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DOMINATION BY POSITIVE WEAK* DUNFORD–PETTIS OPERATORS ON BANACH LATTICES

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Abstract

Recently, H'michane *et al.* ['On the class of limited operators', *Acta Math. Sci.* (submitted)] introduced the class of weak* Dunford–Pettis operators on Banach spaces, that is, operators which send weakly compact sets onto limited sets. In this paper, the domination problem for weak* Dunford–Pettis operators is considered. Let $S, T : E \to F$ be two positive operators between Banach lattices E and F such that $0 \leq S \leq T$. We show that if T is a weak* Dunford–Pettis operator and F is σ-Dedekind complete, then S itself is weak* Dunford–Pettis.

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1. Introduction

Throughout this paper X, Y are Banach spaces and E, F are Banach lattices. We write sol(A) for the solid hull of a subset A of a Banach lattice. The positive cone of E is denoted by E^+. Following Andrews [3] (or Bourgain and Diestel [5]), we say that a norm bounded subset A of X is a *Dunford–Pettis set* (respectively, a *limited set*) whenever every weakly null sequence in X^* (respectively, weak* null sequence in X^*) converges uniformly to zero on A. Clearly, every relatively compact set in X is a limited set, and every limited set in X is a Dunford–Pettis set, but the converses are not true in general. Let us recall that a linear operator $T : X \to Y$ is called a *Dunford–Pettis operator* if $x_n \xrightarrow{w} 0$ in X implies $\|Tx_n\| \to 0$ or, equivalently, if T carries relatively weakly compact subsets of X onto relatively compact subsets of Y. Aliprantis and Burkinshaw [1] introduced a class of operators related to the Dunford–Pettis operators, the so-called weak Dunford–Pettis operators. A bounded linear operator $T : X \to Y$ between Banach spaces is said to be a *weak Dunford–Pettis operator* whenever $x_n \xrightarrow{w} 0$

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in X and $f_n \xrightarrow{w} 0$ in Y^* imply $\lim_n f_n(Tx_n) = 0$ or, equivalently, whenever T carries relatively weakly compact subsets of X onto Dunford–Pettis subsets of Y.

Recently, H'michane *et al.* [11] introduced the class of weak* Dunford–Pettis operators, and characterised this class of operators and studied some of its properties in [10]. Following H'michane *et al.* [11], we say that a bounded linear operator $T : X \to Y$ is a *weak* Dunford–Pettis operator* whenever $x_n \xrightarrow{w} 0$ in X and $f_n \xrightarrow{w^*} 0$ in Y^* imply $f_n(Tx_n) \to 0$ or, equivalently, whenever T carries relatively weakly compact subsets of X onto limited subsets of Y [10, Theorem 3.2].

Recall that in the literature the domination problem for a class C of operators acting between Banach lattices is stated as follows:

- Let $S, T : E \to F$ be two positive operators between Banach lattices such that $0 \le S \le T$. Assume that T belongs to the class C. Which conditions on E and F ensure that S belongs to C?

In [12], Kalton and Saab established that a positive operator from E into F, dominated by a positive weak Dunford–Pettis operator, must also be weak Dunford–Pettis, and they obtained such a result for Dunford–Pettis operators provided the norm of F is order continuous. Later, Wickstead [14] studied the converse for the Kalton–Saab theorem: every positive operator from E into F dominated by a Dunford–Pettis operator is Dunford–Pettis if and only if E has weakly sequentially continuous lattice operations or F has order continuous norm.

Naturally, we come to the case of weak* Dunford–Pettis operators. The main purpose of this paper is to study the domination problem for positive weak* Dunford–Pettis operators between Banach lattices. Let $S : E \to F$ be a positive operator between Banach lattices E and F such that F is σ-Dedekind complete. We show that if S is dominated by a positive weak* Dunford–Pettis operator, then S itself is weak* Dunford–Pettis.

Our notions are standard. For the theory of Banach lattices and positive operators, we refer the reader to the monographs [2, 13].

2. Lattice properties of positive weak* Dunford–Pettis operators

It should be noted that in a Banach lattice (or in its dual) the lattice operations fail to be weakly (respectively, weak*) sequentially continuous in general. Let us recall that every disjoint sequence in the solid hull of a relatively weakly compact subset of a Banach lattice E converges weakly to zero (see [2, Theorem 4.34]). In particular, if (x_n) is a disjoint, weakly convergent sequence in E, then the sequences (x_n), $(|x_n|)$, (x_n^+), (x_n^-) all converge weakly to zero. However, from [9, Example 2.1], we can see that such a property need not be possessed by w^*-convergent disjoint sequences in the dual space.

