60. DIFFUSION OF COMETS FROM PARABOLIC INTO NEARLY PARABOLIC ORBITS

K. A. SHTEJNS
Astronomical Observatory, Latvian State University, Riga, U.S.S.R.

Abstract. It is assumed that the accumulation of small, independent, random perturbations in the reciprocal semimajor axis of the orbit of a comet follows a normal distribution law whose standard deviation is a function of the inclination and perihelion distance and that for nongravitational forces it is a function of perihelion distances only; it is also assumed that secular accelerations do not change into decelerations, and vice versa. The standard deviations given by diffusion theory are in good agreement with the mean values of nongravitational impulses obtained from calculations from short overlapping arcs. The mean lifetime of a comet is found to be one hundred revolutions. To explain why many more near-parabolic comets are actually discovered than are theoretically expected the existence of comets of very short lifetimes must be accepted.

Considerable advances have been made in the study of separate stages of the evolution of cometary orbits and of the structure of cometary nuclei and atmospheres, but almost nothing has been elucidated concerning the actual origin of comets. This is because for some of the stages of cometary evolution we have only hypothetical conclusions and almost no observational data at all.

Let us consider the problem of the diffusion of comets (Oort, 1950). The existence of the cometary cloud seems indisputable to us. We understand the cometary cloud to be an aggregate of comets moving in nearly parabolic orbits. The only assumption made here is that we consider the number of long-period comets not to be restricted to the few hundred that have already been discovered. The number of comets regularly entering the inner part of the solar system is evidently much larger.

The orbits of these comets undergo diffusion, essentially in the following manner. Let us consider a number of comets with parabolic orbits and approaching the Sun for the first time. After passing through the inner solar system, approximately 50% will have elliptical orbits and the other 50% will leave the solar system. When the remaining comets pass by the Sun a second time, the probability of their leaving is much less, since the initial orbits are elliptical. For a given comet the process of accumulation of small perturbations is random, since the revolution period of the comet remains very much longer than that of Jupiter, and whenever the comet approaches the Sun it encounters an entirely different configuration of planets.

We assume that independent random accumulation (i.e., diffusion) takes place when the reciprocal semimajor axis ($1/a$) is in the interval 0 to 0.025 AU^{-1}. We have demonstrated that the perihelion distances are practically invariable under diffusion (Shtejns and Kronkalne, 1964). In the present paper we consider that a normal distribution law holds and that its standard deviation σ is a function of the inclination i and perihelion distance. The effects of nongravitational impulses is taken into account in a similar way. It is assumed that the diffusion process takes place over a very long time and that a sufficient number of comets is involved. The disintegration of comets

Chebotarev et al. (eds.), The Motion, Evolution of Orbits, and Origin of Comets, 347–351. *All Rights Reserved. Copyright © 1972 by the IAU.*

https://doi.org/10.1017/S0074180900006756 Published online by Cambridge University Press

is taken into account by considering that after a particular number N of revolutions around the Sun they perish or become invisible. The number N is assumed to be a function of perihelion distance q.

If there were a sufficiently large number of known comets having the same perihelion distance and whose statistics $n = n(1/a, i, q)$ are not distorted by discovery conditions, we could find N in a purely empirical way. Knowing how the values of $1/a$ are distributed, i.e., knowing the standard deviation of the normal distribution σ for different hypothetical N, we can find $n = n(1/a, i)$. Choosing from all the theoretical $n(1/a, i)$ the one in best agreement with the empirical $n(1/a, i)$, we determine the maximum number of revolutions of the comet as the value of N with which the best theoretical distribution was found. But $n(1/a, i, q)$, even over the very small interval $(q, q + \Delta q)$, must be distorted by discovery conditions. Comets in the planetary zone diminish in brightness in the course of time, and consequently more new comets are discovered than is theoretically expected if discovery selection is ignored. New comets have $1/a \approx 0$. We feel that the most dependable results are obtained for $q \approx 1$ AU, when the observational conditions are the most favourable. Unfortunately, there are not enough known comets over a narrow interval ($\Delta q \approx 0.3$ AU), and if we have to use a longer one, we need to know $N = N(q)$, which is supposed to take the form

