Precise Stellar Radial Velocities
ASP Conference Series, Vol. 185, 1999
J. B. Hearnshaw and C. D. Scarfe, eds.

Exactly What Is Stellar 'Radial Velocity'?

Lennart Lindegren, Dainis Dravins and Søren Madsen

Lund Observatory, Box 43, SE-22100 Lund, Sweden

Abstract. Accuracy levels of metres per second require the concept of 'radial velocity' to be examined, in particular with respect to relativistic velocity effects and spectroscopic measurements made inside gravitational fields. Already in a classical (non-relativistic) framework the line-of-sight velocity component is an ambiguous concept. In the relativistic context, the observed wavelength shifts depend e.g. on the transverse velocity of the star and the gravitational potential at the source. We argue that the observational quantity resulting from high-precision radial-velocity measurements is not a physical velocity but a spectroscopic *radial-velocity measure*, which only for historic and practical reasons is expressed in velocity units. This radial-velocity measure may be defined as cz, where c is the speed of light and z is the observed relative wavelength shift reduced to the solar system barycentre. To first order, cz equals the line-of-sight velocity, but its precise interpretation is model dependent.

1. Introduction

It is known that the interpretation of spectroscopic line shifts in terms of stellar radial velocity becomes quite complicated at an accuracy level below ~ 1 km s^{-1}, due to the many physical effects in stellar atmospheres contributing to the observed wavelength shifts (e.g. Dravins 1998; Dravins et al. 1998). In this paper we are not concerned with shifts caused for instance by stellar convection, but with the fundamental question how the concept 'radial velocity' may be defined. In general, radial velocity cannot be treated separately from the other five coordinates in phase space, or their observational equivalents — the astrometric position and proper motion (in two coordinates each), and parallax. Indeed, our discussion is based on formulations found in textbooks on relativistic astrometry and celestial mechanics, e.g. Murray (1983), Soffel (1989) and Brumberg (1991).

2. Classical Treatment

In a Euclidean metric with origin at the solar system barycentre (SSB) and with t denoting coordinate time, let $\mathbf{r}_0(t)$ be the motion of the star, $\mathbf{v}_0 = \mathrm{d}\mathbf{r}_0/\mathrm{d}t$ its barycentric space velocity, and $\mathbf{u}_0 = \mathbf{r}_0/r_0$ the barycentric direction to the star. Radial velocity is classically defined as the component of $\mathbf{v}_0$ along $\mathbf{u}_0$, or

$$v_R = \mathbf{u}_0 \cdot \mathbf{v}_0 = \frac{\mathrm{d}r_0}{\mathrm{d}t}\,. \tag{1}$$

Due to the finite speed of light (c) we must however distinguish between the time of light emission at the star, t_0, and the time of light reception at the observer, t_1. With $\mathbf{r}_1(t)$ denoting the position of the observer, the events are related by

$$c(t_1 - t_0) = |\mathbf{r}_0(t_0) - \mathbf{r}_1(t_1)| \,. \tag{2}$$

Assume first that the observer is fixed at the SSB ($\mathbf{r}_1 \equiv \mathbf{0}$), so that the barycentric distance is $r_0 = c(t_1 - t_0)$. To calculate v_R we may insert this r_0 into Eq. (1). But the question then arises whether the t in Eq. (1) should be identified with t_0 or t_1. The answer depends on one's point of view. If radial velocity is regarded as a component of the star's space velocity, and thus as a 'property' of the star, it is natural to use t_0; but from an observational point of view t_1 may be considered more natural.[1] The two choices give different expressions for v_R:

$$v'_R \equiv \frac{\mathrm{d}r_0}{\mathrm{d}t_0} = c\left(\frac{\mathrm{d}t_1}{\mathrm{d}t_0} - 1\right) , \qquad v''_R \equiv \frac{\mathrm{d}r_0}{\mathrm{d}t_1} = c\left(1 - \frac{\mathrm{d}t_0}{\mathrm{d}t_1}\right) . \tag{3}$$

The difference, $v'_R - v''_R = v'_R v''_R/c \simeq v_R^2/c$, exceeds 0.1 km s^{-1} for $|v_R| >$ 173 km s^{-1} and 1 km s^{-1} for $|v_R| > 548$ km s^{-1}. We thus have an ambiguity already in the classical definition of radial velocity. Since relative velocities in our Galaxy can reach several hundred km s^{-1}, this ambiguity has practical relevance in the context of precise stellar radial velocities.