The following lemma, which deals with disjoint sequences in the dual of a σ-Dedekind complete Banach lattice, is due to the authors of [9] and is needed in the rest of this paper.

LEMMA 2.1 [9]. *Let E be a σ-Dedekind complete Banach lattice and let (f_n) be a w^*-convergent sequence of E^*. If (g_n) is a disjoint sequence of E^* satisfying $|g_n| \le |f_n|$ for each $n \in \mathbb{N}$, then the sequences (g_n), $(|g_n|)$, (g_n^+), (g_n^-) all weak* converge to zero. In particular, if (f_n) is a disjoint w^*-convergent sequence in its own right, then the sequences (f_n), $(|f_n|)$, (f_n^+), (f_n^-) are all weak* null.*

Let $T : E \to F$ be a positive weak* Dunford–Pettis operator between Banach lattices. For every weakly null sequence (x_n) in E^+ and every weak* null sequence (f_n) in F^*, by the definition of weak* Dunford–Pettis operators we have $f_n(Tx_n) \to 0$. Indeed, we can say more when F is σ-Dedekind complete.

THEOREM 2.2. *Let $T : E \to F$ be a positive weak* Dunford–Pettis operator between Banach lattices E and F with F σ-Dedekind complete. Then, for every weakly null sequence (x_n) in E^+ and every weak* null sequence (f_n) in F^*, we have $|f_n|(Tx_n) \to 0$ (as $n \to \infty$).*

PROOF. Let $\varepsilon > 0$ be arbitrary. First, we claim that there exist $0 \le g \in F^*$ and $N \in \mathbb{N}$ such that

$$(|f_n| - g)^+(Tx_n) < \varepsilon \qquad (*)$$

holds for all $n > N$. Suppose that $(*)$ is false. Then there exists an $\varepsilon_0 > 0$ such that for each $0 \le g \in F^*$ and each $N \in \mathbb{N}$, we have $(|f_k| - g)^+(Tx_k) \ge \varepsilon_0$ for at least one $k > N$. Let us put $g = 4|f_1|$ and $n_1 = 1$. Thus, there exists a natural number $n_2 > n_1$ satisfying

$$(|f_{n_2}| - 4|f_1|)^+(Tx_{n_2}) \ge \varepsilon_0.$$

Also, let us put $g = 4^2 \sum_{i=1}^{2} |f_{n_i}|$. Then

$$\Big(|f_{n_3}| - 4^2 \sum_{i=1}^{2} |f_{n_i}|\Big)^+ (Tx_{n_3}) \ge \varepsilon_0$$

for some natural number $n_3 > n_2$. Proceeding with an inductive argument, we can obtain a strictly increasing subsequence (n_k) of $\mathbb{N}$ such that

$$\Big(|f_{n_{k+1}}| - 4^k \sum_{i=1}^{k} |f_{n_i}|\Big)^+ (Tx_{n_{k+1}}) \ge \varepsilon_0$$

for all $k \in \mathbb{N}$. Let $f = \sum_{k=1}^{\infty} 2^{-k}|f_{n_k}|$ and put

$$g_{k+1} = \Big(|f_{n_{k+1}}| - 4^k \sum_{i=1}^{k} |f_{n_i}|\Big)^+, \quad \tilde{f}_{k+1} = \Big(|f_{n_{k+1}}| - 4^k \sum_{i=1}^{k} |f_{n_i}| - 2^{-k} f\Big)^+.$$

Note that $0 \le g_{k+1} \le \tilde{f}_{k+1} + 2^{-k} f$ and $g_{k+1}(Tx_{n_{k+1}}) \ge \varepsilon_0$ for any $k \in \mathbb{N}$. By [2, Lemma 4.35], $(\tilde{f}_{k+1})$ is a disjoint sequence. Since $0 \le \tilde{f}_{k+1} \le |f_{n_{k+1}}|$ and $f_{n_{k+1}} \xrightarrow{w^*} 0$, in view of Lemma 2.1 we have $\tilde{f}_{k+1} \xrightarrow{w^*} 0$ in F^*. From the weak* Dunford–Pettis property of T,

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it follows that $\tilde{f}_{k+1}(Tx_{n_{k+1}}) \to 0$. However,

$$\begin{aligned}0 < \varepsilon_0 \leq g_{k+1}(Tx_{n_{k+1}}) &\leq (\tilde{f}_{k+1} + 2^{-k} f)(Tx_{n_{k+1}}) \\ &= \tilde{f}_{k+1}(Tx_{n_{k+1}}) + 2^{-k} f(Tx_{n_{k+1}}) \to 0.\end{aligned}$$

This leads to a contradiction. Hence, $(*)$ is true.

