# ON DIELECTRIC MODELS OF INTERSTELLAR GRAINS*

J. MAYO GREENBERG†
*Leiden Observatory, Leiden, The Netherlands*

and

R. STOECKLY
*Rensselaer Polytechnic Institute*

**Abstract.** Silicate, ice, and silicate core-ice mantle particles are considered with a view to describing or predicting ultraviolet features in both extinction and polarization which depend on the dielectric nature of the particles.

## Introduction

With the discovery that silicates [1] probably exist in circumstellar envelopes we have another candidate for the materials constituting the particles which produce interstellar extinction and polarization. However, on the basis of present theories for the formation of the silicates in cool stellar atmospheres and by using reasonable estimates of mass loss per star and the total number of such stars it does not seem possible to account for more than a small fraction of the total interstellar extinction by grains which are exclusively formed by stars.

The full range of possibilities seems to be that the interstellar space contains a mixture of particles in the following categories: (a) ices, (b) silicates, (c) graphite (or soot), (d) silicate cores with ice mantles, (e) graphite cores with ice mantles.

With the already present and future availability of detailed observations in the ultraviolet as far as 1000 Å it will be useful to anticipate criteria for appraising the suitability of the various grain models. Furthermore the theoretical basis on which the model calculations are made may be tested. Such factors as the degree of smoothness of the particles and the degree of their internal homogeneity begin to become important as the wavelength of the probing radiation decreases. These effects are under investigation theoretically and with the aid of the Rensselaer Polytechnic Institute microwave scattering laboratory. In this paper we will primarily limit ourselves to a discussion of theoretical results based on calculations for smooth particles made up of homogeneous materials.

In view of the optical properties of the material we have mentioned we divide the particles into those which are principally dielectric (ices, silicates) and those which are principally metallic (graphite) and look for possible differences in the ultraviolet with the assumption that the visible extinction and polarization are equally well matched. In the next section we review some of the basic model parameters for dielectric and

* Work supported in part by National Aeronautics and Space Administration Grant No. NsG113 and National Science Foundation Grant No. GP7553.
† On leave from Rensselaer Polytechnic Institute 1968–69.

*Houziaux and Butler (eds.), Ultraviolet Stellar Spectra and Ground-Based Observations*, 36–41.

compound particles. In the following section we examine these models for differences from metallic models in the ultraviolet. In Section 4 we discuss briefly some possible polarization features which may be associated with the diffuse 4430 Å line. Finally in Section 5 we give an approximate theoretical calculation which indicates how some model parameters may be modified by including a roughness effect.

## 2. Model Parameters for Dielectric Interstellar Grains

Using a constant index of refraction of $m = 1.33 - 0.05i$ to represent ice it has been shown [2] that we achieve an excellent match (see Figure 1) of theoretical to observed extinction from the infrared to the ultraviolet with a distribution of cylindrical (perfectly aligned) particle sizes given by $n(a) = 49 \exp[-5(a/0.2)^3] + \exp[-5(a/0.6)^3]$. The wavelength dependence of polarization is almost equally well matched. We expect the polarization to be even further improved as a consequence of two factors which reduce the polarization at long wavelength without modifying it significantly at intermediate wavelengths: (a) The larger particles are less well oriented than the small ones [3], (b) If we use a proper magnetic orientation distribution the particle sizes needed to produce the extinction will be slightly reduced (see next section) and the smaller particles produce less polarization at the long wavelengths.

In view of the uncertainties [4] associated with the existence of interstellar ice and also because of the strong suggestion of silicates in interstellar space, it is strongly indicated that the above model should be considered in an alternative manner. Noting that silicate materials have an index of refraction of the form $m \approx 1.66 - im''$

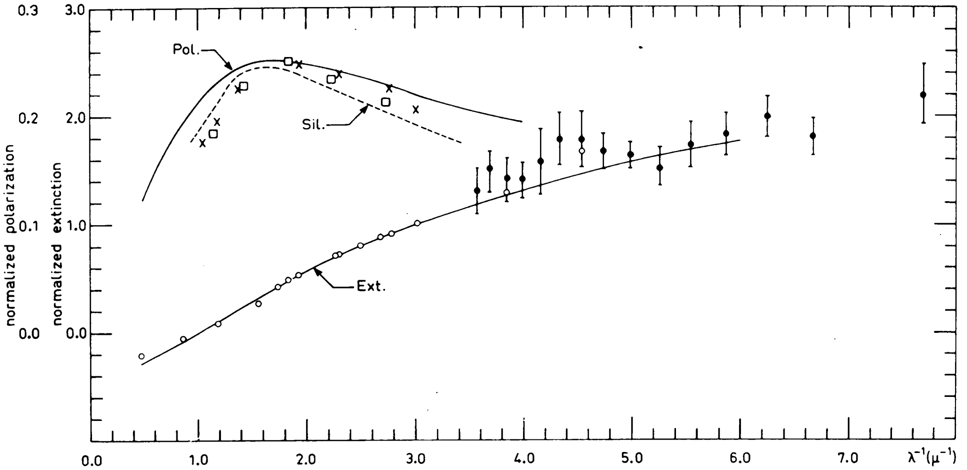


Fig. 1. Extinction and polarization by dielectric grains. Lower curve and upper solid curve (labeled × and Pol. respectively) are extinction and polarization by a size distribution $n(a) = 49 \exp[-5(a/0.2)^3] + \exp[-5(a/0.6)^3]$ of cylinders with $m = 1.33 - 0.05i$. Extinction observations are from [7]. Polarization observations (crosses and squares) are as averaged in [5]. Dashed curve is polarization by a size distribution $n = \exp[-5(a/0.1)^3]$ of cylinders with $m = 1.66$. The theoretical polarization curves are normalized to a difference of 1 magnitude of extinction between 10000 Å and 3330 Å.