It is seen from Eq. (3) that the ambiguity arises when the quantity $\mathrm{d}t_1/\mathrm{d}t_0$ is transformed into a velocity, i.e. when a model is used to interpret the data. $\mathrm{d}t_1/\mathrm{d}t_0$, on the other hand, is a direct, model-independent relation between the basic events of light emission and reception. From an observational viewpoint, we must then regard $\mathrm{d}t_1/\mathrm{d}t_0$ as more fundamental than v_R. Differentiation of Eq. (2) in the general case when the observer is not at the SSB gives

$$\frac{\mathrm{d}t_1}{\mathrm{d}t_0} = \frac{1 + \mathbf{u}\cdot\mathbf{v}_0/c}{1 + \mathbf{u}\cdot\mathbf{v}_1/c} \tag{4}$$

where $\mathbf{u} = (\mathbf{r}_0 - \mathbf{r}_1)/|\mathbf{r}_0 - \mathbf{r}_1|$ is the unit vector from the observer to the source.

3. Relativistic Formulation

In a general-relativistic context the coordinates $(t, \mathbf{r})$ used to describe the light emission/reception processes are essentially arbitrary (metric-dependent) labels of space-time events. Spectroscopy is however about comparing atomic oscillators or clocks, which keep local proper time τ. It is therefore necessary to include the proper time at the source (τ_0) and at the observer (τ_1) in the discussion. Suppose that $n = \nu_0 \mathrm{d}\tau_0$ cycles of radiation are emitted at frequency ν_0 in the interval $\mathrm{d}\tau_0$ of proper time at the source. Then n cycles are received in the interval $\mathrm{d}\tau_1$ of proper time at the observer, who derives the frequency $\nu_1 = n/\mathrm{d}\tau_1 = \nu_0 \mathrm{d}\tau_0/\mathrm{d}\tau_1$. In terms of wavelength ($\lambda = c/\nu$) we can write

$$1 + z_{\rm obs} \equiv \frac{\lambda_{\rm obs}}{\lambda_{\rm lab}} \equiv \frac{\lambda_1}{\lambda_0} = \frac{\mathrm{d}\tau_1}{\mathrm{d}\tau_0} = \frac{\mathrm{d}\tau_1}{\mathrm{d}t_1}\,\frac{\mathrm{d}t_1}{\mathrm{d}t_0}\,\frac{\mathrm{d}t_0}{\mathrm{d}\tau_0} , \tag{5}$$

[1]There is an analogous problem in the definition of proper motion, i.e. the rate of change in direction $\mathbf{u}_0$, but here there is a consensus that proper motion means $\mathrm{d}\mathbf{u}_0/\mathrm{d}t_1$, not $\mathrm{d}\mathbf{u}_0/\mathrm{d}t_0$.

where z has its usual meaning of a relative wavelength shift.

The factors $d\tau_1/dt_1$, dt_1/dt_0 and $dt_0/d\tau_0$ involve the space-time coordinates $(t_i, \mathbf{r}_i)$ and the coordinate velocities $\mathbf{v}_i$ $(i = 0, 1)$, and thus depend on the chosen metric (while $d\tau_1/d\tau_0$ is of course independent of the metric). However, for a metric that is asymptotically flat far away from the masses, the weak-field or post-Newtonian approximation yields (e.g. Soffel 1989)

$$\frac{d\tau_i}{dt_i} = 1 - \frac{\Phi_i}{c^2} - \frac{1}{2}\left(\frac{\mathbf{v}_i}{c}\right)^2 + \mathcal{O}(c^{-4}). \tag{6}$$

Here $\Phi_i = \sum_j Gm_j/r_{ij}$ is the total Newtonian gravitational potential at $\mathbf{r}_i$.

For the derivation of dt_1/dt_0 one needs the relativistic version of Eq. (2), in which the right-hand side is supplemented with a small term representing the relativistic time delay along the photon track:

$$c(t_1 - t_0) = |\mathbf{r}_0(t_0) - \mathbf{r}_1(t_1)| + \Delta(\mathbf{r}_1, \mathbf{r}_0). \tag{7}$$

Formulae for $\Delta(\mathbf{r}_1, \mathbf{r}_0)$ can be found e.g. in Murray (1983). The time derivative of the delay term is usually negligible and the classical expression, Eq. (4), can be used also in the relativistic case. However, the term $d\Delta/dt_0$ could become measurable in special cases, e.g. during microlensing events (Kislik 1985). The effect is expected to be small, and may be overshadowed by other effects such as caused by differential magnification of the stellar disk (Maoz & Gould 1994).

Other factors which might conceivably affect the measured z_{obs} in special situations or over large distances include gravitational waves in the intervening space (Detweiler 1979; Fakir 1994) and cosmological redshift. Although gravitationally bound systems, such as our Galaxy, do not follow the general expansion of the universe (Dicke & Peebles 1964), the local expansion rate is not reduced to exactly zero (Noerdlinger & Petrosian 1971).

