How to Calculate the Temperature Rise Due to Beam Heating

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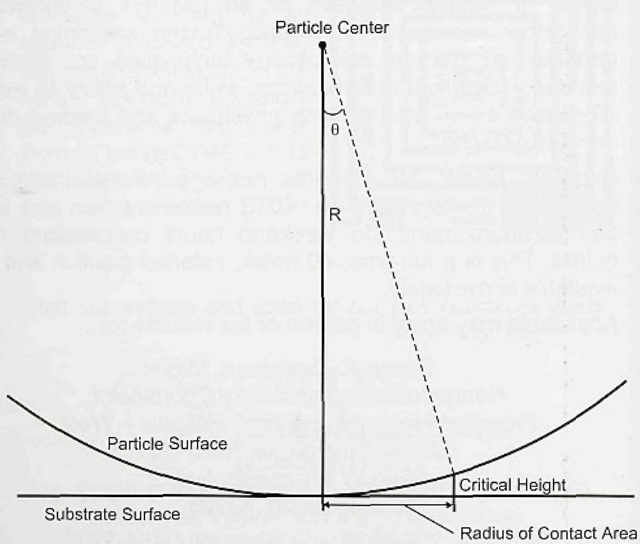
The temperature of a specimen rises when the electron beam interacts with it, producing ionization and excitation of atoms and breaking molecular bonds. Energy loss in bulk materials is ultimately converted to heat, but for small particles some of the energy escapes. Of the energy lost by the electrons in the incident beam, that which is absorbed by the specimen and degraded to heat includes oscillations of valence electrons, the kinetic energy of low-energy secondary electrons, and radiationless recombination of ionized atoms or molecules. Energy not absorbed includes brehmsstrahlung, characteristic x-rays, and the kinetic energy of higher-energy secondary electrons. Glaeser (1979) estimated that 50% of the energy loss is confined to a distance of about 5 nm from the track of the incident electron, while the other 50% is largely due to secondary electrons having 0.5 to 5 keV kinetic energy. Since the range of a 10 keV electron in nickel is about 0.5 μm (Berger and Seltzer, 1982), much of this energy is converted to heat as well. In the following, then, I will assume that any energy loss not resulting in the production of electromagnetic radiation (x-rays) will be absorbed and will contribute to heating.

In order to calculate the heat input, one needs to have the stopping power of the material for electrons of the appropriate energy. For 200 keV electrons in nickel that would be about 1.9 MeV cm^2/g (Berger and Seltzer, 1982). This figure is the collision stopping power. Using the simplifying assumption that the energy absorbed is the same for all points within the particle, the total heat input per unit time, H, is calculated by multiplying the stopping power by the density to get energy deposited per unit length for each electron, multiplying by the electron flux, then integrating over the volume, V, of the particle (equivalent to integrating along the path for each possible path, assuming that the path length is the same as that for an undeflected electron). This gives the energy deposited per electron per unit length as (8.9 g/cm^3) • (1.9 MeV cm^2 /g) = (17 MeV/cm) = (1.7 eV/nm), or H = (1.7 eV/nm) • V • J, where J is the electron flux. For a beam current of 10^3 electrons/s • nm^2, and a particle with a 1 μm radius, the input heat is $(4\pi/3) \cdot 10^9$ nm^3 • 1.7 eV/nm • 10^3 electrons/s • nm^2 = $7 \cdot 10^{12}$ eV/s.

The loss is through radiation and conduction. The former depends on the total surface area, and the latter is a function of contact area (among other variables). The particles can probably be treated as black bodies. At most a constant, k_g, would be put into the black-body equation; the particle is then a gray body. The heat radiated is $k_g \cdot \sigma \cdot A \cdot T_p^4$ for a particle of surface area A, where σ is 5.67 $\cdot 10^{-5}$ erg/s for A in cm^2 and T_p in Kelvins (*Handbook of Chemistry and Physics*). Of course, the particle also absorbs radiation at the rate $k_g \cdot \sigma \cdot A \cdot T_A^4$, where T_A is the ambient temperature, so the net radiation loss is $k_g \cdot \sigma \cdot A \cdot (T_p^4 - T_A^4)$. For k_g = 1 and a particle with a radius of 1 μm at T_A = 300 K, the radiative heat loss at the melting temperature of nickel (1726 K) is 63 erg/s or $4.0 \cdot 10^{13}$ eV/s.



