Preparatory and Primary School Interests

Adams, Miss L. D., (*b*, *c*)	formerly Ministry of Education Inspectorate.
James, Mr. E. J., (*b*)	Redland College, Bristol.
Morris, Miss R. E., (*c*)	Dinorben School, Wallington.
Sutcliffe, Miss E. W., (*c*)	The Maynard School, Exeter.
Talbot, Mr. B. L., (*p*)	St. Peter's School, Seaford.
Theakston, Mr. T. R., (*g*)	City of Coventry Training College.

Special Interests

Riley, Mr. A. W., (*b**, *c*)	Wolverhampton L.E.A. Inspectorate
Robson, Mr. A., (*l*)	formerly Marlborough College.
Rollett, Mr. A. P., (*g*)	Ministry of Education Inspectorate.
Tuckey, Mr. C. O., (*f*)	formerly Charterhouse.

Sub-committees

b Secondary Modern Schools.
c Primary Schools.
d Technical Colleges.
f Sixth-form Analysis.
g Professional Training.
h History of Mathematics.
l Sixth-form Algebra.
p Preparatory Schools.
s Standing Sub-committee.
* Convener or Secretary.

CORRESPONDENCE

THE HIGHER GEOMETRY REPORT

Sir,—I left the discussion on this report very dissatisfied and conversation afterwards indicated that I was not alone in this. That the discussion virtually died and had to be rescued forcibly perhaps confirms this impression. If I am right, may I boldly suggest reasons for this failure which seem worth recording.

First, I think many members present felt the concentration on abstract geometry did not concern them at all. It is small comfort to genuine inquirers on three-dimensional (pedestrian) geometry to be told " Teach them abstract n-dimensional geometry, and then just put $n=3$ ".

Secondly, many of us were suspicious of the actual content of abstract geometry " quite different from physics " and are convinced that this is work for the universities, who frequently and piously decry premature specialization. Personally as a boy I delighted in a spatial course as in Mr. Durell's *Modern Geometry* and *Projective Geometry* and am cave-man enough to think so still. Yet I found the change of ideas at Cambridge not too great, though the course generally duller! Further, many grammar school pupils who later specialize in mathematics do little or no projective or complex geometry at all.

Finally, when questions are set in scholarship papers on abstract geometry, it would make the preponderance of the few interested schools more marked, a step I should deplore.

Yours, etc., H. Ivor Jones.

Sir,—The presidential address and the discussion of the geometry report at the recent annual meeting have encouraged me to put forward some ideas which arise out of my work at a Rudolf Steiner school.

My senior colleagues have now for 25 years taught projective geometry to pupils of varying intellectual ability and I have been fortunate enough to be able to join in this work for the last seven years. It may be that some of the experience gained, although not directly applicable to the grammar schools, may help eventually in the solution of some of the problems raised in the discussion.

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It seemed to me that some of the members, while wholeheartedly welcoming the report as a guiding line for the work with their ablest pupils, were somewhat worried on two counts. They seemed to wonder how this work would stand in relation to geometry teaching earlier in the school and a note of disappointment could be detected that there was no indication how the more systematic treatment of essentially euclidean geometry could be approached now that the strictly axiomatic development has been discarded. The latter seems important because there is a danger that some pupils never make a proper acquaintance with a coherent, logical and deductive edifice of thought.

It seems to me that both these difficulties can be met if " pure " projective geometry finds a place in the school curriculum somewhere at 5th form level.

In the general course of mathematics teaching, one usually goes from the concrete to the abstract and very frequently one follows the course of historic development. The traditional geometry course, for instance, starts with Euclid which forms both historically and logically the basis for work with rectangular cartesian co-ordinates. On these principles alone, the introduction of algebraic projective geometry at 6th form level calls for the teaching of " pure " projective geometry earlier on. I am certain that such concepts as " ideal elements " would have much firmer roots in a pupil's mind if they were first experienced as limiting positions of concrete elements on the drawing board. " Involution " would lose much of its enigmatic quality if actual involution ranges had been constructed ; perhaps on a fixed side of an otherwise variable self polar triangle with respect to a circle. The needs of the future specialist might thereby be met.

