec,  $M$  is the mass of the Galaxy. GPs  $V, U, A, C$  refer to vicinity of the Sun. The model allows to obtain expressions for each GP as a function of MPs:  $y_i = g_i(\mathbf{P})$ ,  $i = 1, \dots, k = 7$ . In addition two descriptive functions are used. Data on the rotation curve of the Galaxy give us estimates just of the rotation function defined as  $\Omega(x) = V(x)/x - V$ ,  $x = R/R_{\odot}$ ,  $V(x)$  is the circular velocity function. Data on the peculiar velocity dispersion as a function of  $z$  allows to estimate the vertical acceleration  $K_z(z)$ . GP  $C$  is a slope of this curve at  $z = 0$ .  $R, z$  are cylindrical coordinates. Two model functions  $g_8(\mathbf{P}, x)$  and  $g_9(\mathbf{P}, z)$  correspond to  $\Omega(x)$  and  $K_z(z)$  respectively.

Observational estimates of GPs lie within *initial intervals* with assumed limits. We ascribe the weight  $w_i$  to the  $i$ -th interval which is inversely proportional to the square of its width. Now we can describe the agreement of the model with observational data by the value of the functional

$$F(\mathbf{P}, \mathbf{Y}, \Omega, K_z) = W_1 \sum_{i=1}^k w_i [1 - g_i(\mathbf{P})/y_i]^2 +$$

$$\begin{aligned}
 & + \frac{W_2}{d_2 - c_2} \int_{c_2}^{d_2} w_2(x) [1 - g_{k+1}(\mathbf{P}, x) / \Omega(x)]^2 dx + \\
 & + \frac{W_3}{d_3 - c_3} \int_{c_3}^{d_3} w_3(z) [1 - g_{k+2}(\mathbf{P}, z) / K_z(z)]^2 dz.
 \end{aligned}$$

$W_l$  are group weights given arbitrarily,  $[c_l, d_l]$  are the ranges of corresponding functions. Weight functions  $w_2(x)$ ,  $w_3(z)$  are calculated similar to  $w_i$ . The best agreement is obtained by minimizing  $F$  with respect to  $\mathbf{P}$  taking into account the constraints on  $MPs$ . Such a problem refers to the constrained nonlinear programming. The solution depends on assumed values of  $\mathbf{Y}, \Omega, K_z$ . Taking the centres of the intervals we obtain the *central solution*. Let us vary the selected  $GP$  or all the values of one function at once taking in turn the values of left and right limits. It gives *separate intervals*. On their basis we construct *united intervals* which include all separate ones.

We take the initial intervals of  $GPs$  so that they would include most of the known observational estimates of  $GPs$  being narrow enough. The separate intervals of  $GPs$  have been calculated for three variants: 1 – equipartition of  $GPs$  and two functions, 2 and 3 – without data on vertical acceleration and on rotation respectively. We conclude that the own variation of  $GPs$  have the greatest effect on the separate intervals. The separate intervals do not extend very much beyond the initial ones. The exception occurs for  $GP m$  in the case of variants 1 and 2 only. It means that the rotation data do not allow high value of the central parsec mass.

United intervals of the function  $V(x)$  are caused most frequently by variation of  $GP M$  for the central part, but they are caused exclusively by variation of  $GP V$  for  $x > 0.2$ . The model circular velocity has two maxima and is flat enough outside the solar circle. United intervals of the function  $K_z(z)$  are caused by variations of the acceleration itself. They coincide with the initial intervals quite satisfactorily.

So, after all the  $MPs$  have been estimated, and the model can be used for calculation of galactic orbits as the potential has relatively simple analytical expression.

## References

- Kutuzov, S.A. (1988) *Kin. i fiz. neb. tel*, 4, p. 39.  
 Kutuzov, S.A. and Ossipkov, L.P. (1989) *Astron. Zhurn.*, 66, p. 965.