OPTICAL AND RADIO OCCULTATION ANALYSIS

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1. Introduction

The study of occultations of radio sources by the Moon has proved a powerful method of studying the structure of radio sources with a resolution limit, in some cases, as small as 0"1 and at the same time of obtaining positions of the radio components with an accuracy of the order of 0"1 to 1". Recent optical observations of occultations suggest that the method is likely to play an important role in the measurement of stellar diameters down to about 0"001 and in the detection and measurement of binary star systems. Over the past several years considerable experience has been gained in the analysis of the occultation curves of radio sources and, since the problems encountered are common to both the optical and radio analysis, our conclusions on how best to analyse occultation curves may be of some interest to the optical workers and also to radio observers who have recently entered the field. Before discussing the methods of analysis and also before discussing some essential differences between the optical and radio work it is useful to consider in some detail the nature of the occultation curve of a simple source of small angular size. It is not proposed here to give a detailed account of the methods of analysis but to indicate the general principles along which the analysis should proceed so as to enable the choice of the most appropriate method in a particular case. A simple treatment of Scheuer's convolution procedure is given and a simple derivation of the resolution limit imposed by the receiver bandwidth.

2. The Nature of the Occultation Curve

Consider an observer O, receiving the radiation from a source S as it is uncovered by a screen M at a distance D, gradually uncovering the source as its edge moves along the y-axis from $y=\infty$ to $-\infty$ (see Figure 1). The diffraction pattern is built up as successive elements in the wavefront are uncovered. The change in amplitude and phase at O as each element is uncovered thus, in principle at least, enables the calculation and phase of each elementary section in the wavefront at a distance D. As the wavefront from $y=\infty$ to $y=-\infty$ contains information on all Fourier spatial components of the source brightness distribution, the occultation curve therefore also contains this information. A spatial component with frequency Z/λ will be weighted according to the number of times the spacing Z appears among the elementary elements in the wavefront. If the source is a point source and the wavefront constant in amplitude and phase, the number of times a spacing Z appears is proportional to $1/Z$ and it follows immediately that the amplitude of each Fourier component is

De Jager (ed.), Highlights of Astronomy, 607–621.

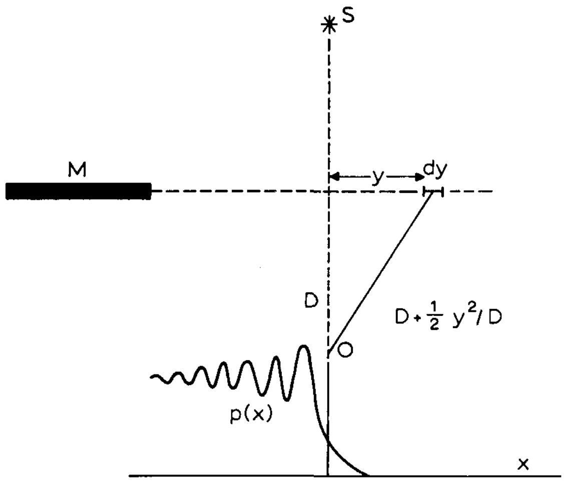

Fig. 1. A sketch showing the generation of the occultation curve as the screen *M*, at a distance *D* from the observer, uncovers the source *S* in moving from $y = \infty$ to $-\infty$. It will be noted that the occultation curve is not the diffraction curve at a straight edge but is reversed along the *y*-axis.

inversely proportional to its frequency. That is, as was first pointed out by Scheuer (1962), the amplitude of the Fourier components are exactly the same as if the diffraction effects were negligible and the occultation curve a step function instead of the familiar diffraction curve at a straight edge. The diffraction fringes are due to a distortion of the relative phases of the Fourier components at the receiving system. The phase displacement of the Fourier components as a function of frequency may be calculated from the geometry of the occultation and thus, in principle, corrections can be applied to the diffraction curve of a point source to remove the diffraction effects. The usual calculation of the diffraction pattern at a straight edge is basically the above operation in reverse.

The occultation curve $(b(x))$ of an actual source is the convolution of the time brightness distribution $(f(x))$ with the diffraction pattern of a straight edge $(q(x))$, where x is measured in a plane perpendicular to the observer and containing the diffracting edge. The Fourier transform of the occultation curve $(B(v))$ is thus related to the transforms of the brightness distribution $(F(v))$ and the occultation curve of a point source $(Q(v))$ by,

$$B(v) = F(v) \times Q(v) \tag{1}$$

The occultation curve thus contains information on all Fourier components of the brightness distribution but weighted and phase shifted as in the straight edge diffraction pattern. In principle the true brightness distribution can be recovered by taking the Fourier transform of the observed occultation curve and suitably correcting the phase and amplitude of the Fourier components. The restoration procedure described by Scheuer and discussed in more detail below is an elegant method of applying these corrections, which makes use of the fact all the required information on the phase and amplitude corrections is contained in the easily calculated straight edge pattern.