we may readily see that if ice-like particles ($m'=1.33$, $m''=-0.05$) of an appropriate size give a match to the extinction curve, then silicate particles of one half this size will serve equally well since the dimensionless parameter $(m'-1)a/\lambda$ will be the same. We then arrive at the possibility that the small particles ($\exp[-5(a/0.2)^3]$ for ices) can be replaced by silicate particles with a size distribution of the form $n(a)=\exp[-5(a/0.1)^3]$. The 'large' particles may then be particles with silicate cores and ice mantles. The mantle size distribution would have a smaller cut-off size than the one given by $\exp[-5(a/0.6)^3)]$ (cut-off radius $\approx 0.6\mu$) for the pure ices because such core-mantle particles act as effectively larger particles – at least in the intermediate ($1 \leqslant \lambda^{-1} \leqslant 3$) region. A mantle distribution with a cut-off below about $0.45\mu$ is entirely consistent with the grain growth mechanism by ice accretion [5].

## 3. Ultraviolet Features

### A. DEPENDENCE OF ULTRAVIOLET EXTINCTION ON POLARIZATION

It was suggested a number of years ago [6] that the wavelength dependence of extinction toward the short wavelengths depends on the degree and kind of orientation of the particles. Furthermore it was noted that this variability of ultraviolet (relative to visible) extinction would be greater for dielectric particles than for metallic or other types of particles. The original calculations were approximate ones for prolate spheroids whose ratio of length to width is 2:1. The essential idea of the interpretation is that: (1) an elongated dielectric particle seen sideways acts for long wavelengths according to its small dimension even though its asymptotic extinction is directly proportional to its long dimension; (2) an elongated particle seen end-on acts for long wavelengths according to its long dimension while its asymptotic ($\lambda \rightarrow 0$) extinction is independent of its long dimension, i.e. it acts larger for large wavelengths and smaller for short wavelengths than its orthogonally oriented counterpart. In Table I we present for illustration the wavelength dependence of extinction for ice cylinders of $0.1\mu$ radius which are in the cases labeled $\psi=0°$ and $\psi=90°$ spinning in perfect Davis and Greenstein orientation about axes perpendicular to and along the line of sight respectively. As expected the particles seen exclusively sideways, ($\psi=90°$) and for which zero polarization is expected, act as if they are smaller than those seen partly sidewise and end-on ($\psi=0°$) for which the polarization would be a maximum. We therefore expect that – other parameters being equal – the extinction in the ultraviolet will be higher for regions with low polarization than for regions with high polarization.

### B. WAVELENGTH DEPENDENCE OF POLARIZATION

As we see in Figure 1 the polarization decreases slowly for dielectric particles in the ultraviolet. Metallic particles which give an equally good representation of the polarization in the intermediate region give significantly less polarization in the ultraviolet.

### C. THE EXTINCTION 'HUMP' AT ABOUT 2200 Å

The extinction hump at about 2200 Å [7] is now apparently well confirmed [8] although the possibility of a selection effect can not be excluded until the same feature is demonstrated for stars other than hot bright nearby young stars.

The fact that the ices have an absorption edge at about this wavelength leads to a *drop* [2, 5] in the extinction at this wavelength with a subsequent rather flat continuation rather than the rise which is observed. This is true for the graphite core-ice mantle particles as well as for the pure ices. The absorptivity ($m''$) of the silicates is not well known in this region, but if it turns out that it remains fairly small beyond the absorption edge (<0.05) then the model proposed in the previous section would satisfy all the necessary requirements. The effect of the absorption edge of the silicates could easily be masked by surface roughness effects which incidentally become more important in the ultraviolet. The small silicate particles would then provide the continuing increase in the extinction on which would be superposed the extinction by the silicate core-ice mantle particles the ice mantles of which produce the hump. Detailed calculations on this model are in progress [9].