The conductive loss is more difficult to calculate. Assuming that any part of the particle closer to the substrate surface than some critical height, h_c will be in contact with the substrate -- *i.e.,* thermal vibrations of atoms of the particle can be efficiently transmitted to atoms of the substrate -- the contact area for a particle of radius R can be calculated. The geometry is shown in the figure. Letting the radius of the contact area be R_c, we have, with very little work, $R_c = R \cdot \sin\theta$ and $h_c = R(1-\cos\theta)$. Eliminating θ gives $R_c = (2Rh_c-h_c^2)^{1/2}$. The contact area, A_c , is $A_c = \pi(2Rh_c-h_c^2)$.

If the contact is with a thick enough substrate (as in the SEM), the conductive loss will be proportional to the contact area, but if the substrate is too thin (as in the TEM), the heat conducted into it will have to diffuse away from the particle, and the loss will be proportional to the circumference of the contact area times the film thickness. Guessing that h_c is about the sum of the Van der Waals radii of the atoms of the particle and substrate (read off of any chart which has bars representing atomic sizes on it, such as the NSRDS (National Standard Reference Data System) chart, 1979 revised edition), and hoping that the effects of surface roughness will not be significant, this gives for a 1 μm nickel particle on aluminum, $A_c = 4 \cdot 10^3$ nm^2, and for the same particle on a carbon film, the circumference of the contact area is $2 \cdot 10^2$ nm, and for a 1 nm film thickness, the area through which heat is conducted is $2 \cdot 10^2$ nm^2.

The conductive loss is $H_c = K \cdot A_c \cdot dT/dz$, where K is the thermal conductivity and A_c is the appropriate area. If the center of the particle is at a temperature T_p and the substrate is at ambient temperature ($T_s = T_A$), the thermal gradient can be approximated as linear, giving $H_c = K \cdot A_c \cdot (T_p - T_s)/R$. The thermal conductivity of carbon varies greatly with its form. Using the value for graphite, and the better defined values for the conductivities of nickel and aluminum (*Handbook of Chemistry and Physics*), and taking T_s = 300 K and T_p = 1726 K, for the usual 1 μm nickel particle the conductive heat loss is about 0.6 mW for the aluminum substrate and about 30 μW for the carbon substrate. This converts to about $4 \cdot 10^{15}$ eV/s for aluminum and $2 \cdot 10^{13}$ eV/s for carbon. Although several questionable assumptions were made in this derivation, the conductive loss is predicted to be larger than the radiative loss for contact with a bulk substrate, and about equal for contact with a thin film under the assumed conditions. Both losses are predicted to be larger than the heat input, which implies that for the example beam current, the temperature of a 1 μm particle would not approach the melting temperature of nickel.

As the radius of the particle increases, the heat input rises faster (∝ R3) than either loss process, and radiative loss rises faster (∝ R2) than conductive loss, which rises very slowly (∝ R or $R^{1/2}$). This predicts that a nickel particle with a radius of several micrometers on a thin carbon substrate will melt in an electron beam whose flux is 10^3 electrons/s • nm^2.