3. Analysis of Occultation Curves

A. LOBE ANALYSIS AND MODEL FITTING

The shape of the diffraction curve of a point source at a straight edge is shown in Figure 2 where the horizontal scale is plotted in units of v (one unit of $v = \sqrt{\lambda/2D}$ radians, the usual dimensionless parameter used in the treatment of straight edge diffraction. It can be seen that in that part of the pattern outside the geometrical shadow the pattern approximates to a damped sinusoidal oscillation whose frequency increases with increasing v. As a source passes through this pattern it will reproduce the diffraction pattern unless its angular size is comparable to the lobe separation, when the pattern will be smoothed out, exactly as the fringes are smoothed out in an interferometer pattern. The pattern in the region remote from the geometrical shadow can thus be considered as a variable spacing interferometer whose effective spacing

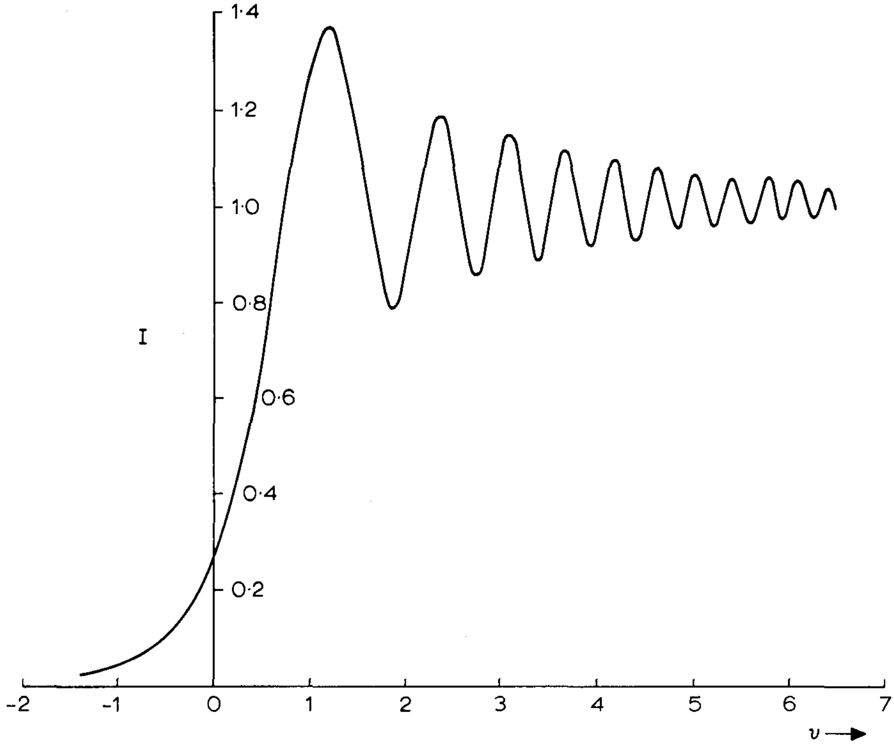

Fig. 2. The shape of the occultation curve for a point source. The horizontal scale is in units of v. The angular scale of the pattern is given by $\theta = v\sqrt{\lambda/2D}$ where λ is the wavelength and D the Moon's distance. For $\lambda = 75$ cm one unit of v corresponds to about 6″ while at $\lambda = 5400$ Å it corresponds to about 0.″005. *Abscissa:* units of v ($v = 0$ correspond to the edge of the geometrical shadow). *Ordinate:* relative flux density. $I = 1$ corresponds to the flux density of the unobstructed wave.

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increases with increasing v. At any part of the pattern the ratio of the observed lobe amplitude to the theoretical amplitude (calculated from the mean change in the received flux during the occultation) is the normalized amplitude of the Fourier component corresponding to the particular lobe separation. However, while in an ideal variable spacing interferometer the signal to noise ratio remains constant with increasing antenna separation, and thus with increasing resolution or lobe separation, in an occultation curve the lobe amplitude decreases as $1/v$. Since increasing v corresponds to increasing resolution this means that the signal/noise decreases for higher frequency Fourier components. It is this decrease of signal/noise which sets a limit to the resolution which can be obtained using the occultation techniques.

