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Frequency shifts of resonant modes of the Sun due to near-surface convective scattering

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Abstract. Measurements of oscillation frequencies of the Sun and stars can provide important independent constraints on their internal structure and dynamics. Seismic models of these oscillations are used to connect structure and rotation of the star to its resonant frequencies, which are then compared with observations, the goal being that of minimizing the difference between the two. Even in the case of the Sun, for which structure models are highly tuned, observed frequencies show systematic deviations from modeled frequencies, a phenomenon referred to as the "surface term." The dominant source of this systematic effect is thought to be vigorous near-surface convection, which is not well accounted for in both stellar modeling and mode-oscillation physics. Here we bring to bear the method of homogenization, applicable in the asymptotic limit of large wavelengths (in comparison to the correlation scale of convection), to characterize the effect of small-scale surface convection on resonant-mode frequencies in the Sun. We show that the full oscillation equations, in the presence of temporally stationary 3D flows, can be reduced to an effective "quiet-Sun" wave equation with altered sound speed, Brünt–Väisäla frequency, and Lamb frequency. We derive the modified equation and relations for the appropriate averaging of 3D flows and thermal quantities to obtain the properties of this effective medium. Using flows obtained from 3D numerical simulations of near-surface convection, we quantify their effect on solar oscillation frequencies and find that they are shifted systematically and substantially. We argue therefore that consistent interpretations of resonant frequencies must include modifications to the wave equation that effectively capture the impact of vigorous hydrodynamic convection.

1. Introduction

Measurements of oscillation frequencies of stars provide some of the strongest constraints on stellar structure models. Recent missions Kepler (Borucki *et al.* 2010) and CoRoT (Auvergne *et al.* 2009) have provided us with high-quality asteroseismic data, which have revealed structural features of Sun-like stars in unprecedented detail. Despite the high precision in measured frequencies, the modeled frequencies display systematic deviations from the observed ones. There are various reasons why these shifts are observed, which were categorized as model and modal effects by (Rosenthal *et al.* 1999) . Some of the possible sources for this bias are as follows:

(*a*) Errors in physics that go into the modeling, including turbulent-pressure contributions (model effect).

(*b*) Non-adiabaticity prevalent in the outer layers of the Sun (model+modal effect).

(*c*) Imprecise modeling of propagation of waves through convective flows near the surface of the Sun (modal effect).

(*d*) Nonlinearity in the wave equation (modal effect).

In this paper, we show that advection of traveling waves by these flows can have a significant impact on the resonant-mode frequencies, and it is therefore important to account for this effect in order to extract the correct physics from measurements.

An early attempt at mitigating the frequency differences was by Gough (1990), whose prescription was in the form of a power-law corrections. Kjeldsen *et al.* (2008) had studied the shifts in the Sun, and with an empirical power law model $\delta\nu = a\left(\nu/\nu_0\right)^b$, were able to correct for the shifts in α Cen A, α Cen B and β Hyi. Ball & Gizon (2014) have shown that a combination of the two effects studied by Gough — the cubic shift and the inverse shift — is able to fit the deviations better than Kjeldsen's power law. These corrections are satisfactory from a model-fitting viewpoint, but by eliminating the frequency differences altogether, we lose information about their origin.

Several authors such as Murawski & Roberts (1993); Duvall *et al.* (1998) had studied the effect of random scatterings by convective flows on surface gravity waves (f-modes), thereby correcting the dispersion relation. Brown (1984) had suggested a correction of the form $\left(1 - \langle v_z^2\rangle/c^2\right)$ to p-mode speeds, by carrying out a horizontal average over the wave equation. An asymptotic analysis of p-mode frequency shifts for small Mach numbers had been carried out by Stix & Zhugzhda (2004), but whether the analysis extends to transonic flows near the solar surface remains unclear.

Despite the high flow speeds, there still exists a small parameter in the wave equation — the granulation length scales are usually much smaller than horizontal wavelengths of low-ℓ modes, which are of the order of the solar radius. Hanasoge *et al.* (2013) had described a perturbative approach by expanding the wave equation in terms of the ratio of the two length scales, followed by averaging over small scales. In this paper, we employ a similar technique to reduce the spatially varying flows to a set of equivalent homogeneous parameters, thereby computing the shifted frequencies and resulting differences.

