PART II

THE EFFECTS OF ROTATION ON STELLAR ATMOSPHERES

THE EFFECTS OF ROTATION ON THE ATMOSPHERES OF EARLY-TYPE MAIN-SEQUENCE STARS

(Review Paper)

GEORGE W. COLLINS, II

Perkins Observatory, The Ohio State and Ohio Wesleyan Universities, U.S.A.

1. Introduction

In discussing the status of the theory of rotating stellar atmospheres, it is necessary to draw upon the contributions of many well established aspects of astrophysics and to interconnect them in a cohesive pattern structured so as to provide insight into a rather specific problem – namely, the structure and characteristics of a surface of the star undergoing axial rotation. Many different connections are possible having varying degrees of emphasis and, of necessity, those given here represent only one such presentation. The discussion could be much simplified if it were not necessary to test the efficacy of the theoretical development by referring to observations. Unfortunately, such a comparison is necessary and the results are at the moment somewhat inconclusive. This unhappy situation arises from the retrospectively obvious fact that axial rotation does not play a dominant role in determining the directly observable properties of stars. Indeed, if rotation were a dominant factor, earlier attempts at describing stellar structure and evolution would have met with little success. However, it is becoming increasingly clear that the structure and final evolution of highly evolved stars are greatly influenced by the total angular momentum which they retain from their earlier history. In order to understand the angular momentum distribution present in the final state, it is necessary to understand the effects of stellar evolution on the total angular momentum and its distribution. But even before this step can be taken, one must first successfully describe the rotational structure of the main-sequence phase as it is this state which provides the initial conditions necessary for any further study. It is further appropriate that we attempt to describe this period of a star's life as the largest body of observational material with which we must test our results exists for these stars. In addition, we should expect a study of the atmospheric structure to be the most fruitful as this is the region of the star which provides the final modification of the radiation we observe.

The observations will be based on the integrated contributions from regions on the surface of the star which differ greatly from one another with respect to the parameters that define the local radiation field. Thus, unlike the theory of stellar atmospheres, our task is not just to determine the variation of state-variables with depth at a specific point on the star, but rather at any point on the star. In principle, we are therefore required to construct many model atmospheres representing the local structure of the star, recognizing that to some extent these models will interact with each other. In

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addition, it is clear that the parameters defining the atmospheric structure will be determined by the underlying interior structure of the star. We shall not attempt to describe the state of the theory of rotating interiors, but only present the results of several investigators and indicate the extent to which observed parameters may be sensitive to their results.

We shall then assume that the theory of stellar interiors provides us with not only the variation of effective temperature and gravity over the surface, but also with the shape of that surface. With this information, we may then estimate the extent to which various points on the surface will affect the model structure at other points. If the extent of this interaction is not large, we may employ the existing theory of stellar atmospheres to determine the state of the local radiation field resulting in both continuous and line radiation. Then, taking into account the Doppler effect, the contribution of the local field to the observed field may be integrated over the surface of the star yielding observed parameters. Many different detailed approaches have been taken to this problem and we shall herein undertake to describe some of the more successful ones and indicate their results.

In the first section, we shall describe the results of interior studies insofar as they enable us to estimate the variations of the effective temperature, surface gravity and shape and then using these results, indicate the extent of the horizontal interaction that may be expected between models. Following this, we shall undertake a brief description of the methods used to determine the continuum and bolometric properties of the radiation field indicating some of the results of these studies. Since the numerical techniques required to investigate the effects of rotation on the formation of spectral lines are essentially the same as those required for the investigation of the continuum, we shall conclude this section with a review of those studies relating to spectral lines. In the next section, we shall undertake to investigate the success which theory has had in describing observational results. Finally, we shall make some concluding remarks about what may be inferred at the present time about the nature of the early-type stars.

2. Specification of Atmospheric Parameters

As we have already indicated, it is necessary to inquire of the results of rotating interior investigations to ascertain the run of effective temperature and gravity over the surface of the star. In addition, we must determine the extent to which atmospheric structure at one point on the surface will influence the structure at another. These problems are not unrelated as the resolution of the latter depends largely on the former.

Ever since the internal structure of stars became of interest to astrophysicists, researchers have demonstrated concern over the effects of rotation upon this structure. A detailed account of the development and present status of model rotating interiors here would be inappropriate. However, it must be noted that in all such models made to date, the distribution of angular momentum within the star is either specified in an ad hoc manner or, within certain constraints, is arbitrary. The central reason for this situation is that the nature of the forces which bring about redistribution of angular momentum tending to an equilibrium value, is not well understood. Indeed, it may well be that if radiative and kinematic viscosity are the only agents available for this re-distribution, the star may not reach an equilibrium state in a time which is less than its main-sequence life-time. The implication of this for the computation of model stars is simply that the apparent arbitrariness now present in existing models may be seen reflected in nature.

In any event, we may specify the local gravity and the surface shape of the star from the combined gravitational and rotational potential. Thus

$$\mathbf{g} = + \nabla\mathbf{\Phi} = + \nabla\left[\Phi_0 + \tfrac{1}{2}\omega^2 r^2 \sin\theta\right] \tag{1}$$

where Φ_0 is the gravitational component of the potential while r and θ are defined by Figure 1.

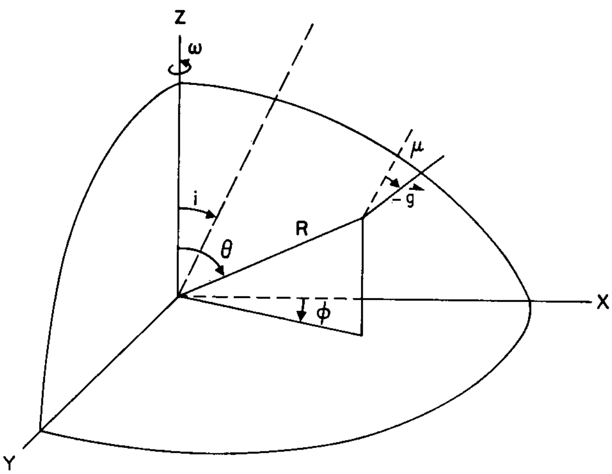

Fig. 1. One octant of a star rotating at critical velocity, indicating the defining coordinate system, local surface gravity, etc. (reprinted courtesy of the *Astrophysical Journal*).

Also

$$R(\theta) = R\left[\Phi = \mathrm{const}\right]. \tag{2}$$

The remaining parameter required to specify an atmosphere is the effective temperature. In order to do this, we must make use of the often misunderstood theorem of von Zeipel (1924), the most lucid presentation of which is due to Eddington (1926). In light of the modern theory of stellar structure, this theorem can be rigorously viewed as a proof by contradiction that stars in both hydrostatic and radiative equilibrium cannot rotate as rigid bodies. However, the more interesting and more used corollary that, a rigidly rotating star in both hydrostatic and radiative equilibrium will have a variation of radiative flux which is proportional to the local gravity, may well hold to a high order of approximation in stars exhibiting differential rotation. Indeed, if the following two conditions are met, then the corollary will hold rigorously in any distorted star:

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(a) the state variables P, T, and ϱ are functions of the potential alone,
(b) the radiation pressure is a function of the local state variables alone.

The first of these conditions may be violated by the existence of any circulation currents generated by differential rotation as can be seen from the equation of hydrodynamic flow for a medium in steady state:

$$\varrho\nabla\boldsymbol{\Phi} + \nabla\mathbf{P} = \varrho\left[\mathbf{u}\cdot\nabla\right)\mathbf{u}\right]. \tag{3}$$

Here local u is the stream velocity in the rotating coordinate frame and the right-hand side of (3) represents essentially the forces exerted on a volume element by the circulation flow. Since the flow is driven by the rotation, we should expect the magnitude of the right-hand side to be small compared to either term on the left-hand side, even in the lower atmosphere. However, the magnitude of the effect of the right-hand side of (3) on the von Zeipel condition that $F \sim g$ still remains to be investigated in a rigorous manner.