This simple concept of the occultation curve as a variable spacing interferometer was used in the original analysis of radio occultations including 3C273 (Hazard *et al.*, 1963). It is applicable when the source size is smaller than the first Fresnel zone and the diameter information therefore contained entirely in the lobe pattern. Since each lobe, or series of lobes, is considered as a section of record which would have been obtained using an interferometer of the correct spacing the well known theory of interferometers applies, from which it follows that the normalized lobe amplitude gives the amplitude of the corresponding Fourier component and the lobe displacement relative to that for a point source gives its phase. In the simple form in which we used this procedure, however, the phase was ignored (except in the case of double sources) since we were estimating sizes of assumed symmetrical models where it is irrelevant. In the case of the close double, 3C245, its double structure was first inferred from the minimum in its lobe pattern corresponding to the source separation and the phase change of π which occurred at this point (Hazard *et al.*, 1966). The resolution at each point in the pattern was obtained from the calculated lobe spacing which to a sufficient degree of accuracy can be obtained from the Moon's mean distance and the time of occultation was obtained from the known distances of the lobes from the edge of the Moon. The change in level as the source passed behind or emerged from the Moon was used to normalize the lobe amplitudes and the height of the first lobe was used to infer the presence of any broader structure. The results obtained in this way (Hazard *et al.*, 1963) were in remarkably good agreement with the results of the more detailed analyses later carried out. It can be seen that the method requires neither a detailed knowledge of the theory of occultations nor of the circumstances of the occultation to yield information on the angular structure. It provides a rapid method of estimating the source size and is particularly useful if no computer is available or the record is available only in analogue form and must be laboriously digitized before processing in a computer. If the source is complex but the components well separated each component is analysed separately. If, however, the component separation is such that interference of the two patterns occurs in the region 0–$2v$ the analysis is more difficult. This is illustrated by the record of 3C273 shown in Figure 3 and to analyse this record it was necessary to resort to a graphical curve fitting procedure. A curve fitting procedure of this type is adequate provided there is some evidence already of the source structure and the source is not too complex; for the more

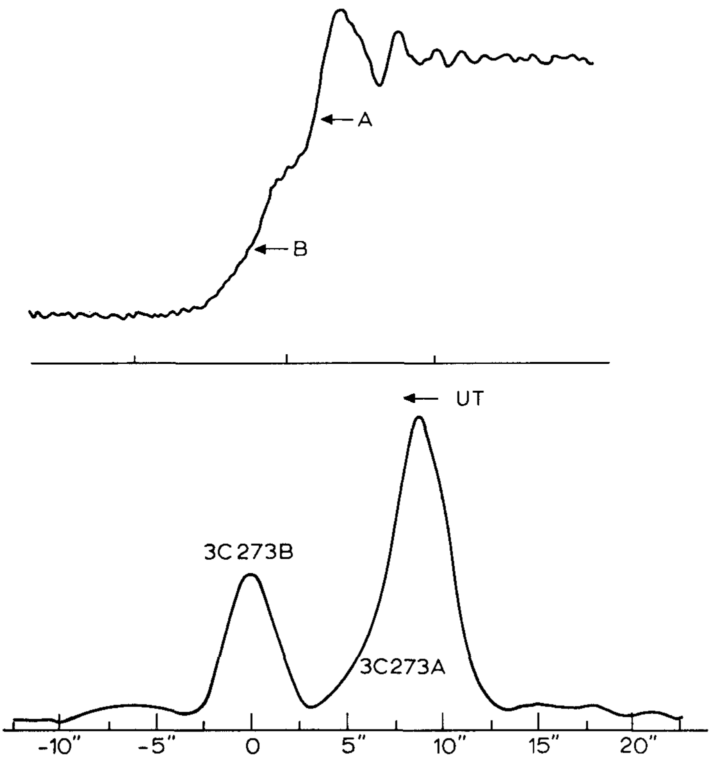

Fig. 3. (a) An occultation of 3C273 where the *A* and *B* components are separated such as to make difficult a simple lobe analysis. *Horizontal scale.* Universal time. *Vertical scale.* Received power in arbitrary units. (b) The brightness distribution across 3C273 obtained from the curve in Figure 2(a) using Scheuer's restoration procedure. *Horizontal scale.* Angular distance from component B.

complex radio sources with several components of different angular sizes it is unlikely to be practical.

An objection to the simple lobe analysis is that it becomes difficult to follow the lobes into the noise. A multiple channel recorder with different time constants on each channel to provide optimum signal/noise over different section of the lobe pattern would make it easier to apply where no computer is available although nowadays this is unlikely to be a problem. Lang (1969) who has discussed the Fourier analysis approach in detail has described a computer analysis which enables all the useful data including the phase of the Fourier components to be obtained from the lobe pattern and thus to achieve higher resolution.