Now let $0 \leq g \in F^*$ and $N \in \mathbb{N}$ satisfy $(*)$. For all $n > N$, we have the inequalities

$$\begin{aligned}|f_n|(Tx_n) &= (|f_n| - g)^+(Tx_n) + (|f_n| \wedge g)(Tx_n) \\ &\leq (|f_n| - g)^+(Tx_n) + g(Tx_n) \\ &\leq \varepsilon + g(Tx_n).\end{aligned}$$

Because $x_n \xrightarrow{w} 0$ in E, it follows that $\limsup |f_n|(Tx_n) \leq \varepsilon$. Since $\varepsilon > 0$ is arbitrary, we have $\lim_n |f_n|(Tx_n) \to 0$, as desired. □

The next theorem describes an important approximation property of positive weak* Dunford–Pettis operators.

Theorem 2.3. *Let $T : E \to F$ be a positive weak* Dunford–Pettis operator between Banach lattices E and F with F σ-Dedekind complete. Let W be a relatively weakly compact subset of E and (f_n) be a weak* null sequence in F^*. Then, for any $\varepsilon > 0$, there exist some $N \in \mathbb{N}$ and some $u \in E^+$ lying in the ideal generated by W such that*

$$|f_n|(T(|x| - u)^+) < \varepsilon$$

for all $n > N$ and all $x \in W$.

Proof. Assume by way of contradiction that the claim is false. That is, there exists an $\varepsilon_0 > 0$ such that for each $N \in \mathbb{N}$ and each $u \geq 0$ in the ideal generated by W we can find a natural number $m > N$ and some $x_m \in W$ satisfying $|f_m|(T(|x_m| - u)^+) \geq \varepsilon_0$. Hence, by an easy inductive argument we can choose a strictly increasing subsequence (n_k) of $\mathbb{N}$ and a sequence $(x_k) \subseteq W$ such that

$$|f_{n_{k+1}}|\Big(T\Big(|x_{k+1}| - 4^k \sum_{i=1}^{k} |x_i|\Big)^+\Big) \geq \varepsilon_0 > 0. \tag{$**$}$$

Let $x = \sum_{k=1}^{\infty} 2^{-k}|x_k|$. Also, put

$$w_{k+1} = \Big(|x_{k+1}| - 4^k \sum_{i=1}^{k} |x_i|\Big)^+, \quad v_{k+1} = \Big(|x_{k+1}| - 4^k \sum_{i=1}^{k} |x_i| - 2^{-k}x\Big)^+.$$

Clearly, $|f_{n_{k+1}}|(Tw_{k+1}) \geq \varepsilon_0 > 0$ and $0 \leq w_{k+1} \leq v_{k+1} + 2^{-k}x$ hold for all $k \in \mathbb{N}$. By [2, Lemma 4.35], (v_{k+1}) is a disjoint sequence in E. Note that $0 \leq v_{k+1} \leq |x_{k+1}|$. It follows that $v_{k+1} \in \mathrm{sol}(W)$ holds for all k. Since every disjoint sequence in the solid hull of a relatively weakly compact set of a Banach lattice converges weakly to zero (see [2, Theorem 4.34]), we see that $v_{k+1} \xrightarrow{w} 0$ in E. Hence, from Theorem 2.2 it follows that $|f_{n_{k+1}}|(Tv_{k+1}) \to 0$. On the other hand,

$$\begin{aligned} 0 < \varepsilon_0 \le |f_{n_{k+1}}|(Tw_{k+1}) &\le |f_{n_{k+1}}|(T(v_{k+1} + 2^{-k}x)) \\ &= |f_{n_{k+1}}|(Tv_{k+1}) + 2^{-k}|f_{n_{k+1}}|(Tx) \to 0, \end{aligned}$$