The dependence of the radiation pressure on the local state variables is nicely satisfied in the lower regions, but of necessity must break down to an increasing extent as one enters the atmosphere. This is evident from the more or less global nature of the equation of transfer appropriate for the atmosphere. That is, in the atmosphere the radiation field, and hence its higher moments, no longer depends on the local physical parameters of the gas but is strongly influenced by the neighboring conditions. The extent to which this condition affects the von Zeipel condition has also never been thoroughly investigated. We may hope that the variation of physical parameters contributing to the majority of the emergent radiation is sufficiently small to allow the use of the von Zeipel result that

$$F = -\frac{4c}{\kappa\varrho}\frac{\mathrm{d}P_r}{\mathrm{d}\Phi}\,g = -f(\Phi)\,g \tag{4}$$

which implies $F \sim g$ on a surface of constant Φ.

Slettebak (1949) has observed that the constant of proportionality appropriate for the integrated flux may then be obtained by integrating the gravity over the surface of the star and equating the result to the total luminosity of the star. So that

$$L(\omega) = \sigma C(\omega) \int_A g\,\mathrm{d}A \tag{5}$$

yielding the effective temperature variations over the surface through

$$T_e^4(\omega, \theta) = C(\omega)\,g\,. \tag{6}$$

This procedure has been used since by Collins (1963, 1965, 1966, 1968a, b), Roxburgh and Strittmatter (1965), and Rubin (1966) while Hardorp and Strittmatter (1968a) preferred to replace the surface integral in (5) with a volume integral of $\nabla^2\Phi$.

It is clear from Equation (5) that any study of rotating stellar atmospheres must rely on interior calculations to provide knowledge relating to the variation of the total luminosity of a star of given mass $L(\omega)$ with ω. Many authors have investigated this variation (Chandrasekhar, 1933; Sweet and Roy, 1953; Roxburgh *et al.*, 1965; Limber and Roberts, 1965; Epps, 1968; Sackmann, 1968; and Roxburgh *et al.*, 1968) by means of perturbation theory and have found the intuitively expected result that at least to first order in ω^2 the luminosity decreases linearly by an amount depending slightly on the detailed nature of the model. At present the effect and indeed even the sign of the second order term is somewhat in doubt. Roxburgh and Strittmatter (1965) find them to be small and of the same sign as the first order terms, while Sackmann (1968) finds them to be comparable to the first order term, but of opposite sign. Her results for $L(\omega_c)$ seem in agreement with Roxburgh *et al.* (1968).

Having discussed the relationship of the effective temperature variation to the total luminosity and the surface gravity, we still must turn to the interior models for the detailed evaluation of Equation (1). The geometrical shape of the equipotential surface is governed only by the internal mass and angular momentum distributions. Most investigators agree that to better than 0.1% the gravitational potential can be represented at the surface by that of a point mass. The apparent arbitrariness of the angular momentum distribution would indicate that greater accuracy than this is unnecessary. A notable exception to this is presented by Mark (1968) wherein he shows that appreciable differential rotation may lead to extreme distortion of the inner core. The scale of the equipotential surface (i.e., that level surface which can be said to define the surface of the star) is given by the boundary conditions of the interior models. The variation of this parameter with rotational velocity is, like the total luminosity, determined by perturbation theory. Again, there is general agreement concerning the behavior of the first order terms, but the nature of the second order terms is subject to some controversy. There does appear to be near general agreement that both $L(\omega)$ and the scale factor (e.g., the polar radius $R_p(\omega)$) are affected in the same sense by a variation of ω. That is an increase in ω for models of constant mass brings about a decrease in both $L(\omega)$ and $R_p(\omega)$. This appears to be true for a large range of models and angular momentum distributions. As pointed out by Collins (1965), this type of behavior for $L(\omega)$ and $R_p(\omega)$ will result in a reduced effect on the distribution of effective temperature over the surface of the star. In essence then, the variation of effective temperature over the surface is rather insensitive to the change in the model parameters $L(\omega)$ and $R_p(\omega)$. Unfortunately, this is not true for the surface gravity or the integrated flux.

Finally, in order to construct a local atmosphere appropriate for any point on a rotating star, we must estimate the extent to which one atmosphere model will interact with its neighbor. This interaction could arise from two causes:

(1) Disruption of radiative equilibrium by circulation currents which convey energy in and out of a given region.

(2) Radiative flux being transported horizontally through the atmosphere driven by a latitudinal temperature gradient.

Von Zeipel's Theorem assures us that both these conditions will prevail, but does not allow us to estimate the effect they will have on atmospheric structure. However, we may crudely estimate the extent to which radiative equilibrium can be disturbed by the meridional circulation by estimating the ratio of energy flow that might be expected from circulation currents to the energy carried by radiation. This ratio is very roughly given by

$$r \ll \tfrac{1}{2}\varrho u^3/\pi\sigma T_e^4 \tag{7}$$

where u is the stream velocity and T_e is some typical effective temperature. It is unlikely that the stream velocity could reach much more than 10% of the equatorial rotational velocity as these currents are driven by the rotation itself. Thus, choosing the generous value for the density in the atmosphere of a rotating early-type star to be 10^{-9} g/cm^3 and a maximum equatorial velocity of 500 km/sec, together with a mid-latitude temperature of 10^4 K, we arrive at $r < 3\%$. Since we have been generous on the side of circulation in our choice of numbers, it is most probable that we can safely ignore the effects of circulation currents on the atmospheric structure. A more sophisticated argument reaching a similar conclusion is given by Strittmatter (1969).

A 'Devil's Advocate' for the case for circulation currents, at this point, would point out that in the equatorial region of Be stars the situation may well be different due to the lower effective temperature and possibly higher u. Although this may be true, the relevance of this argument to the computation of the theoretical spectrum of such a star is diminished by: (a) the substantially lower density, (b) the reduced luminosity of the equatorial region which contributes to the integrated spectrum. A far more serious problem is the sensitivity of the surface temperature distribution in early-type stars to small departures from radiative equilibrium. It may well be that departures from radiative equilibrium of 0.1% that might result in early-type stars may be sufficient to upset the calculation of relatively strong lines. Unfortunately, a thorough study of this effect has never been made.

The final possibility for horizontal interaction of atmosphere models lies in the existence of a horizontal surface temperature gradient. The magnitude of this gradient compared to the radial gradients will provide an estimate of the degree of coupling to be expected between adjacent points on the stellar surface. The radial gradient may be estimated by noting that in most atmosphere models of early-type stars, the temperature drops from 10^5 K to 2×10^4 K in a physical distance of about 10^4 km. Thus the radial gradient should be on the order of 10^{-4} K/cm. However, the horizontal gradient cannot exceed that resulting, ala von Zeipel, from a temperature of 2×10^4 K at the pole to zero at the equator. This yields a temperature gradient of less than 10^{-7} K/cm or three orders of magnitude less than the radial gradient. Thus, we may simply ignore this form of horizontal coupling.

It is also to be pointed out that magnetic fields do exist in most stars and in some may well be strong enough to affect the stellar structure. For a detailed discussion of the influence of such fields, as well as a more comprehensive picture of the present state of rotating stellar interior studies, the reader is referred to review articles by Strittmatter (1969) and Roxburgh (1970).

3. Calculation of Observable Parameters

Having established that we can locally approximate the surface of a rotating star with atmospheres made in accord with present atmospheric theory, we may now turn to the calculation of those parameters accessible to observation. To do this, we must first describe the local radiation field of the atmosphere at any point on the surface of the star. The only major difference between this procedure and the one normally followed in stellar atmospheres is that we shall require knowledge of the emergent specific intensity rather than the local flux. This is necessitated by the fact that the star is no longer spherical and the conditions defining the local radiation field vary over the surface. Thus

$$F_0 = 2\pi \int_A I\mu \, \mathrm{d}A \neq 4\pi^2 R^2 \int_{-1}^{+1} I\mu \, \mathrm{d}\mu$$

as is the case for spherical stars. As a result, the left-hand integral must be evaluated numerically.