I think, however, that this approach would also be of great pedagogical value to the rest of the class, that is, to the vast majority.

The propositions of incidence of points, lines and planes in space can be established quite intuitively. This is in fact an exercise which trains the children's powers of visual imagination, the development of which is often sadly neglected. These propositions can be developed in such a way that then they exhibit most clearly the principle of duality, one of the most important and most beautiful facts of geometry. Moreover, their immediate consequences such as Desargues' triangle theorem and the properties of quadrangles and quadrilaterals are both interesting and surprising. This can also be used as the basis for drawing exercises which can give beautiful results and which require great care and accuracy. The introduction of points and lines at infinity can, of course, cause difficulties. We do, however, find that they can be met by slowly accustoming the children to these ideas.

Perhaps it would be helpful if I gave a rough outline of the geometry curriculum in use at Michael Hall, where I am teaching. Geometry starts seriously when the children are about 11 years old and by the time they are 14 they have been introduced to the main metrical properties of triangles, quadrilaterals and circles. This is done by a combination of experiment and deduction from intuitively obvious facts with great emphasis on drawing. The development in the next three years is then planned to culminate in projective geometry. Here teachers vary in their method. I myself have used quite a number of different approaches. The most successful is perhaps likely to be as follows :

1st year : Regular polygons and polyhedra giving a chance to revise much of the previous work followed by an introduction to descriptive geometry.

2nd year : Conic sections treated as loci of various kinds. Here the transformation of one form into another is stressed and there is, therefore, a good opportunity to accustom the pupils to points moving through infinity. Concurrently with this, one would take descriptive geometry up to sections of solids by inclined planes, linking up with the work on conics.

3rd year : Projective Geometry : Propositions of Incidence in space, Desargues' Theorem, Quadrangle and Quadrilateral, Conics as products of projective ranges and pencils, Pascal's and Brianchon's theorems leading back to the quadrilateral and pole and polar via degenerate hexagons, Cross ratio, using similar triangles to link up with the previous work.

I have myself never actually done the work quite in that order, but this is how I should do it next time. It must of course be said that we have two very great advantages at Michael Hall which enable us to pursue such a course. First of all, we need not prepare our pupils for external examinations until they are between 17 and 18 and secondly, our timetable arrangements make really concentrated work possible. When it comes to examinations, however, the preparation for the geometry required at Ordinary Level does not take more than a term's work.

Personally, I was delighted with the geometry report. I should, therefore, be very glad if my suggestion to introduce projective geometry earlier in the school curriculum were considered to be in line with the spirit of that report. I hope also that it is in line with the plea from the presidential chair for a widening of horizons in mathematics teaching. I cannot help feeling that it is high time that a larger public should become acquainted with modern thought in the realm of Mathematics. The general conception of mathematics as a complete subject incapable of further development might then disappear and a conception of space might be developed which would make it easier to follow the development of science.

Yours, etc., H. Gebert.

COMBINATORIAL NOTATION

To the Editor of the *Mathematical Gazette*.

Sir,—May I enquire of your readers their opinions on a suggested new mathematical notation? I refer to the use of $(a:b:c:d:e)$ to denote the number of ways of distributing $(a+b+c+d+e)$ objects among boxes labelled Box 1, Box 2, . . . Box 5, so that there are a objects in Box 1, b in Box 2, . . . , e in Box 5. And similarly for any larger or smaller number of boxes.

My original reason for adopting this notation was the ease of typing $(a:b)$ as compared with current alternatives for binomial coefficients. But subsequent experience in teaching the theory of combinations for probability purposes made me think it simplifies the presentation of this theory. One reason is, that the notation helps to emphasise the symmetry between the boxes.

To develop the theory, we consider first two boxes. Since their labels can be interchanged, we have

$$(a:b)=(b:a) \quad \text{...(1)}$$

while if all objects are to go into Box 1, this can be done in only one way, so that

$$(a:0)=1 \quad \text{...(2)}$$

If there are a objects, and only one is to go into Box 2, this one can be chosen in a ways, so that

$$(a-1:1)=a, \quad \text{...(3)}$$

Next, we observe that, to obtain $(a:b)$, we can take $(a+b-1)$ objects and

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