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ESSAYS ON STRONG AND WEAK APPROXIMATIONS OF STOCHASTIC DIFFERENTIAL EQUATIONS

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The thesis is composed of two projects on approximations of stochastic differential equations. In the first project, we present a method to construct positivity-preserving strong approximation schemes for jump-extended CEV and CIR processes where the jumps are governed by a compensated spectrally positive α-stable process with $\alpha \in (1, 2)$. To the best of our knowledge, the proposed scheme is the first of its kind, that is, a positivity-preserving scheme for α-stable-extended CEV processes, and it has the advantage that at each discretisation step, an explicit form of the scheme is available and it is given by the positive solution of a quadratic equation. We show that the proposed scheme converges and theoretically achieves a strong convergence rate of at least $\frac{1}{2}(\frac{1}{2}\alpha_- \wedge \alpha^{-1})$, which is faster than that obtained for an Euler scheme by Frikha and Li [2].

The second project is on the weak approximation and density estimates for a skew diffusion with coefficients depending on its local time at zero. In the existing literature, the parametrix method has been applied to obtain density estimates for skew diffusion processes (see Kohatsu-Higa *et al.* [3]) and, more recently, for Itô diffusion processes with coefficients depending on local time (see Frikha and Li [1]). The goal of the second project is to extend the class of processes to which one can apply the parametrix method. As our main contribution, we obtain an explicit representation and Gaussian estimates of the joint transition density, which can lead to an exact simulation method for the skew diffusion process and its local time at zero.

The structure of the thesis is as follows. In Chapter 2, we gather and present the necessary results on general stochastic processes, Lévy processes, the skew Brownian motion and skew diffusion processes that will be used in the rest of the thesis.

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In Chapter 3, we present the derivation of our positivity-preserving numerical scheme, and show in Theorem 3.16 that the strong rate of convergence in the case of an alpha-CEV process with $k > 0$ is at least $\frac{1}{2}(\frac{1}{2}\alpha_- \wedge \alpha^{-1})$. We also show in Theorem 3.25 that in the case of an alpha-CIR process, the strong rate of convergence is of polynomial order, which improves the logarithmic rate obtained in the preliminary investigation [5]. We include an appendix at the end of the chapter to collect auxiliary estimates and proofs.

In Chapter 4, we apply the parametrix method to obtain, in Theorem 4.3, a semigroup expansion of the skew diffusion and its local time at zero and give, in Theorem 4.9, a representation and a Gaussian upper estimate of the joint transition density of the skew diffusion and its local time at zero. Similar to the previous chapter, we include an appendix at the end to collect auxiliary estimates and proofs.

Finally, as future work, we propose two research projects in Chapter 5. One is on a positivity-preserving approximation scheme for a jump-extension of the *Aït–Sahalia model*, where the jumps are governed by the α-stable process. We show in Theorem 5.2 the existence and uniqueness of such an extension of the Aït–Sahalia model, and briefly discuss the techniques and difficulties in proving strong convergence of the backward Euler–Maruyama scheme. The other project aims to extend the results obtained in Chapter 4, and to apply the parametrix method to obtain density estimates for the skew diffusion, its local time at zero and its occupation time of the positive half-line.

Some of the research has been published in [4].

References

[1] N. Frikha and L. Li, 'Weak uniqueness and density estimates for SDEs with coefficients depending on some path-functionals', *Ann. Inst. Henri Poincaré Probab. Stat.* **56**(2) (2020), 1002–1040.

[2] N. Frikha and L. Li, 'Well-posedness and approximation for some one-dimensional Lévy driven non-linear SDEs', *Stochastic Process. Appl.* **132** (2021), 76–107.

[3] A. Kohatsu-Higa, D. Taguchi and J. Zhong, 'The parametrix method for skew diffusions', *Potential Anal.* **45** (2016), 299–329.

[4] L. Li and G. Liu, 'A positivity preserving numerical scheme for the mean-reverting alpha-CEV process', Preprint, 2023, arXiv:2103.13002.

[5] L. Li and D. Taguchi, 'On a positivity preserving numerical scheme for jump-extended CIR process: the alpha-stable case', *BIT* **59**(3) (2019), 747–774.

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