The basic approaches for evaluating the integral of the specific intensity over the surface of a star are independent of whether or not one is dealing with radiation arising in the continuum or in a region dominated by spectral lines. Indeed, calculation of the net state of polarization of the integrated radiation may be done using basically the same techniques if care is taken always to refer the state of polarization to an external coordinate frame. The details of the techniques used vary from author to author, but usually involve the use of some general two-dimensional quadrature technique. The exact nature of the quadrature scheme is not terribly important unless computer time is a scarce commodity. In this case, the Gauss-type quadrature schemes are to be preferred due to their much higher efficiency. Regardless of the precise nature of the chosen quadrature scheme, one must be careful to use a sufficient number of points to inspire confidence in the accuracy of the result.

One numerical technique is worthy of note as it greatly simplifies the formulation of the integration limits and reduces the number of atmospheres which must be calculated. Collins (1965) noted that if the function to be integrated exhibits both axial and equatorial plane symmetry, the integral over the apparent surface is formally equal to the integral having limits of 0 and π in the polar angle and $\pm\pi/2$ in the azimuthal angle. This is true regardless of the angle of inclination. Of course, since the angle of inclination explicitly enters into the calculation of μ, the integral, $\int_{-1}^{+1} I(\mu)\mu \, \mathrm{d}\mu$, will be angle dependent. However, a corollary of this symmetry property is that any scalar function which has the proper symmetry and is uniquely defined on the surface of the star will have an area-average which is independent of the angle of inclination. This is the case for such functions as the effective temperature and the scalar value of the local gravity. Since this result is contra-indicated by both theoretical calculation of the spectral energy distributions and observations of rotating stars, one may conclude that arithmetical area-average values of T_e and g are inappropriate for describing the observed properties of these stars.

One may divide the results of studies of the theoretical properties of the radiation field of rotating stars into two broad areas:

(a) Results relating to the continuum.

(b) Results relating to line profiles and strengths.

We shall discuss the results from these two areas separately and in the last section, attempt to relate them to observational results.

A. CONTINUUM RESULTS

The effects of rotation upon the continuum radiation of stars have been studied by a number of authors since 1963. However, the earliest effort appears to be due to Sweet and Roy (1953) where, on the basis of interior models, they estimated the effect on the bolometric magnitude to be on the order of one magnitude. Their calculation relates to the total energy output of the star and not to that quantity that would be determined by an observer. The difference lies in the fact that the radiation field of such a star is not isotropic and any 'absolute' quantity measured by an observer will depend on the orientation of the star.

Attempts to measure the effects of varying the aspect of the star upon the observed bolometric magnitude were first made by Zhu (1963) and Collins (1963). In the absence of the interior model studies, both authors made assumptions about the variation of total luminosity and polar radius with angular velocity. Subsequent interior studies have shown that these variations are not important insofar as the estimation of the aspect effect is concerned. Zhu (1963), assuming that $F=\sigma T_e^4$, calculated the variation in the observed bolometric magnitude due to aspect effects for stars rotating at the critical velocity. Collins (1963) investigated this case while also studying the effect on the spectral energy distribution when the atmosphere was assumed to be locally gray. In addition, he considered the variation of these effects with rotational velocity. These studies, as well as all others since (with the exception of Rubin, 1966), have assumed that the gravitational part of the effective potential is given by that of the Roche Model. Rubin (1966) carried out calculations similar to Zhu (1963) with a homogeneous mass distribution rather than a Roche model. The resulting Maclaurin Spheroids, being more highly distorted than a Roche Model with corresponding angular velocity, yielded a larger maximum aspect effect of 1.36 magnitude as opposed to about 0.80 magnitude for the Roche Model. The surprising result is the close agreement between these two vastly different models. Thus, one may expect about 1 magnitude variation in the observed bolometric magnitude to be present in the most rapidly rotating stars as a result of aspect effects alone.

Roxburgh and Strittmatter (1965) repeated some of the calculations of Collins (1963) including the variation of total luminosity and polar radius with rotational velocity which are predicted by the interior calculations of Roxburg *et al.* (1965). Allowing for the variation of total luminosity, the two studies are in excellent agreement and indicate that a star will be displaced to be right and above the main sequence on a color-magnitude diagram (see Figure 2). Strittmatter (1966) states that this displacement will be proportional to the square of the equatorial velocity. However,

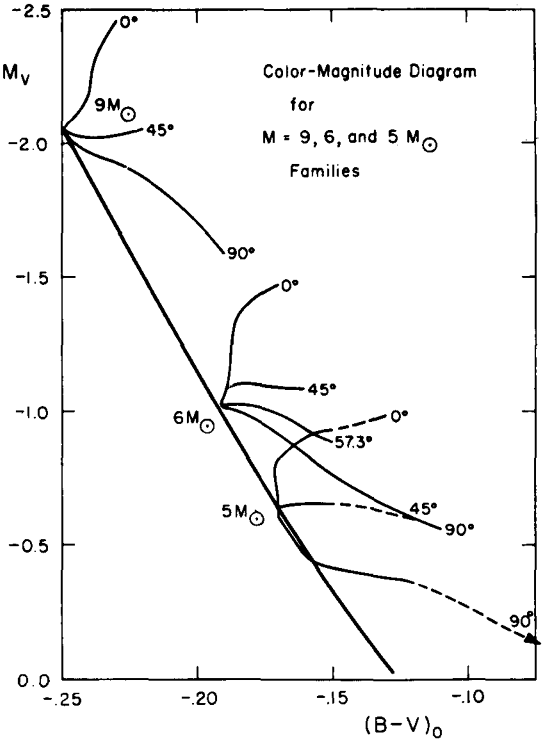

Fig. 2. A color-magnitude diagram indicating the effect of rotation upon early-type main-sequence stars. A model moves along any given line a distance roughly proportional to ω^2 with the end points representing models for which $\omega = \omega_c$. The dotted lines depict models in which there is some uncertainty (reprinted courtesy of the *Astrophysical Journal*).

Mander (1968) finds observationally that the displacement is proportional to V^α where α varies with spectral type. Golay (1968) finds that the constant of proportionality is also a function of spectral type. Replacing the gray atmosphere with a non-gray atmosphere and attempting to reconcile the critical angular velocity models of Roxburgh *et al.* (1965) with the first-order perturbation theory of Sweet and Roy (1953), Collins (1965) concludes that higher-order terms may defy any simple description and in addition, that the amplitude of the displacement is color-dependent. Collins and Harrington (1966), on the basis of an extended non-gray atmosphere study, find that rotation has little or no effect on photo-electric color-color diagrams. This is clearly indicated in Figure 3. This leads to the result that reddening effects may be distinguished from color changes introduced by rotation on a color-color diagram.