$$N = Aq^{1/2},$$

A being a constant. To avoid dependence of n on q we have determined the maximum number of revolutions according to the arithmetic mean value of $1/a$ and considered it independent of the discovery conditions. The data on $1/a$ have been taken from Porter (1961), although the original values have been replaced with those given by Sekanina (1966). Unlike our earlier study (Shtejns, 1964), parabolic and hyperbolic orbits are not considered here. As is well known, a parabolic orbit is usually determined from observations over a comparatively small arc, and it is not possible to detect a deviation from a parabola with sufficient reliability. Including parabolic orbits undoubtedly diminishes the mean value of $1/a$, since many of these comets must really have quite large positive values of $1/a$. Determination of the arithmetic mean of the original $1/a$ values also causes N to be underrated because these values have been determined mainly for orbits with negative values of $1/a$ near perihelion, the calculations having been made in order to prove Strömgren's idea of the nonexistence of hyperbolic original orbits.

For the determination of N we have taken comets with $0.9 < q < 1.9$ AU, the relevant data being summarized in Table I. It can be seen that there are 43 comets with direct motion and 27 with retrograde motion, and that this difference is due mainly to comets of large $1/a$. This phenomenon was discovered and explained by Oort. We have defined it as the first law of diffusion, namely: As a result of the diffusion of comets, orbits with the greater reciprocal semimajor axes have smaller inclinations; i.e., they concentrate towards the plane of Jupiter's orbit.

With the aid of the method of independent tests we have obtained theoretical values of $n = n(1/a, q, N)$. It should be noted that, in deriving the average value with respect to i, account has been taken of the fact that the normals to the orbital planes intersect

https://doi.org/10.1017/S0074180900006756 Published online by Cambridge University Press

TABLE I

Comets for which $0.9 < q < 1.9$ AU

Interval of $1/a$ (AU^{-1})	Number of comets with direct motion	retrograde motion
$0 < 1/a < 0.0015$	18	13
$0.0015 < 1/a < 0.015$	15	14
$0.015 < 1/a < 0.025$	10	0
Mean value	0.0069	0.0033
$1/a < 0$	4	3
Mean value	0.0063	0.0028

the celestial sphere uniformly. The results of these calculations are given in Table II, from which it may be seen that the arithmetic mean of $1/a$ is almost identical for $q=1$ and $q=2$. As for the observed comets, for $i<90°$ there is a small decrease from $q=1$ to $q=2$. Therefore, the hypothesis $N=Aq^{1/2}$ is valid within this range. Comparing the data of Tables I and II, we come to the conclusion that $N=100$ revolutions. This number should be considered as a minimum, because selection results in the brighter comets being discovered, and these have smaller values of $1/a$.

TABLE II

Dependence of $1/a$ on N and q

Maximum number of revolutions N		Mean value of $1/a$ (AU^{-1}) $q = 1$ AU	$q = 2$ AU
$i < 90°$	7	0.0014	0.0013
	60	0.0047	0.0050
	100	0.0063	0.0064
	180	0.0072	0.0072
$i > 90°$	7	0.0009	0.0009
	60	0.0033	0.0032
	100	0.0038	0.0040
	180	0.0061	0.0060

An estimate of mean cometary lifetime can be made over a definite interval $\Delta(1/a)$ according to the relative number of comets whose theoretical values for direct and retrograde motion are given in Table III. Comparing these data with the observational data of Table I, we see that considerably fewer comets with $1/a \approx 0$ and 0.02 are actually discovered than theoretically expected. According to Table III the most probable lifetime of a comet is also 100 revolutions.