It is shown in standard texts on diffraction theory that the distance of the *p*th minimum (V_p) and its height (H_p) are given by,

$$V_p = \sqrt{4p - \tfrac{1}{2}}, \qquad H_p = I_0\left(1 - \frac{1}{\pi(8p-1)^{1/2}}\right)^2 \tag{2}$$

while the corresponding values for the maxima are,

$$V_p = \sqrt{4p - \tfrac{5}{2}}, \qquad H_p = I_0\left(1 + \frac{1}{\pi(8p-5)^{1/2}}\right)^2. \tag{3}$$

It is easily shown from these equations that the lobe separation ($\delta v_p \approx 2/v_p$) and amplitude both decrease inversely as v which is to be expected since, apart from a phase distortion, the FT of the diffraction pattern corresponds to the FT of a step function. The envelope of the pattern thus corresponds to the FT of a step function and it follows that the differential of the occultation curve gives an oscillating pattern of constant amplitude corresponding to the FT of a δ-function. The envelope of the differentiated occultation curve on the side remote from the geometrical shadow thus represents the amplitude of the FT of the source brightness distribution.

It may also be shown from Equations (2) and (3) that the interferometer spacing corresponding to a given v is that which corresponds to a spacing equal to the distance of the Moon from the line joining the source to the observer (i.e. x in Figure 1) as has been pointed out by Lang (1969). It is of interest why the angular size information is apparently confined to one half of the diffraction pattern, corresponding to that half of the wavefront on the side remote from the geometrical shadow, when the half of the wavefront uncovered as the Moon moves from ∞ to 0 in Figure 1 contains similar information to the half subsequently uncovered in moving to $-\infty$. Consideration of the Cornu spiral shows that there is indeed information in both halves of the occultation curve but that while the sourve is obscured no phase information is available. The lobe pattern develops when half the wavefront has been uncovered to act as a phase reference for those parts later uncovered. In the geometrical shadow the information available corresponds to that from an intensity interferometer while outside the shadow it corresponds to a Michelson interferometer.

B. SCHEUER'S METHOD OF ANALYSIS

As indicated earlier the true brightness distribution can be recovered from an occultation curve by suitably correcting the phase and amplitude of its components. The methods employing lobe analysis make use of the fact that the occultation curve yields the amplitude of the higher order lobes directly. Scheuer's method is of more general application and make use of the fact that all the phase and amplitude corrections are contained in the easily calculated straight edge pattern.

From the previous section it follows that differentiating the occultation curve will restore the amplitudes of the Fourier components and leave only a phase distortion. If $\mathrm{d}(v)$ is the differentiated occultation curve, $p(v)=q'(v)$ the differentiated occultation curve of a point source and $f(v)$ the source brightness distribution,

$$\mathrm{d}(v) = f(v) * p(v) \tag{4}$$

and in the transform plane

$$D(v) = F(v) \times P(v). \tag{5}$$

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Since the amplitude of the Fourier components in $P(\nu)$ is constant, the phase may be corrected by multiplying by $P(-\nu)$.

Thus,

$$\begin{aligned} D(\nu) \times P(-\nu) &= F(\nu) \times P(\nu) \times P(-\nu) \\ &= F(\nu) \times |P(\nu)|^2 \\ &= \text{constant}\, F(\nu). \end{aligned}$$

That is neglecting the constant term,

$$\mathrm{d}(v) * p(-v) = f(v). \tag{6}$$

It may be noted that auto-correlation of the differentiated occultation curve will also remove the phase distortion since,

$$D(\nu)\, D(-\nu) = F(\nu) \cdot F(-\nu) \cdot P(\nu) \cdot P(-\nu) = \text{constant}\, |F(\nu)|^2$$

but at the loss of the phase of the spatial components of the brightness distribution.

Since $p(v)$ is the differential of the occultation curve observed as a point source passes through the diffraction pattern, $p(-v)$ is the differentiated diffraction pattern itself. It therefore follows from Equation (6) that convolution of the differentiated occultation curve with the differential of the straight edge diffraction pattern will recover the true source distribution. For practical reasons Scheuer proposed instead convolution of the observed curve with $p'(-v)$, the double differential of the calculated pattern, and it is this method which is generally adopted. As will be shown below it is superior and not equivalent to the convolution of the differentials of the observed and calculated patterns.

The amplitude of $p'(-v)$ increases as v, as is required to restore the amplitudes of the Fourier components in the occultation curve where they are decreased as v. To apply this procedure in practice it is therefore necessary to force $p'(-v)$ to converge and Scheuer proposed convolution with a gaussian beam, $g(v)$, equivalent to convolving the same brightness distribution with a gaussian beam of the same width. The width of $g(v)$ is decreased in successive restorations until the signal/noise of the restored curve becomes too high. However, the optimum converging function depends on the questions being asked about the source and on its actual brightness distribution. The problems of restoration in the presence of noise in the general case have been discussed by Bracewell (1965) but a few comments will be given here.