All work on the continuum which has attempted to predict the spectral energy distribution has lead to the conclusion that the largest effects will occur in the vicinity of the energy maximum. Thus, in the B stars, one would expect rather large effects in the far ultraviolet. However, as Hardorp and Strittmatter (1968a) point out, not only is the ultraviolet flux sensitive to aspect, it is also very sensitive to $(B-V)_0$. Thus,

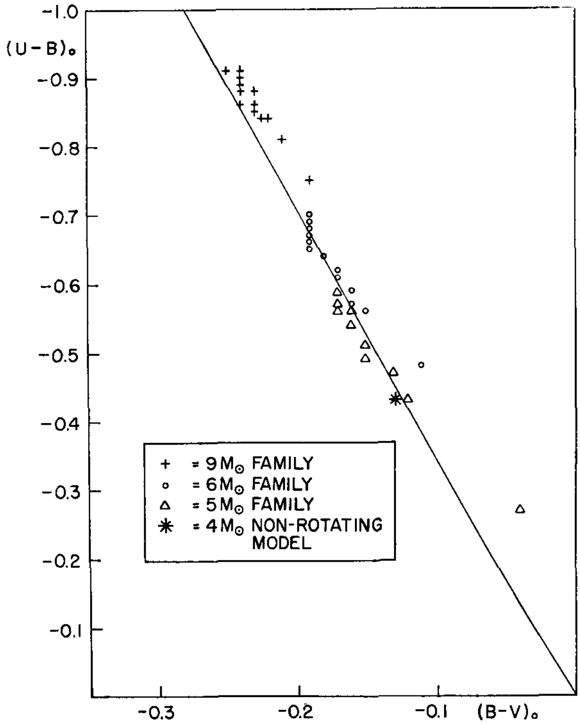

Fig. 3. A color-color diagram for various rotating models indicating the displacement along the Johnson-Morgan zero-age main sequence (reprinted courtesy of the *Astrophysical Journal*).

they predict no more than 0.5 mag. variation in m_{1350} due to rotation for stars of the same $(B-V)$. In this study, they also carry out theoretical calculations for A and early F stars which heretofore had been largely neglected. They found, as did Kraft and Wrubel (1965), that a change in the Strömgren c_1 index of about 0.2 magnitudes may be expected to result from rotational effects.

We may therefore summarize the effects of rotation upon the spectral energy distribution of stars as predicted by the numerous theoretical studies carried out since about 1963 as follows:

(i) large effects (about 1 mag.) in the observed bolometric magnitudes (and hence in bolometric corrections) may be expected.

(ii) rotation may generate between 0.5 and 1 mag. changes in M_v depending on V, aspect and $(B-V)_0$.

(iii) for small rotation, we may expect these changes to be roughly proportional to V^2.

(iv) rotation has little or no effect on the color-color diagram.

(v) the effect of rotation on color is such as to displace the star to the right and above the main sequence.

(vi) approximately 0.2 magnitude increase in the Strömgren c_1 index for a star may be introduced by extreme rotation.

Before we turn to the effects of rotation upon spectral lines, there is one further aspect of the continuum radiation field that should be discussed. Chandrasekhar (1946) and later Code (1950) indicated that polarization would arise in atmospheres of early-type stars due to the presence of free electrons and the anisotropy of the radiation field near the surface of the star. Normally, this polarization would average to zero when the contribution from all parts of a spherical star were summed up. However, as this zero result arose from the spherical symmetry and not from any physical properties of the atmosphere, it was to be expected that a rotationally distorted star should show a net intrinsic polarization. This possibility was investigated by Harrington and Collins (1968) using the gray atmosphere development of Chandrasekhar (1946) and it was found that up to 1.8% net polarization might be expected in the continuum of stars undergoing extreme rotation and seen nearly equator-on. However, further investigation by Collins (1970) employing non-gray atmospheres ascertained that the wavelength dependence of the polarization was such that it could only be detected in the line-free sections of the far ultraviolet and that the amount to be expected in the visible is essentially zero. There appears to be some difficulty in reconciling this result with recent observations and we shall return to this problem in the last section. Both Harrington (1969) and Collins (1970) point out that polarization arising from Rayleigh Scattering in the atmospheres of late-type giants may yield an amount considerably in excess of that to be expected from the gray studies of Chandrasekhar (1946) and Code (1950). Indeed, if these stars are distorted by some mechanism (i.e., rotation, magnetic fields, or non-radial pulsation), intrinsic polarization of an amount comparable with that presently observed may well be present.

B. ROTATIONAL EFFECTS ON LINE PROFILES AND EQUIVALENT WIDTHS

In contrast to the rather short time during which the rotational effects upon the continuum have been studied, the history of the investigation into the effects of rotation upon spectral lines dates back 50 years. The theoretical study of rotational profiles initiated by Shapley and Nicholson (1919) was expanded and formalized by Carroll (1928, 1933) and later applied to actual stars by Carroll and Ingram (1933) and others. An essentially geometrical formulation of the problem was given by Shajn and Struve (1929) and it is this form which has been most frequently used.

However, the problem as formulated by both Carroll (1928, 1933) and Shajn and Struve (1929) neglects effects of limb-darkening, 'gravity-darkening' (resulting from von Zeipel's condition), and shape distortion. Unsöld (1955) describes an analytic method for including the effects of limb-darkening in the method of Shajn and Struve (1929). Slettebak (1949) using essentially this method did include effects of limb-darkening, shape distortion and gravity darkening separately, but did not estimate the effects of varying aspect. For a more detailed account of these first efforts to analyze rotationally-broadened lines, the reader is referred to the very excellent review article by Huang and Struve (1960).

All of the initial efforts quite properly focused primary attention on the effects of the rotational Doppler broadening alone. Thus, the nature of the line profile that is formed locally on the surface of such a star received little or no attention. This somewhat historical breakdown of the problem also serves as a useful astrophysical delineator for its investigation.

To investigate the effects of rotational Doppler broadening alone, one has usually assumed a form for the local line profile and then, correcting for the rotationally induced Doppler shift, has convoluted these profiles over the surface of the star to obtain the observed profile. Initially, the locally assumed profile had some simple analytic form such as a delta function or a Doppler profile. Except for the limb-darkening correction of Unsöld (1955), the profile was assumed to be the same at all points on the disk. Slettebak (1949) and later Stoeckley (1968a, b, c) assumed that the local profile could be accurately determined by using carefully measured empirical profiles for known sharp-line stars. This has the advantage that in using the theoretically resulting profiles to interpret observed profiles for rotating stars, the instrumental broadening is automatically included in both results. It has the disadvantage that one cannot be sure that a profile obtained from a sharp-line star will be at all related to the profile generated locally on the surface of a rotating star. For determining first-order effects such as the value of $V_e \sin i$, the approach is probably justified for all but the most rapidly rotating stars. However, for investigating second-order effects such as differential rotation, there appears to be some doubt as to the effect of such an assumption.

More recently a variety of authors (e.g., Collins and Harrington 1966, and Hardorp and Strittmatter, 1968b) have attempted to remove the assumption of the form of the local line profile shape. This is done by calculating a non-gray atmosphere appropriate for the local surface condition and then, in accordance with the theory of radiative transfer, calculating a specific intensity profile at each point on the disk. Thus, provided with the local profile shape, the results are then integrated over the apparent disk so as to yield a flux profile. This provides a self-consistent means for estimating the importance of various effects resulting from the rotation of the star on a resulting profile.

Rather than delineate the findings of each of the many investigators in this field, it is perhaps more appropriate to indicate the nature of the effects and discuss what is presently known about the importance of each upon the resulting profile. Starting from a purely theoretical base, one naturally inherits all of the uncertainties inherent in calculating a line profile for an ordinary star. In addition, the importance of these uncertainties will vary over the surface of the star and therefore the ultimate effect on the final result is more difficult to ascertain than in the non-rotating case. As the nature of the assumptions which result in these uncertainties have been widely discussed in the literature, we shall only briefly list them with some qualitative comments relating to their estimated importance.