which contradicts ($**$). Therefore, the proof is complete. □

3. Domination by positive weak* Dunford–Pettis operators

Let us recall that a Banach space X is said to be a *Gelfand–Phillips space* whenever all limited sets in X are relatively compact. It is well known that all separable Banach spaces and all weakly compactly generated spaces are Gelfand–Phillips spaces. Note that a σ-Dedekind complete Banach lattice E is a Gelfand–Phillips space if and only if the norm of E is order continuous (see [6]). Further, X has the *Dunford–Pettis property* (respectively, the *Dunford–Pettis* property*) whenever every relatively weakly compact set in X is a Dunford–Pettis set (respectively, a limited set), in other words, for each weakly null sequence (x_n) in X and each weakly null sequence (respectively, weak* null sequence) (f_n) in X^*, $\lim_n f_n(x_n) = 0$. The Dunford–Pettis* property, introduced first by Borwein *et al.* [4], is stronger than the Dunford–Pettis property. Carrión *et al.* [8] showed that X has the Dunford–Pettis* property if and only if every bounded linear operator $T : X \to c_0$ is a Dunford–Pettis operator.

It should be noted that if either X or Y has the Dunford–Pettis* property, then every bounded linear operator from X into Y is weak* Dunford–Pettis. Also, If Y is a Gelfand–Phillips space, weak* Dunford–Pettis operators from X into Y and Dunford–Pettis operators between them coincide.

Now, since $L_1[0, 1]$ does not have weakly sequentially continuous lattice operations and the norm of c is not order continuous, by the converse for the Kalton–Saab theorem proved by Wickstead [14] we know that there exists a positive operator from $L_1[0, 1]$ into c, which is dominated by a Dunford–Pettis operator, which is not Dunford–Pettis. On the other hand, since c is a Gelfand–Phillips space, every weak* Dunford–Pettis operator from $L_1[0, 1]$ into c is Dunford–Pettis. Therefore, there exists a positive operator from $L_1[0, 1]$ into c dominated by a weak* Dunford–Pettis operator which is not weak* Dunford–Pettis. It should be noted that c is not σ-Dedekind complete. In case the range space is σ-Dedekind complete, we have the following domination result for positive weak* Dunford–Pettis operators.

THEOREM 3.1. *Let E, F be two Banach lattices such that F is σ-Dedekind complete. If a positive operator $S : E \to F$ is dominated by a positive weak* Dunford–Pettis operator, then S itself is weak* Dunford–Pettis.*

PROOF. We shall follow the plan of Kalton and Saab in their weak Dunford–Pettis version, but we have to make an effort to overcome some obstacles on our way since the behaviour of weak* sequential convergence is quite different from that of weak sequential convergence in general.

Assume that F is σ-Dedekind complete and that $T : E \to F$ is a positive weak* Dunford–Pettis operator satisfying $0 \le S \le T$. Let $x_n \xrightarrow{w} 0$ in E and let $f_n \xrightarrow{w^*} 0$ in F^*. To prove that S is weak* Dunford–Pettis, we have to show that $f_n(Sx_n) \to 0$. To this end, put $e = \sum_{n=1}^{\infty} 2^{-n}|x_n| \in E^+$ and let A_e be the ideal generated in E by e. Consider the operators $0 \le S \le T : \bar{A}_e \to F$, where $\bar{A}_e$ is the norm closure of A_e in E. Clearly, $x_n \xrightarrow{w} 0$ in $\bar{A}_e$, and $T : \bar{A}_e \to F$ is likewise weak* Dunford–Pettis. Let $\varepsilon > 0$ be fixed. By Theorem 2.3, there exist some $N_1 \in \mathbb{N}$ and some $u \in E^+$ lying in the ideal generated by (x_n) such that

$$|f_n|(T(|x_n| - u)^+) < \varepsilon$$

for all $n > N_1$. Note that $u \in A_e$. Since F is σ-Dedekind complete and $f_n \xrightarrow{w^*} 0$ in F^*, there exists $0 \le g \in F^*$ lying in the ideal generated by (f_n) in F^* such that

$$(|f_n| - g)^+(Tu) < \varepsilon$$

holds for all $n \in \mathbb{N}$ (see [7]; see also [2, Theorem 4.42]).