Consider first a source being restored using $p'(-v)$ convolved with a gaussian much narrower than the true source distribution, such that its length is effectively infinite. All spatial Fourier components of brightness distribution will be restored to their true values and phases but it follows from what has been said earlier that the signal/noise of each component will decrease with its Fourier frequence (ν) since the noise contribution will increase linearly with ν. Now suppose that the source has a brightness distribution which has a FT which is approximately triangular with no Fourier components beyond ν_2 and that it is required to estimate its size. An estimate of the size basically requires that we distinguish between its FT and the transforms of sources with angular sizes some given fraction larger and smaller. The restored components beyond ν_2 contribute only to the noise and yield no diameter information. Between $\nu=0$ and $\nu=\nu_2$ the separation of adjacent transforms will increase linearly with ν as also does the noise. The ability to distinguish between adjacent transforms will be independent of ν, each Fourier component contributing information with the same weight regarding the source size. For such a source all Fourier components should be restored to their true values out to a frequency ν_2 where the restoration function should be terminated. It is for such a case that the suggestion of Lang (1969) that convolution of $p'(-v)$ with a sinc function would be most appropriate. If, however, we wish to keep all the diameter information in the restored curve but have optimum signal/noise on the restored record we should weight each component according to its signal/noise in the occultation curve, that is as is well known, we should convolve with a function whose FT is that of the source. A source with a triangular transform is not physically realisable and the application of a sinc function in the general case, while making full use of all the Fourier components out to its cut off would produce serious side-lobes. While useful for estimating source sizes of simple sources (and the sizes of components of sources of known structure) or limits to their size it is not suitable for a preliminary study of the structure of complex sources.

If the source has a gaussian distribution the same considerations as used above show that little or no size information is contributed near the origin and the maximum information just below the half-amplitude point in the transform. In such a case a function weighted to emphasize components near the half-amplitude points would be preferable. Convolution with a gaussian does not weight in this way but has the property that for a gaussian source equal in width to the effective restoring beam each component is weighted according to its signal/noise and thus it produces a record of optimum signal/noise with no side lobes. No information is lost and the diameter can be calculated from the broadening of the restored curve. Whatever the source distribution no side lobes are produced and it is therefore useful for investigating sources with complex structure. After convolution of $p'(-v)$ with the gaussian $g(v)$, it is still, of course, necessary to terminate the new function and for practical reasons the shorter the length of the function used for a given resolution the better. In the programme developed at Arecibo by S. Gulkis, J. Sutton, A. D. Bray and myself, after forming $p'(-v)*g(v)$ the function could be terminated over a few lobes at any chosen value of v. It was found that if a resolution 'av' was required the restored curve for a point source did not differ from the assumed gaussian if the termination occurred at a distance $3/av$ (on the side remote from the geometrical shadow). The length of the restoring function could, however, be reduced to $1.5/av$ with no appreciable broadening and no significant side lobes; for more drastically terminated restoring functions side lobes appeared since the convolving function then approaches a sinc function. For the investigation of complex sources where we had no knowledge of the source structure and wished to investigate all weak structure the longer restoring function was used. At high revolutions ($<0\overset{''}{.}5$) were we were in general attempting

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to set a limit to the source size the shorter function, which is a compromise between the gaussian and sine function was used.

It should be noted that when we stated earlier that the optimum signal/noise is achieved using a function whose FT is matched to that of the source, we were considering the case where all Fourier components are retained. As von Hoerner (1964) has pointed out, from computer analysis of simulated curves, the optimum signal/noise on the restored record increases with the width of the effective restoring beam. It might appear from his statement, and the assumption that convolution of $p'(-v)$ with $g(v)$ is equivalent to convolution of the source distribution with $g(v)$, that after restoration with a narrow $g(v)$ the signal/noise could be improved by then convolving with a wider beam. This, however, is not the case and the apparent paradox arises because we have not considered the constant term in the transform of the step function. This constant term appears as the change in level as the source appears from behind the Moon and it is this change in level which provides the best measure of the source flux and the best means of detecting weak sources. It is obvious that in an ideal record the longer the time constant used to observe the occultation the weaker the source which can be detected. When forming $p'(v)*g(v)$, the wider $g(v)$, the wider the effective time constant and the greater the length of the step which is used. Any given $g(v)$ determines the amount of information on the source flux in the restored record. Further information cannot be obtained without going back to the original record since after convolution with $p'(-v)*g(v)$ the occultation curve has been differentiated destroying all further information on the change in level. If the occultation curve is first differentiated and then convolved with $p(-v)$ the information on the change in level is destroyed at the beginning of the analysis and thus differs from the case where the observed curve is convolved with $p'(-v)$.