Introducing a factor $1 + X$ to take into account such exotic phenomena, the complete expression for the observed wavelength shift becomes, in the post-Newtonian approximation,

$$1 + z_{\text{obs}} = \frac{1 - \Phi_1/c^2 - \mathbf{v}_1^2/2c^2}{1 + \mathbf{u}\cdot\mathbf{v}_1/c}\,(1 + X)\,\frac{1 + \mathbf{u}\cdot\mathbf{v}_0/c}{1 - \Phi_0/c^2 - \mathbf{v}_0^2/2c^2}. \tag{8}$$

It may be useful to recall the meaning and typical size of the various terms in Eq. (8). The term containing Φ_1 accounts for the gravitational blueshift due to the potential at the observer, while $\mathbf{v}_1^2$ includes the transverse Doppler effect from the motion of the observer; each term contributes $\simeq -3$ m s^{-1} for an observer on the Earth. $\mathbf{u}\cdot\mathbf{v}_1$ is the component of the observer's motion along the line of sight, which for a terrestrial observer may amount to $\pm 30\,000$ m s^{-1}. Similarly, $\mathbf{u}\cdot\mathbf{v}_0$ represents the radial velocity of the star, Φ_0 determines its gravitational redshift ($\simeq +300$ to 1000 m s^{-1} for main-sequence stars, but ranging from +30 to 30 000 m s^{-1} for other stellar types; Dravins et al. 1998), and $\mathbf{v}_0^2$ its transverse Doppler effect ($\simeq +100$ m s^{-1} for fast-moving stars). Only the first factor on the right-hand-side of Eq. (8) is accurately known; the rest of the expression depends on quantities (other than the stellar radial velocity) which are generally unavailable to the observer, viz. Φ_0, $|\mathbf{v}_0|$ and X.

To compare different observations they should be standardised through transformation to a fictitious observer located at the SSB, but unaffected by the gravitational field of the solar system. This corresponds to having $\Phi_1 = 0$ and $\mathbf{v}_1 = \mathbf{0}$ in Eq. (8). The resulting barycentric wavelength shift z is given by

$$1 + z = (1 + z_{\mathrm{obs}}) \frac{1 + \mathbf{u} \cdot \mathbf{v}_1/c}{1 - \Phi_1/c^2 - \mathbf{v}_1^2/2c^2} . \tag{9}$$

This z (or, in velocity units, cz) is conceptually a well-defined result of the measurement, but it cannot readily be interpreted as a precise physical velocity.

4. Conclusions

Somewhat surprisingly, we find that the naïve notion of radial velocity as the line-of-sight component of the stellar velocity is ambiguous already in a classical (non-relativistic) formulation. In a relativistic framework the observed shift depends on additional factors, such as the transverse velocity and gravitational potential of the source and, ultimately, the cosmological redshift. Since these factors are generally not (accurately) known to the spectroscopic observer, it is impossible to convert the observed shift into a precise kinematic quantity.

What *can* be derived from spectroscopic radial-velocity measurements is the wavelength shift z reduced to the solar system barycentre through Eq. (9). For convenience, the shift can be expressed in velocity units as cz. Although this quantity approximately corresponds to radial velocity, its precise interpretation is model dependent and one should therefore avoid calling it 'radial velocity'. We propose the term *radial-velocity measure* for cz, emphasizing both its connection with the traditional spectroscopic method and the fact that it is not quite the radial velocity in the usual sense.

Acknowledgments. This work is supported by the Swedish National Space Board and the Swedish Natural Science Research Council.

References

Brumberg, V.A. 1991, Essential Relativistic Celestial Mechanics, Bristol: Adam Hilger

Detweiler, S. 1979, ApJ, 234, 1100

Dicke R.H., Peebles, P.J.E. 1964, Phys.Rev.Lett, 12, 435

Dravins, D. 1998, these proceedings

Dravins, D., Gullberg, D., Lindegren, L., Madsen, S. 1998, these proceedings

Fakir, R. 1994, Phys.Rev.D, 50, 3795

Kislik, M.D. 1985, AZh, 62, 905 (= Soviet Ast., 29, 524)

Maoz, D., Gould, A. 1994, ApJ, 425, L67

Murray, C.A. 1983, Vectorial Astrometry, Bristol: Adam Hilger

Noerdlinger, P.D., Petrosian, V. 1971, ApJ, 168, 1

Soffel, M.H. 1989, Relativity in Astrometry, Celestial Mechanics and Geodesy, Berlin: Springer