Considerable emphasis has been placed on the effects that departures from L. T. E. may have on the formation of spectral lines (cf. Underhill, 1966, 1969). In all likelihood, we need not be concerned about non-L.T.E. influences on the continuum field as Mihalas (1967) has shown that these departures are negligible for non-rotating models. However, as the core of strong lines are formed much higher up in the atmosphere, departures from L.T.E. may be important in their formation. Among main-sequence stars, the most likely candidates for having a physical condition which might lead to departures from L. T. E. are the early B-type stars – particularly those which are rapidly rotating. These departures have two basic origins which are somewhat related. Firstly, a breakdown in the Saha-Boltzman equation resulting from a distortion of the free electron velocity distribution from that expected of a gas in thermal equilibrium. This results in a variation in the population of the various atomic levels from the thermal equilibrium values. This effect, although investigated for non-rotating stars, has never been dealt with for rotating atmospheres. However, it must be pointed out that those regions where the pressure and density are low enough to bring about significant departures from L.T.E. are the equatorial regions where due to the von Zeipel condition, the radiative flux is relatively low. Thus, the contribution to the surface integral of the intensity should be minimized. It is also this region where one would expect effects of meridional circulation, a breakdown of the von Zeipel condition, etc., to have the largest effect.

Secondly, a deviation of the line source function from the Planck function resulting from non-coherent resonance scattering is important when investigating the effects of rotation upon the low-order members of the Balmer series of hydrogen as well as any resonance line. This effect was included by Collins and Harrington (1966) and it was found to lead to approximately a 20% difference in the intensity of the core region of Hβ. Since it is the core radiation that dominates most of a rotating hydrogen line profile, the inclusion of this effect appears to be important and may explain some of the qualitative differences between the calculations of Collins and Harrington (1966) and Hardorp and Strittmatter (1968b). Strom and Kalkofen (1966, 1967) have considered the effects of departure from L.T.E. upon the spectral energy distribution and suggest an observational test for determining the extent to which they are present. They found (1967) that the Paschen- to Balmer-jump ratio was insensitive to rotation, but systematically affected by departures from L.T.E. However, a very carefully designed observational program would be required to detect the effect, since it is possible that other factors may influence this ratio.

In addition to departures from L.T.E., one should also be concerned with the possible existence of small departures from radiative equilibrium which can lead to large variations in the surface temperature distribution in B stars, effects of line blanketing, and the choice of broadening theory to use for hydrogen lines. Although the relevance of none of these problems to rotating line profiles has been investigated, one may take some solace in the fact that at least the last two do not seem to have a major effect on the profiles of single stars. However, since the former may only arise because of the presence of axial rotation, its importance will be much harder to assess. Additional problems may arise from the exclusion of an important opacity source in the theoretical models. Strom and Strom (1969) have been lead to the conclusion that silicon is important in determining not only the shapes of hydrogen line profiles, but also the continuum energy distribution. If this is so, then great care must be exercised in understanding the importance of heretofore ignored elements in early-type stars.

Most of the current investigation has been directed toward estimating the global effects of rotation upon the line shape. The early work of Shajn and Struve (1929) showed that the profile resulting from a uniformly rotating and undarkened star of an infinitely sharp line would be elliptical or 'dish-shaped'. This basic form will be modified by limb-darkening, 'gravity-darkening', shape distortion, differential rotation, the variation of ionization equilibrium over the surface and the aspect which the star presents to the observer. The inclusion of limb-darkening as formulated by Unsöld (1955) is only appropriate for spherical stars not affected by 'gravity-darkening'. The more recent studies of Collins and Harrington (1966) and Hardorp and Strittmatter (1968b) automatically include this effect as well as 'gravity-darkening' through the nature of the locally non-gray atmospheres. Two papers of Collins (1968a, b) were specifically aimed at estimating the relative importance of limb-darkening and aspect effects, 'gravity-darkening' and the variation of ionization equilibrium over the surface. Although the simplified expression for the center-limb variation of pure absorption and scattering lines resulting from the Milne-Eddington atmosphere approximation together with an ad-hoc dependence of the line absorption coefficient on temperatures were used, the results were sufficient to indicate the relative importance of these three effects on the equivalent width. Even in cases where the temperature dependence of the line absorption coefficient was large (i.e., $\eta_\nu \sim T_e^{10}$) the dominant factor in determining the variation of line strength with rotational velocity and aspect appeared to be the nature of the center-limb variation. However, the sign of the temperature dependence was of importance in that a line with a mass absorption coefficient decreasing with increasing temperature was less affected by other rotational effects than one with the opposite dependence on temperature (see Figure 4). Thus, one would expect that for B stars, the equivalent width of lines such as the Balmer lines to show fewer effects of rotation than say He I lines. This result seems to be borne out by the more detailed studies of Collins and Harrington (1966) and Hardorp and Strittmatter (1968b). However, it must be pointed out that all line effects seem to have a somewhat stronger dependence on angular velocity than ω^2 which might have been expected from the first-order interior studies. Indeed, noticeable effects on the equivalent width of spectral lines appear to be present only for stars rotating faster than 80% of their critical velocity. This led Collins (1968b) to conclude that only stars undergoing extreme rotation might be assigned a spectral type that was different from a similar non-rotator. It was also found that the qualitative effect of rotation upon the equivalent width was almost totally insensitive to the choice of the temperature gradient required by the Milne-Eddington model. This would lead one to hope that some of the problems mentioned earlier leading to errors in the surface temperature distribution (e.g., departure from L.T.E.) would not affect the equivalent width of the spectral lines of similar stars differing only in rotational velocity and aspect. This will probably not be the case for the line profiles.

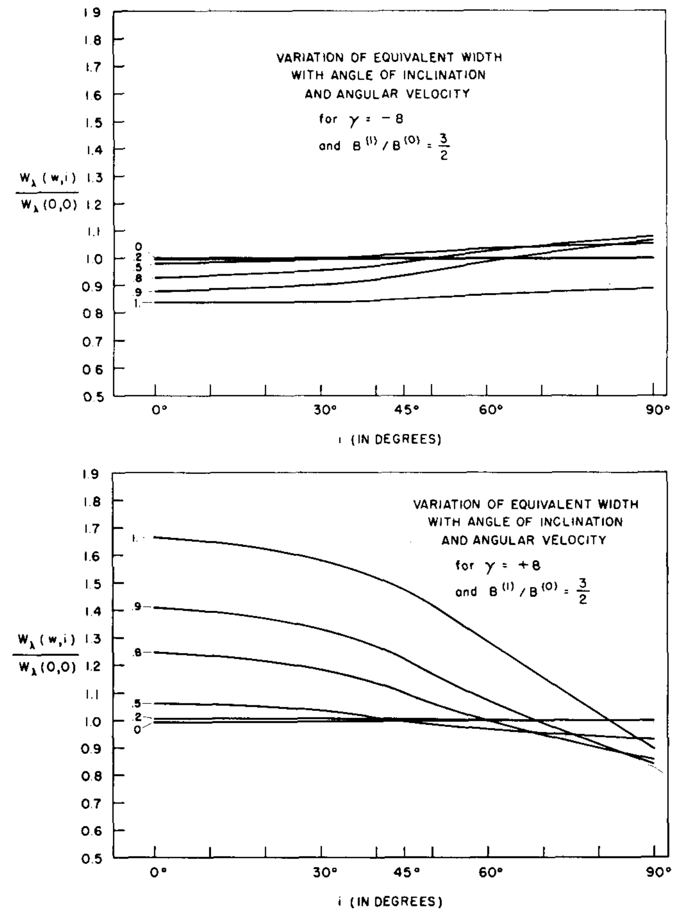

Fig. 4. Variation of equivalent width with fractional rotational velocity and angle of inclination for lines where $\eta_\nu \sim T^{\gamma}{}_e$ and $\gamma = \pm 8$. The factor $B^{(1)}/B^{(0)}$ specifies the source function gradient for the Milne-Eddington type atmosphere (reprinted courtesy of the *Astrophysical Journal*).