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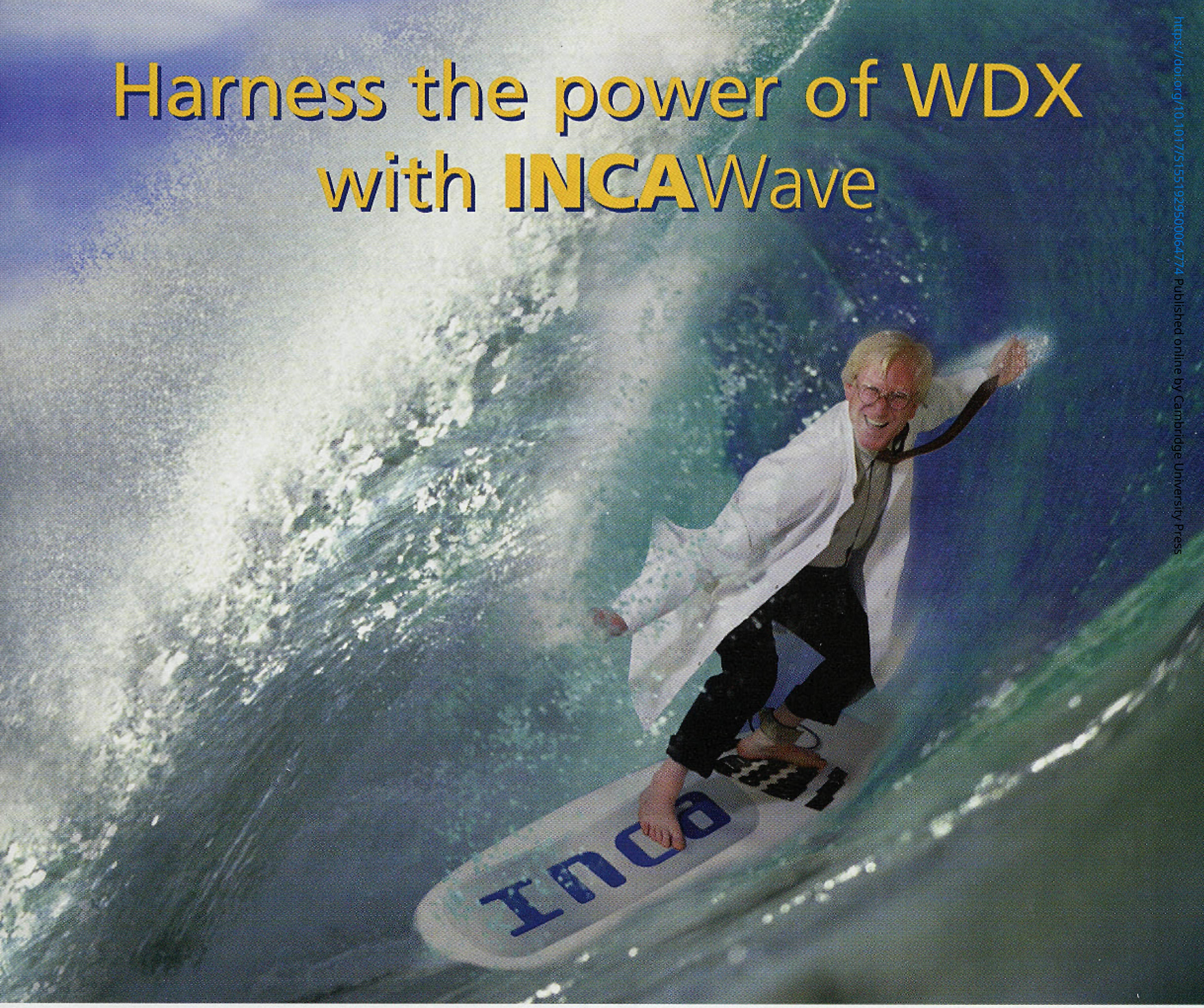
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As the beam current increases, the heat input increases proportionally. The conductive loss increases in proportion to the temperature rise, which in turn is proportional to the heat input, so the steady-state temperature should be linear with the beam current for the case where radiative heat loss is negligible. Radiative loss, however, is a steeply increasing function of temperature, so when radiation is the dominant loss process, the temperature increases only very slowly with beam current. This predicts that an electron flux of 10^4 electrons/s • nm^2 would melt a 1 µm nickel particle on a thin carbon substrate. Melting a nickel particle on an aluminum substrate would require a much larger particle and/or beam current (unless the thermal contact between particle and substrate was so poor that conduction losses were much less than I've calculated).

Finally to calculate the steady-state temperature of a particle under the electron beam, use the fact that at steady state the heat input is equal to the heat lost.

$$P \left.\frac{dE}{dx}\right|_{col} \frac{4\pi}{3} R^3 J = k_g \sigma 4\pi R^2 (T^4_p - T^4_s) + K A_c \frac{T_p - T_s}{R}$$

Where: p is the density

$\left.\frac{dE}{dx}\right|_{col}$ is the collision stopping power

k_g is a "gray body" constant

σ is the Stefan-Boltzmann constant

K is the thermal conductivity

A_c is the appropriate contact area in nm^2

R is in nm

J is in electrons/nm^2

T_s is in Kelvins

Note that the heat capacity of the particle does not enter the equation for the steady-state temperature. ■

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