2. Spatial Homogenization

Low-degree resonant modes on the sun have horizontal wavelengths (λ) comparable to the solar radius, which is much larger than the typical length scales of granulation on the Sun (L). The ratio of scales $\varepsilon = L/\lambda$ allows us to isolate the dynamics at each scale. To study the dynamics in the asymptotic limit $\varepsilon \ll 1$, we introduce two spatial coordinates that are scale separated: a slow coordinate $\mathbf{x} = (x, y, z)$ and a horizontal fast coordinate $\mathbf{x}'_h = (x/\varepsilon, y/\varepsilon)$. Note that the vertical component of mode wavelength can be comparable to the scale of flows, so the advection terms might not be scale separable vertically. Bearing this in mind, scale separation is being carried out strictly in the horizontal directions, labeled by $x - y$ or equivalently by $\mathbf{x}'_h$. In terms of these new coordinates, the spatial gradient can be rewritten as

$$\boldsymbol{\nabla} = \boldsymbol{\nabla}_{\mathbf{x}} + 1/\varepsilon \boldsymbol{\nabla}_{\mathbf{h}}. \tag{2.1}$$

We assume a static background medium characterized by its pressure $p\left(\mathbf{x}\right)$, density $\rho\left(\mathbf{x}\right)$ and sound speed $c\left(\mathbf{x}\right)$. In presence of a flow field $\mathbf{u}\left(\mathbf{x}\right)$, the material derivative of a displacement $\boldsymbol{\xi}\left(\mathbf{x}, t\right)$ is defined as

$$\dot{\boldsymbol{\xi}} = \partial_t \boldsymbol{\xi} + \mathbf{u} \cdot \boldsymbol{\nabla} \boldsymbol{\xi}. \tag{2.2}$$

The acoustic wave equation, to linear order in the perturbations, is given by

$$\partial_t \left(\rho \dot{\boldsymbol{\xi}}\right) + \boldsymbol{\nabla} \cdot \left\{\rho \mathbf{u} \dot{\boldsymbol{\xi}} + \mathbf{I}\left(p - \rho c^2\right) \boldsymbol{\nabla} \cdot \boldsymbol{\xi} - p \left(\boldsymbol{\nabla} \boldsymbol{\xi}\right)^T\right\} + \rho \boldsymbol{\xi} \cdot \boldsymbol{\nabla}\boldsymbol{\nabla} \phi = \mathbf{0}, \tag{2.3}$$

where T in the exponent denotes the transpose operator (Webb *et al.* 2005) We expand the wave field $\boldsymbol{\xi}$ as a perturbative series as

$$\boldsymbol{\xi}\left(\mathbf{x}, \mathbf{x}'_h, t\right) = \boldsymbol{\xi_0}\left(\mathbf{x}, \mathbf{x}'_h, t\right) + \varepsilon \boldsymbol{\xi_1}\left(\mathbf{x}, \mathbf{x}'_h, t\right) + \varepsilon^2 \boldsymbol{\xi_2}\left(\mathbf{x}, \mathbf{x}'_h, t\right) + \mathcal{O}\left(\varepsilon^3\right). \tag{2.4}$$

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We substitute Eq. (2.4) in Eq. (2.3) and use Eq. (2.2) to write down the equations order by order in ε. We introduce the following tensors and tensorial relations:

- The double-dot product of two tensors $\mathbf{A}$ and $\mathbf{B}$ defined as $\mathbf{A} : \mathbf{B} = A_{\dots ij} B_{ij\dots}$, where repeated indices are summed over.
- A fourth-order tensor $\tilde{\mathbf{T}}$ defined as $\tilde{\mathbf{T}}_{ijkl} = \delta_{il}\delta_{jk}$.
- A fourth-rank tensor $\tilde{\mathbf{I}}_4$, defined as $\tilde{\mathbf{I}}_{4ijkl} = \delta_{ik}\delta_{jl}$
- A fourth-rank tensor $\tilde{\mathbf{\Gamma}}$, defined as

$$\tilde{\mathbf{\Gamma}} = \rho\left(\mathbf{uu}\cdot\tilde{\mathbf{I}}_4 - c^2\mathbf{II}\right) + p\left(\mathbf{II} - \tilde{\mathbf{T}}\right) \tag{2.5}$$