The effects of limb-darkening and 'gravity-darkening' on the line profile itself have been investigated by several authors. Slettebak (1949) found that neglecting both limb-darkening and 'gravity-darkening' led to a change in the shape of the line profile such as to lead to an underestimate of $V_e \sin i$. However, he also noted that only in the case of extreme rotation was the effect of 'gravity-darkening' important. Hardorp and Strittmatter (1968b) find a much larger discrepancy (about 40%) between $V_e \sin i$'s determined from gravity-darkened and undarkened profiles for stars rotating within 5% of their critical velocity. Nothing can be said about the far larger number of more slowly rotating objects as their study does not include models of moderate rotation. Similar results are indicated by Friedjung (1968) with a maximum error attributable to gravity-darkening at the critical velocity of 25%. However, his calculations extend to lower velocities and it appears that the effect quickly disappears. At least for $V_e < 300$

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km/sec., he shows that the errors introduced by his assumptions regarding the nature of the local line formation exceed the difference introduced by gravity-darkening.

Since it is guaranteed by the von Zeipel Theorem that stars rotate differentially and it is observed to be the case with the sun (Abetti, 1955), it is logical to inquire into the effects of differential rotation on the shape of spectral lines. This problem has been investigated by a number of people from several different points of view.

Slettebak (1949) concluded that the effect of differential rotation upon line profiles would be difficult to detect observationally. Similar conclusions have been reached by other authors (see Huang and Struve, 1960; also Qin-Yur and Jia-bing, 1964). More recently, Stoeckley (1968a), using the assumption that the local line profile shape could be empirically determined, presented a theoretical method for determining $V_e \sin i$, and a differential rotation parameter he calls S. However, even with observational line profiles which surpass those of Slettebak by nearly two orders of magnitude in accuracy, he is forced to conclude that "No general conclusion can be reached at this time, because many of the observations, although accurate by normal standards, should be much improved".

The importance to astrophysics of observationally determining the extent to which rotating stars differentially rotate cannot be minimized, but great care must be exercised when that determination rests on a theory where the nature of the second-order effects are not well understood. One cannot help but wonder if the uncertainties in the theoretical structure of rotating stellar atmospheres previously mentioned justify the pursuit of such an elusive quantity as differential rotation which apparently requires observational accuracy better than 0.1% in the line profile.

There remain other uncertainties than those already mentioned which cannot be ignored if a highly accurate theory of rotational line broadening is to be formulated. For example, it is not clear that turbulence, induced by rotation, might not be present in stars which rotate. This added complication to the velocity field might vitiate all other attempts to account for second-order effects. Thus great care must be exercised in drawing conclusions based on these second-order effects and indeed only those of a qualitative nature can be given any chance of surviving future inspection. In spite of the uncertainties, we may attempt the following qualitative conclusions resulting from line investigation:

(i) no unambiguous effects other than geometrical broadening are apparent in spectral lines except for the most rapidly rotating stars (i.e., $\omega > 0.8\omega_c$).

(ii) the determination of $V_e \sin i$ for the most rapid rotators is probably underestimated by between 10% and 40% by neglecting numerous 'second-order' effects.

(iii) for the most rapidly rotating stars, the center-limb variation is probably the dominant parameter in determining the line strength and the effect is most noticeable for lines with a mass absorption coefficient which increases with temperature.

(iv) in spite of its importance to astrophysics, no definitive conclusions can be drawn regarding the extent or even the existence of differential rotation. Due to the impressive increases in observational accuracy now possible in determining observational line profiles, it would appear that the greatest insight into this problem could be gained from an improved theoretical study of the conditions determining the surface velocity field in these stars.

4. Comparison of Theoretical Effects with Observations

In this section, we shall review the attempts that have been made to verify the predictions of the theory of rotating stellar atmospheres in the wealth of observational material which exists for these stars. Due to the volume of material and the existence of several other review papers in this colloquium dealing with this subject, we shall omit from discussion all of the material relating to multiple-star systems and most of the material on clusters.

There are several difficulties to be encountered in relating theory to observation. Firstly, except for the rotational Doppler broadening of spectral lines, all the directly observable effects we would hope to detect are small and quite near the limit of detection. Secondly, virtually all of the observational material which is at our disposal has been gathered for other purposes. Thus, it is not surprising that rotational effects have been largely ignored in developing systems to observationally specify stars. For instance, we shall find photometric systems whose standards are chosen without regard to rotation. Since a set of standards is by definition without systematic error, we have the possibility then that rotational effects will be largely lost from the system or at most appear as a systematic error in non-standard stars. Only the morphological system of spectral classification deliberately attempts to avoid a systematic bias introduced by rotational broadening by choosing a dispersion sufficiently low that rotational broadening is difficult to detect in any but the most rapidly rotating stars.

It would appear logical to divide our discussion of observational effects in much the same way as we did the theoretical predictions, but such is not the case. The nature of a star is usually specified by referring exclusively to either properties of the continuum such as M_v and color or to properties of absorption line spectra such as luminosity class and spectral type. Once the specifications have been made, then one looks for variance within that specification attributable to axial rotation. However, this is at best misleading as it is most likely that rotation will affect the nature of the specification to a degree equal to or larger than the variance to be expected within that classification. Thus, it is most appropriate that we first consider the evidence for effects of axial rotation on the parameters normally used to observationally specify an early-type star. The method of specifying the nature of a star for which there is a vast amount of observational material and which is probably the oldest is spectral type. In spite of the high degree of reliability and self-consistency that an individual investigator may acquire, the basic nature of the classification procedure is highly subjective and in many cases not fully specified. Indeed, it appears that the original specification of the system by Morgan *et al.* (1943) is not exactly followed by many investigators. Thus, without knowing in advance which line ratios and line strengths will be used by a given investigator, it is impossible for the theoretician to predict precisely what the effect of rotation may be upon the spectral class assigned any given star. In addition, it is not obvious that the concept of line strength is related to the predictable quantities of equivalent width and central depth, although if the dispersion is low enough, one would expect it to be more strongly correlated with the former than the latter.

The only hope of using this information for rotation studies arises from the result that equivalent widths of stellar lines will only be affected at a detectable level for spectral classification in the most rapid rotators (i.e., $\omega < 0.8\ \omega_c$) and thus we may assume that the spectral class assigned a star will be essentially unaffected by aspect or rotation effects even for stars of moderately large rotational velocity. However, Collins (1968b) did ascertain that if the original Morgan *et al.* (1943) criteria were adhered to, that the most rapidly rotating stars in the middle B region might well be classified a tenth of a class earlier than their non-rotating counterparts. Thus, he considers that a systematic difference between spectral type and photometric color should exist for the Be stars. A careful search for this effect has never been carried out as the photometric color must be corrected for the presence of any emission lines lying within the filter band pass.

A similar investigation led Collins (1968b) to conclude that again, except for the most rapid rotators, the luminosity class of a B star should be relatively unaffected by rotation. Unfortunately, his investigation did not extend to the A and F stars as the classification criteria became too complicated, but it would not be surprising if such a study resulted in similar conclusions for these stars as well.

A second and increasingly common method of specifying the nature of a star is through the use of various photometric indices. We shall not attempt to discuss the effects of rotation upon all of the numerous photometric systems in use today, but rather concentrate upon two types of systems in the hope that they will be illustrative of the types of effects that might be expected in other photometric systems of similar nature.

First, let us consider the rotational effects on the Johnson *UBV* system. As we have already seen, a theoretical investigation into these effects demonstrates that both the absolute magnitude M_v and photometric colors $(U-B)_0$ and $(B-V)_0$ will be affected both by the degree of rotation and the aspect presented by the star. This will be true even for moderate rotational velocities (e.g., $\omega = 0.5\ \omega_c$). The investigation of these effects is severely complicated by three different problems. Firstly, and most fundamental is that effects of rotation on the continuum radiation field are very similar to effects produced by evolution. Thus to clearly separate the two effects, an observational program should be designed to investigate stars in the same state of evolutionary development with varying rotational characteristics, or vice versa. Secondly, the *UBV* standard stars have not, in general, been selected with any regard to their rotational properties. Finally, the absolute characteristics of the *UBV* system seem to be difficult to reproduce (see Code, 1960) and it seems likely that the published definition of the filter transmission functions is in error. In light of these difficulties, the only clear statements that can be made are:

(i) The V magnitude should be affected by rotation by an amount varying with spectral type of between 0.8 and 1.5 mag. This appears to be verified from cluster

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studies, notably Roxburgh *et al.* (1966), Strittmatter and Sargent (1966) and Strittmatter (1966).

