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Some classes of monomial groups

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This thesis is concerned with nM-groups (and sM-groups), finite groups whose complex irreducible characters are all induced from linear characters of normal (subnormal) subgroups. By a classical theorem of Taketa, all such groups are solvable. Our aim is to find group theoretic properties of these groups (that is, properties which are not defined in terms of characters). Isaacs and Passman proved that all metabelian groups are nM-groups. We show that all abelian by nilpotent groups are sM-groups. The class of all nM-groups (or sM-groups) is closed under taking factor groups, direct products, or normal Hall-subgroups. Normal subgroups and subdirect products of nM-groups need not be nM-groups, but all subgroups of nM-groups are sM-groups. The corresponding questions concerning sM-groups are still open.

We prove that if K/L is a complemented chief factor of an sM-group G, then all elements of K/L have subnormal centralizers in G. The p-length of an sM-group is at most 1, for each prime p. All subgroups of G are sM-groups if and only if all chief factors of G (not only the complemented ones) satisfy the subnormal centralizer condition mentioned above, and every non-nilpotent section of G has a non-central minimal normal subgroup.

A (finite solvable) group G is an nM-group if and only if all its factor groups H satisfy the following condition: if A is an abelian normal subgroup of maximal order in H, if g is an element of H outside A, and C a subgroup of A such that A/C is cyclic, g

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normalizes C and acts trivially on A/C, then C must contain some non-trivial normal subgroup of H. If G is an nM-group, then each element of G acts on each chief factor of G either trivially or fixed point free; all subgroups of G are sM-groups; the Frattini factor group of G is a subdirect product of cyclic groups and of Frobenius groups whose kernels are abelian and whose complements have cyclic derived groups; the Fitting factor group G/F of G is metabelian, supersolvable, and the odd order Sylow subgroups of G/F are abelian. These conclusions say nothing when G is a p-group; all we can do is to present examples which show, for each prime p, that there exist non-metabelian p-groups which are nM-groups, but not all p-groups are nM-groups.

An A-group is a (finite solvable) group whose Sylow subgroups are all abelian. We determine precisely which A-groups are nM-groups or sM-groups. In particular, an A-group is an nM-group if and only if it is a subdirect product of Frobenius groups. The class of those A-groups which are nM-groups (sM-groups) is closed under taking subgroups, factor groups, direct products; if the Frattini factor group of an A-group is in this class, so is the group.

We construct an A-group of derived length 5 which is an sM-group. It should be possible to build sM-groups of arbitrary nilpotent length by the same method.