At order ε^{-2} we obtain

$$\mathcal{L}^{(-2)}\boldsymbol{\xi}_\mathbf{0} + \rho\boldsymbol{\xi}_\mathbf{0}\cdot\mathbf{\nabla_h}\mathbf{\nabla_h}\phi = \mathbf{0}, \tag{2.6}$$

where

$$\mathcal{L}^{(-2)}\boldsymbol{\xi} = \mathbf{\nabla_h}\cdot\left(\tilde{\mathbf{\Gamma}} : \mathbf{\nabla_h}\,\boldsymbol{\xi}\right). \tag{2.7}$$

We shall assume henceforth that the gravitational potential ϕ is independent of the fast coordinate $\mathbf{x}'_h$, and drop any term like $\mathbf{\nabla_h}\phi$ or $\mathbf{\nabla_h}\mathbf{\nabla_x}\phi$. Under this assumption, Eq. (2.6) becomes

$$\mathcal{L}^{(-2)}\boldsymbol{\xi}_\mathbf{0} = \mathbf{0}. \tag{2.8}$$

A particular solution to Eq. (2.8) is a zeroth-order field $\boldsymbol{\xi}_\mathbf{0}$ that is independent of the fast coordinate $\mathbf{x}'_h$. This assumption fits into our picture of perturbations about a homogeneous background, and we shall proceed under it.

Collecting terms of the order ε^{-1}, we obtain

$$\mathcal{L}^{(-2)}\boldsymbol{\xi}_\mathbf{1} + \mathcal{L}^{(-1)}\boldsymbol{\xi}_\mathbf{0} = \mathbf{0}, \tag{2.9}$$

where

$$\mathcal{L}^{(-1)}\boldsymbol{\xi} = \mathbf{\nabla_h}\cdot\left(\tilde{\mathbf{\Gamma}} : \mathbf{\nabla_x}\,\boldsymbol{\xi}\right), \tag{2.10}$$

Following Bensoussan *et al.* (1978), we substitute

$$\boldsymbol{\xi}_\mathbf{1} = \tilde{\mathbf{J}}\left(\mathbf{x}'_h, z\right) : \mathbf{\nabla_x}\,\boldsymbol{\xi}_\mathbf{0} \tag{2.11}$$

where $\tilde{\mathbf{J}}\left(\mathbf{x}'_h, z\right)$ is a third-rank tensor, and demand that $\tilde{\mathbf{J}}$ satisfies

$$\mathcal{L}^{(-2)}\tilde{\mathbf{J}} = -\mathbf{\nabla_h}\cdot\tilde{\mathbf{\Gamma}}. \tag{2.12}$$

At order ε^0 we get the equation of motion for the zeroth-order field $\boldsymbol{\xi}_\mathbf{0}$

$$\begin{aligned}\rho\,\partial_t^2\boldsymbol{\xi}_\mathbf{0} + \rho\mathbf{u}\cdot\partial_t\left(\mathbf{\nabla_x}\,\boldsymbol{\xi}_\mathbf{0} + \mathbf{\nabla_h}\,\boldsymbol{\xi}_\mathbf{1}\right) + \mathcal{L}^{(0)}\boldsymbol{\xi}_\mathbf{0} + \mathcal{L}^{(1)}\boldsymbol{\xi}_\mathbf{1} & \\ +\mathcal{L}^{(-1)}\boldsymbol{\xi}_\mathbf{1} + \mathcal{L}^{(-2)}\boldsymbol{\xi}_\mathbf{2} + \rho\,\boldsymbol{\xi}_\mathbf{0}\cdot\mathbf{\nabla_x}\mathbf{\nabla_x}\phi &= \mathbf{0},\end{aligned} \tag{2.13}$$

where

$$\mathcal{L}^{(0)}\boldsymbol{\xi} = \mathbf{\nabla_x}\cdot\left\{\tilde{\mathbf{\Gamma}} : \mathbf{\nabla_x}\,\boldsymbol{\xi}\right\}, \quad \mathcal{L}^{(1)}\boldsymbol{\xi} = \mathbf{\nabla_x}\cdot\left\{\tilde{\mathbf{\Gamma}} : \mathbf{\nabla_h}\,\boldsymbol{\xi}\right\}.$$