(ii) The effect of the rotation upon stars plotted on a color-color diagram is to move them down the main sequence with respect to their non-rotating counterparts. This theoretical result has at the present time not been verified primarily due to the intrinsic difficulties of the *UBV* system and small size of the effect.

We turn now to the observed effects on an intermediate band system such as that defined by Strömgren (1956). The effects on the *uvby* colors should theoretically be essentially the same as those predicted for the Johnson system with the possible exception of the *u* filter. Accurate theoretical estimates of rotational effects on this part of the spectrum require atmospheres which accurately represent the Balmer decrement. An analysis of this nature has recently been carried out by Golay (1968).

Hardorp and Strittmatter (1968a) compute the change of the c_1-index with rotation and aspect. They find that for A and early F-type stars, one should expect changes due to aspect and extreme rotation of as much as 0.16 in c_1 above the average value to be expected for these stars. Kraft and Wrubel (1965) calculated a similar effect and indicated that they felt an anomalously large spread in c_1 on a c_1-$(b-y)$ diagram existed for stars in the Hyades. This they attributed to rotation. The more careful study of Golay is in qualitative agreement with these earlier studies, but indicates that they may have overestimated the size of the effect. However, Strömgren (1967) points out that at that time, no evidence existed for the presence of any rotational effects on c_1, m_1 or the β index as defined by Crawford (1958, 1960, 1964) for early F and A stars. It remains to be seen how much of this difference is attributable to theory and how much to observation. For stars earlier than spectral type A, Hardorp and Strittmatter (1968a) predict a spread of 0.2 in c_1 to be introduced by rotation.

It is worth noting one remaining aspect of the continuum radiation field. The work of Harrington and Collins (1968) implied the possible existence of intrinsic polarization in rapidly rotating B stars. The extension of this study to non-gray atmospheres by Collins (1970) showed that this was not the case in the visible and this result has been confirmed observationally by Serkowski (1968, 1969). However, several observers (Coyne and Gehrels, 1967 and Serkowski, 1968, 1969), have inferred the existence of intrinsic polarization in Be stars and the results have been directly confirmed by observation of differing polarization between two members of several visual binary systems by Bottemiller (1969). This result, combined with that of Collins (1970) implies that the polarization present in these stars must arise above the atmosphere, presumably in a circumstellar shell.

Finally, to conclude our discussion of observed rotational effects, let us turn to effects which appear to be present in the line spectrum of rotating stars. As previously mentioned, the most obvious observational aspect of stellar rotation is the Doppler broadening of lines. This broadening has been used by numerous workers to determine the $V \sin i$ appropriate for many stars. A catalogue of $V \sin i$'s for 2558 stars as determined by various investigators has been compiled by Boyarchuk and Kopylov (1964) and recently extended to 3951 stars by Uesugi and Fukuda (1970). Since the method of determination of $V \sin i$ varies from author to author, some care must be exercised in the use of these catalogues and one must always be watchful for systematic differences arising from both observational sources and methods of reduction. It is probably safe to say that the best determinations of $V \sin i$ have an internal consistency of no better than 5% while 10% is far more common. Indeed, for the most rapidly rotating stars, it has been suggested that 20% might not be an unreasonable observational error to expect. We have already seen that there exists considerable doubt concerning the interpretation of the $V \sin i$ measures for the most rapidly rotating stars and that systematic errors of 10% to 40% exist. It is therefore surprising that Slettebak (1966) finds such a smooth relationship between the maximum rotational velocity of stars and spectral type. This could only result if the measured parameter was determined with a relatively high degree of internal consistency and represented something fundamentally connected with the structure of these stars. This latter fact must be true regardless of the precise interpretation of the term $V \sin i$. However, it is true that the measured 'critical velocities' do fall below those to be expected from existing main-sequence models. Whether this results from the presence of second-order effects as suggested by Hardorp and Strittmatter (1968b), or from the absence of stars rotating at the verge of instability is not clear. This specific point is of extreme importance for future observational studies of stellar rotation.

A possible resolution of this problem could perhaps be found in the detection of the presence of second-order effects on the equivalent width of absorption lines. Guthrie (1963) suggested that such effects were present in the photoelectric indices related to the equivalent widths of Hβ and Hγ. Collins and Harrington (1966) showed that their results were perfectly consistent with Guthrie's (1963) estimates of the behavior of Hβ and Hγ for rotating stars provided the observed $V \sin i$ scale was somewhat increased. However, Strömgren (1967) indicated that there were no rotation effects evident in the A and F stars studied with this filter system. Crawford and Mander (1966) also conclude that no rotation effects were present in the measurements of the β index for the sample of B stars they investigated. Petrie (1964), in revising his Hγ equivalent width-luminosity classification (Petrie, 1958), indicates that no rotational effects on Hγ are present in line studies. Bappu *et al.* (1962) also found good agreement between Hγ luminosities and independent determinations for Orion and NGC 2362 stars.

How may this apparently conflicting observational material be understood? Theory (e.g., Collins and Harrington, 1966; Hardorp and Strittmatter, 1968b), unambiguously indicates that the equivalent width of Hβ and Hγ should decrease roughly as $(V \sin i)^2$. Collins (1968b) indicates that the helium lines should show a strong positive correlation with $V \sin i$ as was found by Deeming and Walker (1967a, b). It is not likely that this ambiguity between theory and some observations and the apparent conflict within the observations themselves has a simple solution. Indeed, in order to understand this result, one must examine each case with great care in order to ascertain the reason why rotation effects were or were not detected. However, several general considerations must be kept in mind when examining any of these observational studies.

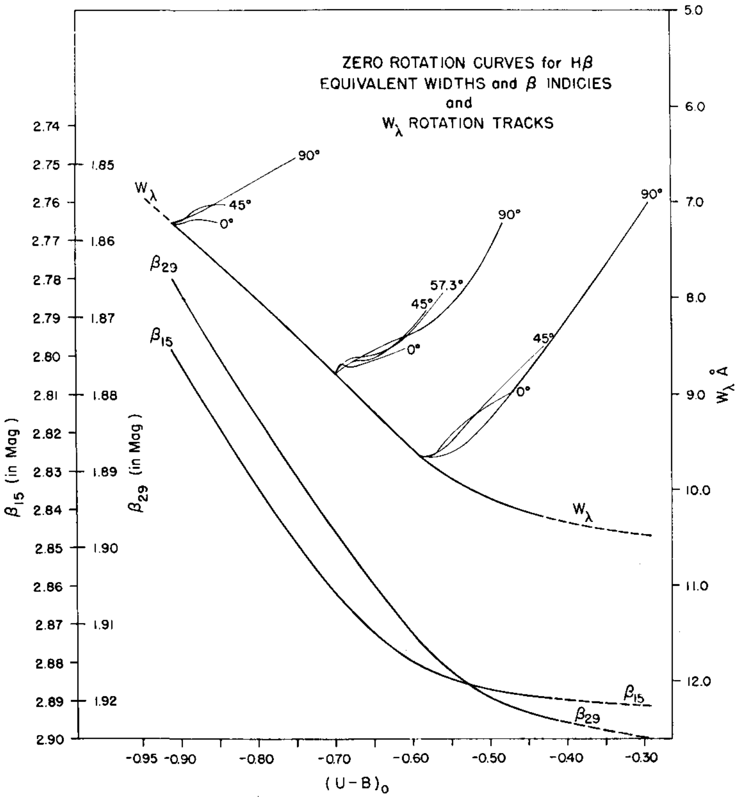

Fig. 5. Rotational effects predicted for the photometric β index for three model stars seen at different angles of inclination. The displacement along the various curves is roughly proportional to $(V \sin i)^2$ (reprinted courtesy of the *Astrophysical Journal*).

