PIXEL LENSING: THE KEY TO THE UNIVERSE

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Abstract.

Pixel lensing, the gravitational microlensing of unresolved stars, is potentially a powerful method for detecting and measuring microlensing events. Two groups are currently refining this method in observations toward M31. I show that the technique has wide application, from searching for intracluster Machos in the Virgo cluster, to improving the accuracy of follow-up observations of Galactic microlensing events, to measuring the star-formation history of the universe. I derive the equation for the pixel lensing event rate $\Gamma = (2/Q_{\rm min}^2)\tau N_{\rm res}\Gamma_{\bar{m}}\xi$ where $Q_{\rm min}$ is the minimum signal to noise for detection, τ is the optical depth, $N_{\rm res}$ is the number of telescope resolution elements in the field, $\Gamma_{\bar{m}}$ is the photon detection rate from a fluctuation magnitude star, and ξ is a suppression factor.

1. Introduction

We have heard talks which were described as unabashedly observational and unabashedly theoretical. This talk is unabashed advocacy. Pixel lensing is microlensing of unresolved stars. Or at any rate, it is usually thought of in these terms. Actually, as I will try to convince you, pixel lensing has wide-ranging applications that go far beyond this limited context.

In classical lensing, such as proposed by Paczyński (1986) and now being carried out by MACHO (Alcock et al. 1993, 1995), EROS (Aubourg et al. 1993, 1995), OGLE (Udalski et al. 1994), and DUO (Allard 1995), one monitors the flux of some star with unmagnified flux F_0. If a microlensing event occurs, the flux is magnified by an amount $A(x) = (x^2 + 2)/\sqrt{x^2(x^2 + 4)}$

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where x is the source–lens separation in units of the Einstein ring radius, θ_e. If the motion of the observer, source, and lens are all uniform, then $x(t; t_0, \beta, \omega) = \sqrt{\omega^2(t-t_0)^2+\beta^2}$ where t_0 is the time of maximum, β is the impact parameter in units of θ_e and $\omega^{-1} \equiv t_e$ is the characteristic time of the event. From the measured light curve $F(t_i)$, one can hope to fit for the four parameters F_0, t_0, β, and ω according to

$$F(t_i) = F_0 A[x(t_i; t_0, \beta, \omega)]. \tag{1}$$

Crotts (1992) and Baillon et al. (1993) have begun to look for lensing toward M31. Here the potentially lensed stars are much fainter than the integrated light from all M31 stars within the point spread function (PSF) of the lensed star. Hence they look for the change in light within the PSF,

$$\Delta F(t_i) = F_0\{A[x(t_i; t_0, \beta, \omega)] - 1\}, \tag{2}$$

in their ongoing observations toward M31.

Equations (1) and (2) look quite similar, but they have very different statistical properties. The correlation coefficients among the parameters are much larger in (2). The basic reason for this can be seen by focusing on the case (typical for pixel lensing toward M31) when ΔF can be detected only for $x \ll 1$. In this case $\Delta F \simeq F_0[\omega^2(t-t_0)^2+\beta^2]^{-1/2}$ $= (F_0/\beta)[(\omega/\beta)^2(t-t_0)^2+1]^{-1/2}$. Clearly, the non-degenerate parameters which can be extracted are F_0/β, βt_e, and t_0.

2. Why Pixel Lensing Has A Bad Name

Thus, it seems clear that t_e, which is a key piece of information about individual events that can be extracted from classical lensing, cannot be directly measured in pixel lensing. This is very important because the optical depth can be determined simply by summing the observed time scales (appropriately weighted by the efficiencies). In addition, the time scales can be used to estimate the mass function of the lenses (Han & Gould 1995). Instead, in pixel lensing what one measures is the product of the time scale and the impact parameter, a random variable.