We substitute (2.11) in Eq. (2.13) and average over the fast coordinate $\mathbf{x}'_h$ to obtain

$$\langle\rho\rangle\partial_t^2\boldsymbol{\xi}_\mathbf{0} + 2\langle\rho\mathbf{u}\rangle\cdot\mathbf{\nabla_x}\,\partial_t\,\boldsymbol{\xi}_\mathbf{0} - \mathbf{\nabla_x}\cdot\left[\langle\rho\tilde{\mathbf{C}}\rangle : \mathbf{\nabla_x}\boldsymbol{\xi}_\mathbf{0}\right] + \boldsymbol{\xi}_\mathbf{0}\cdot\langle\rho\mathbf{\nabla_x}\mathbf{\nabla_x}\phi\rangle = 0, \tag{2.14}$$

where

$$\tilde{\mathbf{C}} = -\frac{1}{\rho}\tilde{\mathbf{\Gamma}} : \left(\tilde{\mathbf{I}}_4 + \mathbf{\nabla_h}\tilde{\mathbf{J}}\right) \tag{2.15}$$

resembles a wave speed squared term that governs wave propagation on large scales. Further assuming that the flow is antisymmetric about the cell center, we set $\langle\rho\mathbf{u}\rangle = \mathbf{0}$

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and obtain

$$\langle\rho\rangle\partial_t^2\boldsymbol{\xi}_\mathbf{0} - \boldsymbol{\nabla}_\mathbf{x}\cdot\left[\langle\rho\tilde{\mathbf{C}}\rangle : \boldsymbol{\nabla}_\mathbf{x}\,\boldsymbol{\xi}_\mathbf{0}\right] + \boldsymbol{\xi}_\mathbf{0}\cdot\langle\rho\boldsymbol{\nabla}_\mathbf{x}\boldsymbol{\nabla}_\mathbf{x}\phi\rangle = \mathbf{0}. \tag{2.16}$$

Eq. (2.16) is the homogenized wave equation, wherein we have replaced the spatially fluctuating background medium by an effective homogeneous, anisotropic one.

3. Impact on solar frequencies

In this section, we start from Model S (Christensen-Dalsgaard *et al.* 1996) which uses the mixing-length formulation of convective flux. We correct wave speeds using flow profiles obtained from simulations by Schüssler (private communication) using the MURaM code (Vögler *et al.* 2005). The flow-speed profiles have been plotted in Figure 1. We start from Eq. (2.16) and work with low-degree modes ($\ell \leqslant 3$), which are predominantly radial. We assume that the Mach number is low enough, and retain only the dominant component of $\tilde{\mathbf{C}}$ which goes roughly as $c^2 - u^2$. The exact form of u that goes into the correction depends on the direction of advection at every depth; for our purposes we shall restrict ourselves to the special cases of radial flow $\left(\mathbf{u} = \mathbf{u}_\parallel\right)$ and tangential flow $(\mathbf{u} = \mathbf{u}_\perp)$. We refer to this modified model as Model Hom, and we compute the eigenfrequencies corresponding to this model assuming the same boundary conditions as Model S. We compare these frequencies with observed solar frequencies obtained by the Birmingham Solar Oscillation Network (BiSON, Chaplin *et al.* 2007). We find that the presence of flow advection in the wave equation leads to a reduction in mode frequencies, which reduces the observed difference between observed and Model S frequencies.

To study the impact of modal versus model corrections on frequencies, we compare the Model Hom frequencies with another hydrostatic model (Basu & Antia 1994, henceforth called Model BA), which uses the Canuto-Mazzitelli formulation of convective flux (Canuto & Mazzitelli 1991). The frequency differences between these models and the observations by BiSON have been plotted in Figure 2. We find that the modal frequency shifts due to the presence of advective flows are comparable to those obtained from modeling differences. Both the effects need to be looked into while fitting solar frequencies.

