Neutron Star Astrophysics at the Crossroads: Magnetars and the Multimessenger Revolution
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Force-free and twisted, relativistic neutron star magnetosphere

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Abstract. In this poster we present the structure of an axisymmetric, force-free magnetosphere of a twisted, aligned rotating dipole within a corotating plasma-filled magnetosphere. We explore various profiles for the twist. We find that as the current increases more field lines cross the light cylinder leading to more efficient spin-down. Moreover, we notice that the twist cannot be increased indefinitely and after a finite twist of about $\pi/2$ the field becomes approximately radial. This could have implications for torque variations of magnetars related to outbursts.

Keywords. stars: neutron, (magnetohydrodynamics:) MHD, magnetic fields.

Magnetar fields are believed to be strongly twisted due to shearing of the crust by internal magnetic stresses. A twisted magnetic field tends to inflate in the radial direction (Lynden-Bell and Boily 1994; Parfrey et al. 2013). We find such solutions by applying the method of simultaneous relaxation (Contopoulos et al. 1999) for the magnetic field inside and outside the light cylinder, demanding a smooth crossing of the field lines on the light cylinder, using a suitably modified version of a previously developed numerical code (Gourgouliatos and Lynden-Bell 2019). The pulsar equation is the following (Goldreich and Julian 1969):

$$(1-R^2)\left(\frac{\partial^2\Psi}{\partial R^2}-\frac{1}{R}\frac{\partial\Psi}{\partial R}+\frac{\partial^2\Psi}{\partial z^2}\right)-2R\frac{\partial\Psi}{\partial R}=I(\Psi)\frac{dI(\Psi)}{d\Psi} \tag{1}$$

where Ψ is the magnetic flux function, I is the electric current distribution and the light cylinder is at $R=1$. We assume that the geometry of the generated magnetic field is that of an aligned rotating star. For closed field lines the electric current distribution, I, is given by the expression: $I(\Psi)=\kappa(\Psi(R_{in},Z_{in})-\Psi_0)^n$ where κ, n are free parameters and R_{in}, Z_{in} describe the domain of the magnetosphere within the cylinder and Ψ_0 is the magnetic flux function of the last closed magnetic field line. The present model cannot be modified for the case of an oblique rotator as the computational code does not take into account the time evolution of the system. There is great freedom on the relation between I and Ψ, the one we follow comes from the relationship (Viganò 2013; Tong 2019):

$$I(\Psi)=\int_0^{\Psi}\alpha(\Psi')d\Psi', \quad \alpha=\frac{\kappa}{R_\star}\left(\frac{\Psi}{\Psi_0}\right)^q \tag{2}$$

with κ being the dimensionless parameter related to the twist, and q a free parameter in correspondence with the parameter n, $n=q+1$.

Figure 1 (a) shows the magnetospheric structure in the presence of a poloidal current distribution in the closed magnetic lines with $\kappa=2.75$, $n=1$. Note that the magnetic field line in red was the last closed field line of the untwisted solutions and it has become open and smoothly crosses the light cylinder. We prescribe the twist of the closed field lines

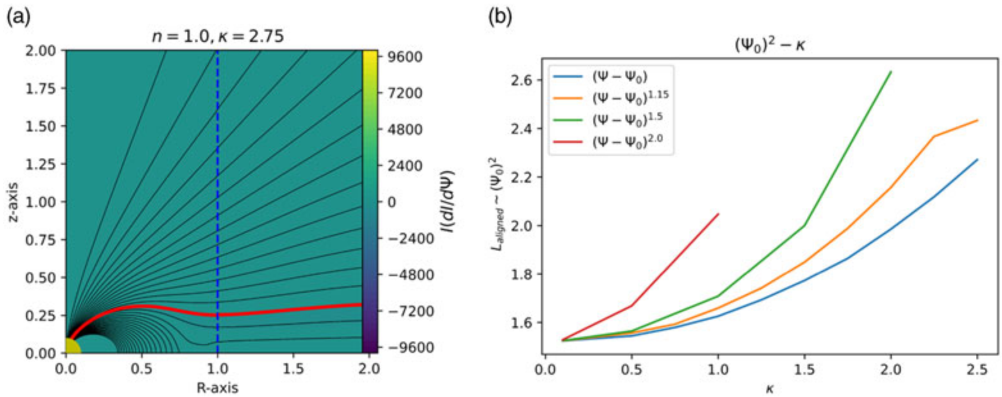

Figure 1. (a) Field inflation for a twisted magnetosphere with $\kappa = 2.75$ and $n = 1$, for a linear current function. The currents are shown in colour. The black lines are lines of constant flux, the blue line is the light cylinder and the red line is the corresponds to the last open field line of the untwisted relativistic magnetosphere. (b) The twist of the closed magnetic lines of the magnetosphere under the application of linear and non-linear current functions as a function of the numerical factor κ. As the current increases the twist increases and so do the open field lines.

by the appropriate electric current function and we notice that by increasing the twist, a larger fraction of the magnetic field lines cross the light cylinder and become open, leading to large polar caps which may correspond to the hot spot during outburst phase. Our calculations reach a limiting twist about $\pi/2$ which is consistent with previous solutions (Gourgouliatos and Lynden-Bell 2008; Akgün et al. 2018). If more twist is added, the field becomes radial and prone to reconnection. Finally, we calculate the spin-down rate, using the relation $L \propto \Psi_0^2$, where L is the spin-down luminosity, and we find that it increases if more twist is added. It is possible that variations in the timing behavior of magnetars before and after outbursts can be associated to the magnetospheric twist. We conclude that by twisting the closed magnetic field lines the field becomes more radial, a larger fraction of magnetic flux crosses the light-cylinder and the spin-down rate becomes higher. Furthermore, we notice that the twist cannot be indefinitely increased, as a finite amount of shearing can lead to radial field lines.

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