On the other hand, by [2, Theorem 4.82], there exist positive operators $M_1, \ldots, M_k$ on $\bar{A}_e$ and order projections $P_1, \ldots, P_k$ on F^{**} satisfying

$$\Big\langle g, \Big|S - \sum_{i=1}^{k} P_i T M_i\Big| u\Big\rangle < \varepsilon \quad \text{and} \quad 0 \le \sum_{i=1}^{k} P_i T M_i \le T.$$

(The proof of the extension of each positive multiplication operator M_i on A_e to a positive operator on $\bar{A}_e$ can be found in [2, Part(b), page 269].) Let us put $R = |S - \sum_{i=1}^{k} P_i T M_i|$. Obviously,

$$\langle g, Ru\rangle < \varepsilon \quad \text{and} \quad R = \Big|S - \sum_{i=1}^{k} P_i T M_i\Big| \le S + \sum_{i=1}^{k} P_i T M_i \le 2T.$$

For each $n > N_1$,

$$\begin{aligned}
\langle |f_n|, R|x_n|\rangle &= \langle |f_n|, R(|x_n| - u)^+\rangle + \langle |f_n|, R(|x_n| \wedge u)\rangle \\
&\le \langle |f_n|, R(|x_n| - u)^+\rangle + \langle |f_n|, Ru\rangle \\
&\le 2|f_n|(T(|x_n| - u)^+) + \langle |f_n|, Ru\rangle \\
&\le 2|f_n|(T(|x_n| - u)^+) + \langle (|f_n| - g)^+, Ru\rangle + \langle g, Ru\rangle \\
&\le 2|f_n|(T(|x_n| - u)^+) + 2(|f_n| - g)^+(Tu) + \langle g, Ru\rangle \\
&< 2\varepsilon + 2\varepsilon + \varepsilon = 5\varepsilon.
\end{aligned}$$

This implies that

$$\begin{aligned}
|f_n(Sx_n)| &\le \Big|\Big\langle f_n, \Big(S - \sum_{i=1}^{k} P_i T M_i\Big)x_n\Big\rangle\Big| + \sum_{i=1}^{k} |\langle f_n, P_i T M_i x_n\rangle| \\
&\le \Big\langle |f_n|, \Big|S - \sum_{i=1}^{k} P_i T M_i\Big| |x_n|\Big\rangle + \sum_{i=1}^{k} |\langle f_n, P_i T M_i x_n\rangle|
\end{aligned}$$

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$$= \langle |f_n|, R|x_n|\rangle + \sum_{i=1}^{k} |\langle f_n, P_i T M_i x_n\rangle|$$

$$< 5\varepsilon + \sum_{i=1}^{k} |\langle f_n, P_i T M_i x_n\rangle|$$

holds for all $n > N_1$. To prove that $\lim_n f_n(Sx_n) = 0$, we need only show that $\langle f_n, P_i T M_i x_n\rangle \to 0$ (as $n \to \infty$) for $i = 1, 2, \ldots, k$. Note that each P_i is an order projection on F^{**}. For each $f \in F^*$, we define $Q_i f$ by

$$(Q_i f)y = \langle f, P_i y\rangle$$

for all $y \in F$. We can easily see that $Q_i f \in F^*$ and $Q_i : F^* \to F^*$ is a bounded linear operator. Now we claim that Q_i is sequentially w^*-continuous on F^*. If this claim is true, then $f_n \xrightarrow{w^*} 0$ in F^* implies that $Q_i f_n \xrightarrow{w^*} 0$ in F^*. It turns out that, by the weak* Dunford–Pettis property,

$$\langle f_n, P_i T M_i x_n\rangle = \langle T M_i x_n, Q_i f_n\rangle \to 0$$

(as $n \to \infty$), since $M_i x_n \xrightarrow{w} 0$ (as $n \to \infty$) in E (or $\bar{A}_e$).

So, the key point is to prove the claim that each Q_i defined above is sequentially w^*-continuous on F^*. To this end, assume that $f_n \xrightarrow{w^*} 0$ in F^* and let A_F denote the ideal generated by F in F^{**}. Then $f_n \xrightarrow{\sigma(F^*, A_F)} 0$ in F^*, since F is σ-Dedekind complete (see [2, Theorem 4.43]). Let us recall that P_i is an order projection on F^{**}. Given $y \in F^+$, since $0 \le P_i y \le y$, we can see that $P_i y \in A_F$. Hence,

$$\langle y, Q_i f_n\rangle = \langle f_n, P_i y\rangle \to 0$$

(as $n \to \infty$), which implies the sequential w^*-continuity of Q_i. The proof is finished. □

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