We have indicated that for different purposes different restoring functions emphasizing different Fourier components may be appropriate. These functions can be generated by convolving $p'(-v)$ with the appropriate functions such as a gaussian, a sinc function etc. but for more complicated weighting of the Fourier components the required convolving functions may not be so obvious. Fortunately $p'(-v)$ can be modified appropriately without convolution since we know that the Fourier components are separated along the v-axis which thus represents the transform plane. Thus to use a gaussian restoring beam we simply multiply $p'(-v)$ by a gaussian centred at the zero of the diffraction pattern (the diffracting edge) while to simulate the effect of a sinc function we do not convolve but simply abruptly terminate $p'(-v)$ at the chosen Fourier frequency. Any desired modification of the Fourier components is thus easily achieved.

A further point in which the Arecibo occultation analysis differs from the procedures described by von Hoerner and by Scheuer is that it takes account of the curvature of the Moon's limb (Hazard *et al.*, 1965, 1966). This curvature produces a nonlinearity in the observed occultation curve and a progressive phase shift between the outer lobes of the observed curve and the lobes of the restoring function. To take this curvature into account the occultation curve was rescaled to remove these nonlinearities before performing the convolution. At high resolutions it was also found necessary to adjust the time scale of the restoring function to allow for deviations of the true limb from the hypothetical limb (Hazard *et al.*, 1965). We have only applied this procedure assuming a constant change in slope over the region of the limb of interest but it could easily be extended to more complicated limb profiles.

C. MODEL FITTING METHODS OF ANALYSIS

We have referred to the simple model fitting procedures used in the early radio analysis. Recently Nather *et al.* (1970) have used more sophisticated model fitting techniques using computers. It should be clear from what has been said that these represent alternative methods of using the phase and amplitude information on the spatial Fourier components which are present in the diffraction pattern. They are not as general as Scheuer's method but particularly suitable when a general solution is not required, when some knowledge of the source structure is already available, the source is not too complex and the lobe structure not extensive.

One advantage of the method is that it is easy to calculate the patterns for different shapes of the libm although as indicated in the previous section more complex limb shapes can also be taken into account using the convolution method of analysis.

4. Resolution Limitations

The methods of analysis discussed above are basically equivalent and subject to the same resolution limitations. The more important of these limitations have been pointed out in earlier work (Scheuer, 1962; von Hoerner, 1964; Hazard, 1965) and are,

(a) the bandwidth of the receiving system
(b) the finite size of the receiving aperture
(c) finite signal/noise ratio of the observed occultation curve
(d) irregularities in the Moon's limb.

A. BANDWIDTH LIMITATION

From Figure 1 we see that the amplitude received at O from the element of wavefront $\mathrm{d}y$ at a distance y from the line joining the source to the observer is given, at a wavelength λ by,

$$\mathrm{d}A_\lambda \propto e^{i\pi y^2/\lambda D} \tag{7}$$

and from what has been said earlier this represents the contribution to the spatial Fourier component with frequency y/λ.

At a wavelength $\lambda + \Delta\tau$ where $\Delta\lambda \ll \lambda$ we have,

$$\begin{aligned} \mathrm{d}A_{\lambda+\Delta\lambda} &\propto e^{-i\pi y^2/\lambda D} e^{-i\pi y^2/\lambda D\, \Delta\lambda/\lambda} \\ &\propto \mathrm{d}A_\lambda\, e^{-i\pi y^2/\lambda D\, \Delta\lambda/\lambda} \end{aligned} \tag{8}$$

Thus the change in wavelength simply produces a shift in the phase of the Fourier component. The total contribution to each Fourier component is the occultation

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curve is obtained by summing the contributions from all elements in the bandwidth. If the power gain of the receiver is represented by $f(\lambda)$ (we sum powers since the contribution over the bandwidth are incoherent) and $\Delta\lambda = \lambda - \lambda_0$ where λ_0 is the adopted central frequency the total contribution at the Fourier frequency y/λ_0 is given by

$$F(y) \propto \int_{-\lambda_0}^{\infty} f(\lambda)\, e^{-i\pi y^2/\lambda_0 D \cdot \Delta\lambda/\lambda_0}\, \mathrm{d}\Delta\lambda$$

which for a symmetrical bandwidth and replacing y by θD reduces to,

$$F(\theta) \propto \int_{-\lambda_0}^{\infty} f(\lambda) \cos \frac{\pi\theta^2 D}{\lambda_0} \cdot \frac{\Delta\lambda}{\lambda_0}\, \mathrm{d}(\Delta\lambda). \tag{9}$$

The FT of the occultation curve is thus multiplied by $F(\theta)$ which is equivalent to convolving the source brightness distribution $f(\theta)$ by the function whose FT is $F(\theta)$.