Firstly, the effect of rotation upon the hydrogen line strengths is not terribly large unless one includes the most rapid rotators (see Figure 5). In that event, the situation may be greatly confused by the presence of emission. Secondly, the effect as predicted by Collins and Harrington (1966), may not be as obvious when the comparison is made between line strengths and spectral type rather than intrinsic color due to the subjective nature of spectral classification. Although the photoelectric line-strength measures (e.g., γ and β indices) admit the possibility of great accuracy, the precision obtained in practice often falls short of that expected. All observing systems now in use suffer from the difficulty that the defining standards of the system have been chosen without regard to rotation. Depending on the distribution function of $V \sin i$ for the standards, this may introduce a systematic error as well as a variance in studies carried out with the system. Although one would expect both the systematic error and the variance to be small, it could be expected to be of the same size as the rotational effects in question and thus mask them in any statistical search.

The disagreement existing between observers of Hγ equivalent widths is somewhat more difficult to understand. However, it must be remembered that equivalent widths are very difficult to measure, with the results depending strongly on the dispersion used. Buscombe (1969), using high-dispersion spectra, finds a clear effect in the M_v vs. $W_{\gamma\delta}$ relationship due to the value of $V \sin i$ and presents a detailed discussion relating to the difference between his result and that of Petrie (1964).

Finally, some mention must be made regarding observations of the helium lines where the effect of rotation should be larger (Collins, 1968b). Unfortunately, very little information is available on this matter. However, what is available appears to support the theory. Deeming and Walker (1967a), find a positive correlation of He I (4471) equivalent width and $V \sin i$. Strittmatter and Sargent (1966) find that certain sharp-line stars in Orion exhibit weak helium lines and on grounds other than the low value of $V \sin i$, feel that they are intrinsically slow rotators. In addition, Buscombe (1969) finds that a correlation between weak helium lines and slow rotation is compatible with his results.

In light of this discussion, we may draw the following conclusions:

(i) The subjective criteria of spectral classification are probably less affected by rotation than differences between observers.

(ii) *UBV* photometry is of limited usefulness in studying rotational effects; however, the effect of rotation upon the observed absolute magnitude predicted by theory appears to exist.

(iii) Narrow band photometry should provide an excellent tool for studying rotation effects, but to date, the results have been partly negative. However, this does not conclusively demonstrate that the effects do not exist.

(iv) An observed discrepancy exists between the theoretical value for the critical rotational velocity and the largest observed values of $V \sin i$. However, the existence of second-order effects on line profiles is more than sufficient to explain the difference.

(v) Present photometric data on the hydrogen and helium lines tend to confirm the theoretical predictions and hence the existence of second-order effects on line profiles due to rotation.

5. Concluding Remarks

In this paper we have attempted to describe the present status of the theory of rotating stellar atmospheres. The central question to which all our discussion points is: 'To what extent are the effects of extreme axial rotation present in stars?'. It is imperative that this question be answered if one wishes to proceed to the next logical question of: 'How do these stars evolve?'. This is necessary from two points of view. Firstly, the angular momentum will be important in determining the evolved structure of the stellar interior. Secondly, the extent to which observational anomalies existing among early-type stars can be explained as evolutionary effects depends on the existence and importance of rotational effects.

Considering the uncertainties present in the theory and the difficulties encountered in observing second-order effects, it appears that excellent agreement now exists between theory and observation at least on a qualitative basis. Thus, we may conclude that rotational effects must be considered wherever observations are to be interpreted in light of the theory of stellar evolution. The question that remains is a quantitative one. The answer to the quantitative problem will require substantial advances in both theory and observation. Theoretically, we need to know the most probable distribution of velocities that should be expected on the surface of a rapidly rotating star. Observationally, we require carefully designed programs specifically aimed at accurately measuring differences between similar stars resulting from rotation. If rotational effects in equivalent widths and line profiles are present, then stars exist which are rotating at or near their critical velocity and most of the anomalies in early type stars may largely be due to rotation or rotationally induced effects. If these effects are not present, then we may infer the existence of processes preventing stars from ever arriving at a state where they would rotate at or near their critical velocity. In addition, most all of the observed anomalies and differences existing in early-type stars would have to result from differences in their evolutionary state.

Unfortunately, at the moment, there is not one piece of observational evidence which unambiguously confirms the existence of stars rotating at the critical velocity. Within their inherent errors, all present observations are consistent with either of two pictures. Firstly, very rapidly rotating stars exist but their detection is made difficult by the presence of second-order effects on line profiles. Alternatively, such stars do not exist and the second-order effects on line profiles are not present. Either view is tenable but the implications of each for astrophysics are quite different.

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Discussion

Jaschek: I am a little bit surprised that you omitted entirely a phenomenon which after all is well known to exist – namely, the emission lines in rapidly rotating B stars. How do they come into the picture?

Collins: Although it is clear that the Be phenomenon is correlated and probably closely related to rotation, the emission lines are most likely generated outside the atmosphere. Thus I considered the phenomenon itself to be outside the scope of this paper.

Mark: I would like to comment that where a star is differentially rotating the effects of the aspect angle would be very much reduced because the effective gravity does not vary by more than four as compared to the infinite ratio of values for the Roche Model. If we allow a variation of angular velocity comparable to the observed differential rotation of the sun, the stellar models differ markedly from a uniformly rotating model. In particular, the luminosity changes by up to two magnitudes for emission stars as compared to non-rotating ones. However, luminosity and effective temperatures averaged over aspect angle stay near to the non-rotating main sequence. Aspect effects are smaller as mentioned. Thus, small changes in photometric calibrations are expected.

Roxburgh: That cannot be so. If you just give the relative variation in angular velocity to be the same as observed on the sun, then there will still be a limit to the amount of angular momentum that can be stored in a star. Indeed, there will be very little change in the observed properties over the effects of uniform rotation. A 20% variation in ω, particularly to make ω larger at the equator, can do very little.

Collins: I tend to agree with Dr. Roxburgh. However, even should your picture be correct and the second-order effects be reduced, this would only tend to support the picture I have presented where the second-order effects are already small. Your comment regarding ratio of polar gravity to equatorial gravity is at best misleading. Although it is true that this ratio formally goes to infinity as ω approaches ω_c, if one accepts the von Zeipel condition, the equatorial region contributes nothing to the integrated properties of the star. Thus one should consider not this ratio but rather some mean value of $\bar{g}$. I think you will find most reasonable mean gravities to be similar for both the differential and rigid models. This is partially borne out by the variation of 2 magnitudes in bolometric magnitude for your model. The Roche model gives a value of between 1 and 1.5 magnitudes depending somewhat on the spectral type.

Deutsch: I want to address the question of aspect determination from high-dispersion spectrograms, as distinguished from ordinary spectral classification. Consider two sharp-line A stars with weak, zero-volt Fe I lines that have the same equivalent widths. Suppose that one star has $V = 300$ km/sec and $i \simeq 0°$, while the other has $V = 10$ km/sec and $i \simeq 90°$. Now consider the behavior of weak lines from high levels of Fe I, or from Fe II or Si II. Can one predict that these lines also will have equivalent widths that are the same within a few percent?

Collins: The question is not simply answered for it requires a detailed knowledge of the mass absorption coefficient dependence on temperature for the lines in question. A velocity of 300 km/sec is only about 50% or 60% of the critical velocity; thus, it might well be the case that rotational effects would only be a few percent. Unless the velocity is considerably higher I feel the effect would be difficult to detect.

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