In fact, this difference between classical and pixel lensing is an illusion. In classical lensing one must take account of possible blended light B (from a binary companion to the source star, from the lens star, or from some random star in the field). Hence the flux should be written, $F = F_0 A + B$. When this additional parameter is included, the uncertainties in the time scale determination increase dramatically. For comparison, in pixel lensing one must also accurately measure the background flux B', so the true pixel lensing equation is $\Delta F = F_0(A-1)+B'$. Since these two equations are formally identical, the only real difference between pixel lensing and classical lensing is signal to noise (S/N). If one compares pixel lensed stars in M31 with classically lensed stars in the LMC, then of course the classical lenses will have better signal to noise. But the real question is how do pixel lensing and classical lensing compare when applied to the same stars, or more fundamentally: what is the photon-noise limit of pixel lensing and how can it be achieved?

3. Achieving the Pixel Lensing Photon Limit

Crotts (1992) and AGAPE (Baillon et al. 1993) advocate two different methods of pixel lensing. Crotts convolves his best-seeing image with the current image and then subtracts the result from the current image. He then searches the difference image for PSFs (negative or positive) which would be a signature of a variable star. Note that these PSFs actually extend over several pixels. The variables are then classified into ordinary variables and lensing candidates. AGAPE follow individual pixels (or groups of "super pixels") and looks for variation. Melchior (1995) applied this technique to archival EROS I CCD data (Aubourg et al. 1995) and actually found variable stars that are 2 mag fainter than those followed by EROS in their original lensing study. I should mention, however, that in order to find these variables, she had to first take out the correlation between the pixel variation and the seeing. Seeing, it is clear, is the main problem to be overcome in pixel lensing.

Here I wish to propose a radically different approach to pixel lensing. In lensing studies, one usually accumulates over time dozens or even 100s of images of the same field. Classify these images according to the seeing. Take for example the (say 50) images which have seeing of $1''.30 \pm 0''.05$ and form their median. Then subtract this median from each of the individual images. The difference should contain only photon noise plus PSFs from any variable stars. Of course, in making this assertion I have implicitly assumed that the seeing is adequately characterized by only one number, the FWHM. In general, of course, the seeing disk will also have some ellipticity and may be variable in its radial profile as well. Nevertheless, I can report that I have performed the experiment of subtracting two images of M31 with the same FWHM taken by AGAPE. The seeing disks were not exactly the same, one was elongated vertically, the other horizontally. The result was very striking. The difference looked almost perfectly flat. There were two PSFs from variable stars that were very difficult to detect by eye simply by comparing the two images. Besides these, the remainder of the image looked almost perfectly flat except for photon noise and for an occasional "butterfly", a quadrupole residual where the foreground stars were imperfectly subtracted. This was a very minor problem in the M31 image.

It would be more severe in the LMC where there are more recognizable point sources. Nevertheless, the butterfly residuals are easily distinguished from PSFs and present no fundamental obstacle to finding the variables. I should emphasize that to apply this method the data should be oversampled. In the case of the AGAPE data, there was $1\rlap{.}''5$ seeing and $0\rlap{.}''3$ pixels, so the images could be very well aligned to a small fraction of a seeing disk using linear interpolation.

4. Applications of the Photon Limit I: M87

Once the photon limit is achieved, pixel lensing opens up many previously inaccessible regions of parameter space. I first consider pixel lensing of M87 or more generally, pixel lensing in the limit where the flux L from typical star is much fainter than the surface brightness S of the galaxy integrated over a resolution element $\Omega_{\rm PSF}$, $L \ll S\Omega_{\rm PSF}$. Now, in this case the signal is $L*(A-1)\alpha t_*$ where t_* is the integration time and α is the number of photons collected by the telescope per unit time per unit flux. The noise is $\sqrt{S\Omega_{\rm PSF}\alpha t_*}$. Only events with $\beta \ll 1$ will be detectable. For these, the maximum signal will be $\sim L\alpha t_*/\beta$ while the width of the peak will be $\sim \beta/\omega$. Hence the total S/N of the event Q will be $Q^2 \sim \pi L^2\alpha t_e/(\beta S\Omega_{\rm PSF})$. Note that for fixed minimum S/N, $Q_{\rm min}$, the maximum impact parameter β for which the event can be detected $\propto L^2 t_e$. Since the event rate $\Gamma_0 = (2/\pi)\omega\tau$, where τ is the optical depth, the detectable event rate $\Gamma = \beta\Gamma_0$ is given by

