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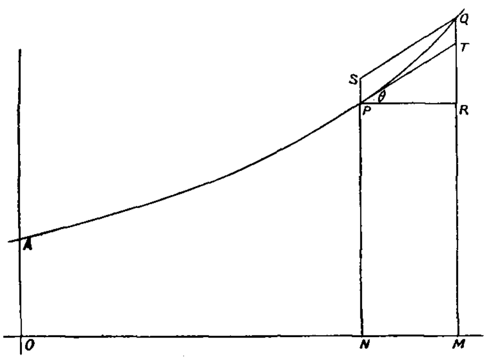
Geometrical illustration of the terms in Taylor's theorem.—The following is an extension of the note by Mr W. M. Roberts in the *Gazette* (215 [v. 4, c], 1906); or of Mr A. H. Barker's discussion on p. 163 of his "Graphical Calculus."



Let ONPA $=f(x)$ and NM $=dx$ so that NP $=f'(x)$ and RT $=dx\tan\theta=dxf''(x)$. Then

$$\text{OMQA}=\text{ONPA}+\text{NMRP}+\text{PRT}+\text{TQP}$$

i.e. $$f(x+dx)=f(x)+dxf'(x)+\tfrac{1}{2}dx^2f''(x)+\text{PTQ}.$$

Assume arc PQ to be parabolic. Then

$$\text{PTQ}=\tfrac{1}{3}\text{PTQS}=\tfrac{1}{3}\text{PR}\,.\,\text{TQ}=\tfrac{1}{3}\text{PR}\,.\,\frac{\text{PT}^2}{4a\sec^2\theta}=\tfrac{1}{3}\,.\,\frac{dx^3}{4a}.$$

But the equation of PQ is of form $4ay=x^2+px+q$ where $y=f'(x)$. Hence $4af'''(x)=2$ and

$$\text{PTQ}=\tfrac{1}{6}dx^3f'''(x)$$

which provides a geometrical meaning for the fourth term in Taylor's theorem.

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