4. Conclusion

Observed solar oscillation frequencies can differ from modeled frequencies because of an inaccurate background model, as well as incomplete description of mode physics. The presence of near-surface convective flows on the Sun results in mode frequencies that are different from ones computed starting from a quiet background. Three-dimensional convection simulations generally result in an increased size of the acoustic cavity, thereby reducing the frequencies (Rosenthal *et al.* 1999; Piau *et al.* 2014). In this paper, we have shown that the impact of advection on waves might result in frequency shifts of similar magnitudes. In the presence of flows, the appropriate wave speed is not the horizontally averaged sound speed; rather, it is a combination of sound speed and an appropriate projection of the flow speed in the direction of propagation.

The impact of time evolution of flows, missing in the present analysis, needs to be studied in detail. The spatial scales of waves and background flows are significantly different, thereby allowing a two-scale analysis. The timescales of granule evolution are, however, not very different from time periods of the wave. Overlap of scales generally results in strong scattering and forcing; thus, a picture of flows evolving in time might be necessary to capture frequency shifts and damping. Added to this are the impacts of super-adiabaticity and magnetic fields, which have been ignored in the present analysis

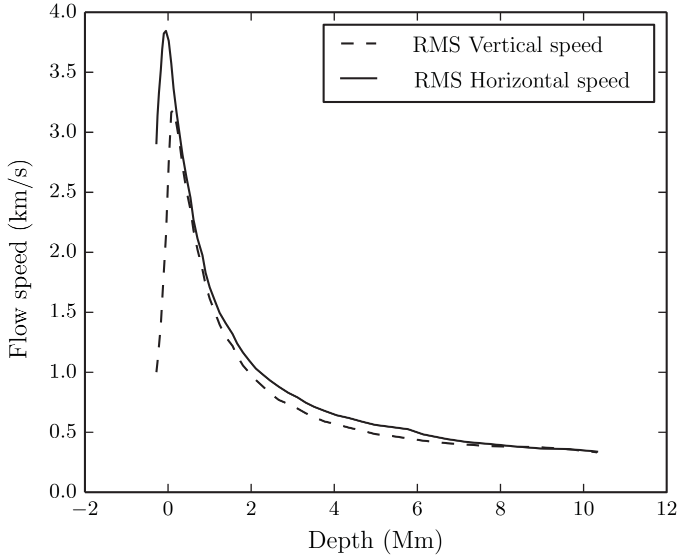

Figure 1. Root-mean-square flow speeds obtained from three-dimensional simulation of the near-surface layers of the Sun (Vögler *et al.* 2005).

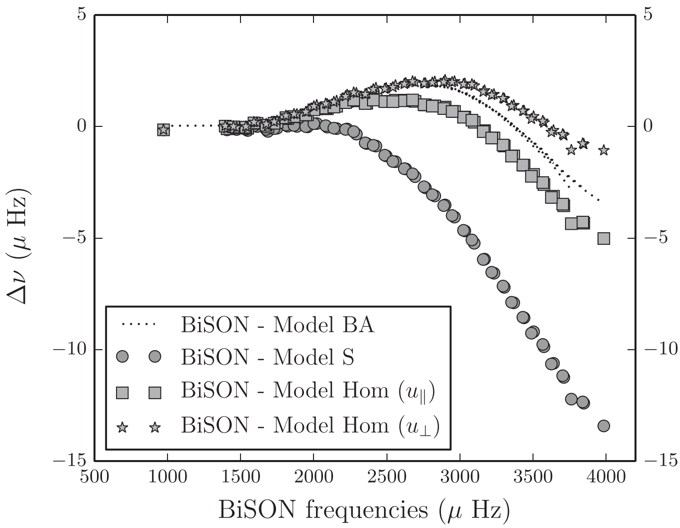

Figure 2. Eigenfrequencies corresponding to Model S (Christensen-Dalsgaard *et al.* 1996, circles) compared with the modified models. The squares indicate a model where advection is predominantly radial and the stars indicate a model where advection is predominantly tangential. The dotted line represents eigenfrequencies corresponding to Model BA (Basu & Antia 1994). We find that the modified models display frequency shifts comparable to Model BA, which shows that modal effects can be significant near the surface and need to be taken into account.

but might be important in governing the spectrum. Nevertheless, the present analysis provides a useful starting point to model wave propagation through convective flows and resultant frequency shifts.

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