Equation (7) was first derived by Scheuer (1965) who gave no derivation but gave the appropriate convolving functions for different assumed bandwidths. (A misprint in one of equations was pointed out by Hazard *et al.* (1965) and by Sutton (1966) but who also gave no derivation). Alternative methods of deriving the equation have been given by Cohen (1969) and by Lang (1969). For an accurate determination of source diameters corrections must be applied for the bandwidth using Equation (9) which requires that the band pass be accurately known. For more approximate estimates it is sufficient to note that the approximate effect of a gaussian bandwidth is to convolve the source distribution with a gaussian of half-width equal to 0.6 $(\Delta\lambda/D)^{1/2}$ (Hazard *et al.*, 1966) where $\Delta\lambda$ is the half-power bandwidth and D the Moon's distance. An alternative approximate approach is to note that a rectangular bandwidth Δf wide will produce cancellation of the fringes for a fringe spacing $\Delta\theta$ given by

$$\Delta\theta = \alpha \sqrt{\frac{\Delta f}{f}}$$

where α is the width of the first Fresnel zone (Scheuer, 1962).

B. FINITE RECEIVER APERTURE

The finite receiver aperture produces a finite beam which limits the length of wavefront studied and thus limits the resolution. Alternatively it may be considered as a probe which averages over the diffraction pattern at the Earth. The result is that at a lobe separation equal to the antenna size the pattern falls to zero. For a rectangular bandwidth of size 'd' complete cancellation occurs at a lobe spacing d/D as it would with a uniform strip source of this size (Hazard, 1965). An obvious misprint in this paper gives the limit as the angle subtended by the Earth at the Moon's distance instead of the angle subtended by the Aerial.) An exact treatment of the aperture limitation requires a calculation of the modification to the diffraction pattern and the way in which it weights the Fourier components; in making this calculation it must be remembered that the wavefront over the aperure is coherent. No calculations have been published for apertures of different shapes probably because in the radio case it is of only marginal importance and usually smaller than the limitation set by the bandwidth and it is likely to be of importance in the optical region only for the larger telescopes.

C. FINITE SIGNAL/NOISE RATIO

The higher the signal/noise the smaller the apparent broadening of the source which can be detected; thus the higher the source flux the greater the resolution possible. This limitation was first discussed in detail by von Hoerner who pointed out that since the resolution worsens with increasing bandwidth (Δf) as $\sqrt{\Delta f}$ while the signal/noise increases as $\sqrt{\Delta f}$ there is an optimum bandwidth for a given source and receiving equipment, and he gives formulae based on the analysis of simulated curves for estimating this optimum. The problem has also been discussed by Lang (1969). A similar optimum occurs between the aperture size and the signal/noise. Here, however, for a circular aperture the signal/noise increases as its (diameter)^2 while the resolution worsens only as the diameter. Where the shape of a brightness distribution is known and it is only required to measure one point on the transform it may thus in some cases be preferable to go to as large an aperture size as possible.

D. IRREGULARITIES IN THE MOON'S LIMB

Irregularities in the Moon's limb will also tend to smooth out the higher order lobes (Hazard, 1965). The effect is difficult to calculate and will depend on the shape of the irregularities. The effect of various shaped irregularities on the diffraction pattern has been studied by Evans (1970).

5. Comparison of the Analysis of Radio and Optical Occultation Curves

At radio wavelengths we are dealing with wavelengths of about 1 m where the size of one unit of v which defines the scale of the occultation pattern is around 6″ while in the optical region the corresponding scale size is about 0″.005; the time scales are thus of the order 20 sec and 20 m.sec respectively. Apart from this difference in time scale there is, in principle, no difference between the two cases. The differences arise because of the different type of source being studied, the different questions being asked about the source structure and the different effects in the two cases of the limitations discussed above.

Radio sources are in general complex, usually of unknown structure (apart from the fact that the majority are basically double), and with components of unknown angular size. It is, therefore, in general not possible to decide beforehand the optimum receiver parameters. In general, therefore, a series of filters defining different bandwidths are used and the optimum choice made later. It should be noted that the band-

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width does not impose a fundamental limitation at radio wavelengths since the pass band may be split into several narrow bands, each channel analysed separately and the restored curves then combined, as suggested by von Hoerner (1964). In the early work the basic data required from the occultation curve were a reliable picture of the source structure, the sizes and positions of the components and the spectra of the components. Recently the emphasis has changed somewhat. Positions sufficiently accurate for identification are available more readily from interferometer and pencil beam surveys and new aperture synthesis instruments are approaching the occultation resolution limit with no restriction on the choice of source to be studied. Furthermore the structure of many sources is now known to be as small as 0″.001 well below the occultation limit of about 0″.1 and accessible to investigation only using Very Long Baseline interferometry. Its main importance now would appear to be not so much to measure angular sizes, and accurate angular sizes were never of great interest, but to compare the positions of the radio components with that of the optical object, investigate structure at low frequencies where similar resolution is not available using other techniques and to investigate the variations of spectral index over the source, an investigation which requires multiple frequency observations.