$$\Gamma = \frac{2}{Q_{\rm min}^2}\tau\frac{L^2\alpha}{S}\frac{\Omega_{\rm CCD}}{\Omega_{\rm PSF}}, \tag{3}$$

where $\Omega_{\rm CCD}$ is the area of the CCD. Of course, this formula is valid only for one class of star of flux L and we don't even know the flux of the star being lensed (although it can be estimated by measuring its color from the color of the lensing event). However, we can integrate equation (3) over the entire luminosity function, in which case $L^2/S \to \bar{L}$, where $\bar{L} = \int dL\phi(L)L^2/\int dL\phi(L)L$ is the "fluctuation flux", the same empirical quantity which is measured in surface-brightness fluctuation distance measurements (Tonry 1991). Hence the total rate from all stars is

$$\Gamma = \frac{2}{Q_{\rm min}^2}\tau N_{\rm res}\Gamma_{\bar{m}}\xi, \tag{4}$$

where $N_{\rm res} = \Omega_{\rm CCD}/\Omega_{\rm PSF}$ is the number of resolution elements on the CCD and $\Gamma_{\bar{m}}$ is the rate at which the telescope detects photons from a star at the "fluctuation magnitude" (Tonry 1991) (which is equivalent to $\bar{L}$). The correction factor ξ arises from the finite size of the source and the finite size of the Einstein ring and is explained in more detail elsewhere (Gould 1995b).

Equation (4) is important for two reasons. First it shows that the optical depth can be measured directly from measurable quantities. Recall that one criticism of pixel lensing is that because individual event times cannot be measured, it has been thought that the optical depth cannot be determined. Second it shows that lensing studies are potentially far more powerful than their present incarnations. Paczyński (1995) showed earlier this week that lensing events are currently being detected at one per 50–100 observations per τ^{-1}. Equation (4) shows that the limit is one per 50–100 photons.

Pixel lensing observations of M87 would be useful to search for intra-cluster Machos which might have formed in individual Milky-Way-like proto-galaxies in the proto-cluster and then dissolved into the cluster when the proto-galaxy was stripped of its gas (Gould 1995b). Such objects could be detected at a rate $\sim 5\,\mathrm{day}^{-1}$ if they exist. But pixel lensing can be put to less exotic applications.

5. Other Applications

For example, follow up observations of ongoing lensing events are now being made by two groups (Pratt 1995; Sackett 1995) with the aim of finding planetary systems, finding binaries, and measuring the proper motions and perhaps parallaxes of the Machos. At present the data are being analyzed using DoPhot, DAOPhot, and similar crowded-field photometry routines. As is well known, such techniques are generally limited to $\sim 1\%$ precision. But substantially more information could be extracted if more precise photometry could be done. Pixel lensing offers the prospect of photon-limited photometry (on the varying stars only – but that is all we are interested in) so it should be used here in place of traditional techniques. The same goes for the analysis of data from a proposed Macho Parallax Satellite (Refsdal 1966; Gould 1994, 1995a). In such an experiment, it is crucial that differential photometry be carried out in exactly the same way from the satellite and the ground. This is intrinsically very difficult using classical photometric techniques because the satellite PSF is diffraction limited (and hence color dependent) whereas the ground-based PSF is atmosphere limited. However, with pixel lensing it is straightforward.

The pixel lensing technique can be applied very widely. For example, Schechter (1995) has been monitoring close lensed quasar pairs for microlensing and time delays. Such measurements would likely be improved by the pixel technique. Even the basic microlensing searches of the LMC might well profit by applying the pixel technique. Here the argument is not quite so one-sided because the experimenters have generally tried to get critically sampled (not over-sampled) data in order to maximize their area coverage. It is not known how coarsely the data can be sampled before the linear interpolation required for pixel lensing breaks down. However, tests should be done to see if more events can be extracted using pixel lensing.

Finally, I want to mention another application: a search for microlensing among 10^6 quasars. Time prevents me from going into this in detail, but I refer you to my forthcoming paper (Gould 1995c).

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