The effects of nongravitational impulses are taken into account as follows. We consider that there is a normal distribution whose standard deviation σ is a function of q

https://doi.org/10.1017/S0074180900006756 Published online by Cambridge University Press

only and that an acceleration does not change to a deceleration, or vice versa. Calculations for $N=100$, $\sigma=0.00006$ and $0.015<1/a<0.025$ give five comets with direct and one with retrograde motion. For $N=100$, $\sigma=0.00015$, we have eight and four comets, respectively. Comparing this with the data of Tables I–III, we see that the relatively larger number of nearly parabolic orbits and orbits with $a=90$ to 40 AU cannot be explained by the influence of nongravitational impulses on the nuclei of long-period comets. The value $\sigma=0.00006$ is in good accord with the mean value of the nongravitational impulses obtained from calculations over short overlapping arcs. The effects of nongravitational impulses thus lead to smaller values for the maximum lifetimes of comets.

TABLE III

Relative number of comets with direct and retrograde motion

	Direct motion				Retrograde motion			
Interval of $1/a$ (AU^{-1}) $N(q=1\text{ AU})$:	7	60	100	180	7	60	100	180
$0 < 1/a < 0.0015$	28	12	10	8	22	10	8	6
$0.0015 < 1/a < 0.015$	15	30	30	31	5	17	19	19
$0.015 < 1/a < 0.025$	0	1	3	3	0	0	0	2

Let us check the second diffusion law, which states that orbits of large perihelion distance have on the average smaller semimajor axes. This law can be found from observations. The observational data are as follows: for direct motion, if $0.1<q<0.9$ AU, the mean value of $1/a$ is 0.0050 (0.0036 with $1/a<0$), while for retrograde motion it is 0.0036 (0.0014 with $1/a<0$). Comparing these data with those of Table I, we see that the mean value of $1/a$ increases with increasing q. For $q=5.2$ AU we have no observational data. After 350 revolutions the theory yields 0.0100 for direct motion and 0.0054 for retrograde motion.

Appropriate conclusions can be made, not only from the mean value of $1/a$ but also from the relative number of comets in different intervals. The results are of interest for $q=5.2$ AU, where capture takes place. Assuming a maximum number of 350 revolutions and if $0<1/a<0.0015$, we have 5 direct and 5 retrograde comets; if $0.0015<1/a<0.015$, we have 28 direct and 10 retrograde, and if $0.015<1/a<0.025$, 21 direct and one retrograde. The maximum number of revolutions is chosen so as to demonstrate the reality of the third diffusion law, i.e., that there are more new comets with smaller perihelion distances. To explain why many more comets with nearly parabolic orbits are actually discovered than theoretically expected we can assume that some comets have very short lifetimes.

Acknowledgment

I thank I. E. Zal'kalne for statistical calculations and useful discussions.

https://doi.org/10.1017/S0074180900006756 Published online by Cambridge University Press

References

Oort, J. H.: 1950, *Bull. Astron. Inst. Neth.* **11**, 91.
Porter, J. G.: 1961, *Mem. Br. Astron. Assoc.* **39**, No. 3.
Sekanina, Z.: 1966, *Publ. Astron. Inst. Charles Univ.* No. 48.
Shtejns, K. A.: 1961, *Astron. Zh.* **38**, 107.
Shtejns, K. A.: 1964, *Uch. Zap. Latv. Gos. Univ.* **68**, 39.
Shtejns, K. and Kronkalne, S.: 1964, *Acta Astron.* **14**, 311.

Discussion

F. L. Whipple: I am delighted to see this more thorough approach to the problem than I myself attempted a few years ago. We agree concerning the rather short lifetimes for many comets. The random distribution function of $\Delta(1/a)$ as shown by H. A. Newton and H. N. Russell is not quite symmetrical, the negative side having a long tail, so I wondered whether your perturbation function was completely symmetrical.

K. A. Shtejns: I assumed it to be symmetrical, but this does not play an important part in my investigation.

https://doi.org/10.1017/S0074180900006756 Published online by Cambridge University Press