In the optical case, on the contrary, the only alternative to occultation observations as a means of studying stellar diameters is the intensity interferometer at Narrabri which has comparable resolution. The observations are not required for position determinations but to make accurate measurements of stellar diameters and to detect and investigate close binary systems. The majority of the objects studied are relatively simple in structure, that is they are either single or double with no extended structure as often found in radio sources. Furthermore the theory of stellar structure is sufficiently well developed that at least in some cases an estimate can be made of the form of the brightness distribution.

At a frequency of 300 Mc/sec ($\lambda = 1$ m) typical bandwidths range from 100 kc/sec to a few Mc/sec. At 100 kc/sec the bandwidth limits the resolution to about 0″.1 and at 1 Mc/sec it is about 0″.26. For the largest instrument available for occultation work, the Arecibo 1000 ft telescope, the aperture limits the resolution to about 0″.1. With this large aperture adequate signal/noise to approach a resolution of 0″.1 can only be obtained for a reasonable number of sources with bandwidths of the order of 1 Mc/sec so that in general the aperture size is not a problem. At this bandwidth the lobe pattern persists out to about 50 v or 600 lobes. Since an accurate measure of the source size is not required an adequate estimate can often be made from lobe analysis of the first few lobes. For full use of the information, however, Scheuer's procedure or the detailed type of lobe analysis described by Lang must be used. For a detailed investigation of complex sources Scheuer's procedure as modified in the Arecibo programme to take into account the curvature of the limb is recommended. It should be noted that extended sources where the diffraction effects are negligible are best studied using interferometers or aperture synthesis techniques.

In the optical case the situation is very different. For a 16% bandwidth, i.e. 850 Å at 5400 Å the minimum resolution is about 0″.0013 and for an 8% bandwidth about

0″.0009 (Sutton, 1965). The pattern thus extends out to only about 10 v or the order of 16 lobes. For 40 in., 100 in. and 200 in. telescopes the aperture limitations are about 0″.0008, 0″.0015 and 0″.003 respectively so even using narrower bandwidths than those above an extensive lobe pattern would not be observed. Thus in the optical case, especially when the finite size of the source is taken into account, we are never dealing with the extensive lobe patterns possible in the radio case and lobe analysis and lobe fitting methods now become competitive with the restoration procedures, at least as far as the time required for the analysis is concerned. It would also appear that the largest optical telescopes even if available for occultation work are not necessarily better than the smaller instruments, although as pointed out earlier where a source model can be assumed the aperture limitation may be compensated (or more than compensated) by the increased signal/noise. There appears to be no method available of overcoming the bandwidth limitation in the manner suggested by van Hoerner for the radio case. However, there is not a great deal to be gained since the aperture and bandwidth limitations are comparable.

Limb irregularities are also more important in the optical region. One unit of v at $\lambda = 1$ m corresponds to about 10 km while at optical wavelengths it corresponds to only 10 m. The shape of the limb and the time scale of the pattern will thus be determined by the irregularities in the limb rather than the general Moon profile as in the radio case. Moreover the time scale may not be regular. It is in taking into account these irregularities that the model fitting and the Fourier analysis method described by Lang may have advantages over the Scheuer technique. It would appear that for a preliminary investigation the Scheuer technique, being completely objective, is to be preferred particularly for the complex radio sources. However, where the final estimate of source size is required, particularly in the optical case where a reasonable estimate may be made of the brightness distribution across the source components, other methods may be tried in an attempt to overcome the effect of limb irregularities.

In this final analysis the model-fitting approach is likely to be more appropriate for optical occultations. In the radio case where the lobe pattern may be extensive the Fourier analysis approach is likely to be most useful. In setting the limit to the size of a small diameter source we can in general, consider the source to be symmetrical and we are thus interested only in the amplitude of its transform. By splitting the record into sections as described by Lang (1969) and analysing each section over a range of Fourier frequencies around the calculated frequency the Fourier amplitudes can be investigated into the outer regions of the pattern even in the presence of phase irregularies and distortions in the time scale introduced by limb irregularities. It is these phase irregularities and time scale distortions which make difficult the application of the convolution technique to extremely high resolution and it was precisely for this reason that the Fourier analysis was suggested.

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