TABLE I

Extinction by ice cylinders of $0.1\mu$ radius spinning about axes parallel ($\psi = 90°$) and perpendicular ($\psi = 0°$) to the line of sight

| $\lambda(\mu)$ | $\lambda^{-1}(\mu^{-1})$ | $\Delta m$ | |
|---|---|---|---|
| | | $\psi = 0°$ | $\psi = 90°$ |
| 0.14 | 7.15 | 1.497 | 2.221 |
| 0.16 | 6.25 | 2.202 | 3.362 |
| 0.18 | 5.55 | 2.457 | 3.559 |
| 0.20 | 5.00 | 2.259 | 2.917 |
| 0.22 | 4.54 | 2.110 | 2.392 |
| 0.24 | 4.17 | 1.894 | 1.947 |
| 0.26 | 3.84 | 1.615 | 1.589 |
| 0.28 | 3.57 | 1.384 | 1.351 |
| 0.30 | 3.33 | 1.211 | 1.178 |
| 0.32 | 3.12 | 1.086 | 1.035 |
| 0.34 | 2.94 | 0.9956 | 0.9085 |
| 0.36 | 2.77 | 0.9193 | 0.7834 |
| 0.38 | 2.63 | 0.8615 | 0.6770 |
| 0.42 | 2.38 | 0.7778 | 0.5181 |
| 0.45 | 2.27 | 0.7271 | 0.4367 |
| 0.48 | 2.08 | 0.6816 | 0.3805 |
| 0.50 | 2.00 | 0.6502 | 0.3509 |
| 0.60 | 1.67 | 0.4876 | 0.2637 |
| 0.70 | 1.43 | 0.3139 | 0.2026 |
| 0.80 | 1.25 | 0.1991 | 0.1542 |
| 1.0 | 1.00 | 0.0931 | 0.0897 |
| 2.5 | 0.4 | 0.0037 | 0.0043 |
| 5.0 | 0.2 | 0.0050 | 0.0059 |

## 4. Diffuse Lines

Following the procedure described in [5], detailed calculations have been made of the shape of both the extinction and polarization in the region around an absorption band centered at 4430 Å in either small silicate or ice particles. In both cases (the silicates being smaller than the ices) the absorption band becomes highly asymmetric, being even more so in the silicates than the ices. We also find that the degree of polarization follows an identical trend with the absorption, i.e. a higher polarization at the longer wavelengths and a smaller polarization at the shorter wavelengths than one would get with no absorption band present. We expect a similar result for the ultraviolet hump at about 2200 Å.

## 5. Surface Roughness of Small Particles

To illustrate the probable effect of surface roughness we show a theoretical calculation based on a ray approximation [10].

For a smooth sphere with an index of refraction near unity the total cross section is given by

$$C = 8\pi a^2 \int_0^\infty x \sin^2 \eta \, dx \tag{1}$$

where

$$\eta = \frac{\varrho}{2}(1 - x^2)^{1/2}$$

$$\varrho = \frac{4\pi a}{\lambda}(m' - 1).$$

The quantity $\eta$ is simply the phase shift of a ray passing a distance $xa$ from the optic axis. We picture the surface roughness on a distribution of particles as introducing a statistical fluctuation on this phase shift. For simplicity we preserve the cylindrical symmetry of the problem. Furthermore we let the phase fluctuation be independent of $x$. The total phase is then given by $\eta' = \frac{1}{2}\varrho(1-x^2)^{1/2} + \varepsilon\varrho$ where the average value of $\varepsilon$ is set equal to zero. Substituting $\eta'$ into Equation 1 and averaging we obtain

$$\langle Q_r - Q_s \rangle = 2 \langle \sin^2 \varepsilon\varrho \rangle (2 - Q_s), \tag{2}$$

where $Q_r$ and $Q_s$ are the extinction efficiencies, $C/\pi a^2$, for the rough and smooth spheres respectively.

Since the quantity $(2 - Q_s)$ is positive [11] for $\varrho \lesssim 2$ and is negative in the range $2 \lesssim \varrho \lesssim 6$ we see that the rough sphere is first (longer wavelengths) more efficient in extinction and then, in the region of the major resonance, less efficient than the equivalent smooth sphere. This result is very much like that obtained by having introduced a small imaginary part in the index of refraction.

If the particle material has an intrinsically variable absorptivity the effect of surface roughness would be to smooth over this variability because its effect is larger when the *actual* absorptivity is smaller. This could be important, as noted previously, in masking the onset of absorption edges in the ultraviolet.

## References


[1] Knacke, R. F., Gaustad, J. E., Gillett, F. C., and Stein, W. A.: 1969, *Astrophys. J.* **155**, L189.
[2] Greenberg, J. M., and Shah, G. A.: 1969, *Physica* **41**, 92.
[3] Greenberg, J. M.: 1969, *Physica* **41**, 67.
[4] Knacke, R. F., Cudaback, D. D. and Gaustad, J. E.: 1969, *Astrophys. J.* **158**, 151.
Johnson, H. L.: 1968, *Astrophys. J.* **154**, L125.
[5] Greenberg, J. M.: 1968, in *Stars and Stellar Systems*, Vol. VII (ed. by B. M. Middlehurst and L. H. Aller), University of Chicago Press, Chicago and London, Chapter 6, p. 221.
[6] Greenberg, J. M.: 1960, *Lowell Obs. Bull.* **4**, 285.
Greenberg, J. M.: 1960, *J. Appl. Phys.* **31**, 82.
Wilson, R.: 1960, *Monthly Notices Roy. Astron. Soc.* **120**, 51.
[7] Stecher, T. P.: 1965, *Astrophys. J.* **142**, 1683.
[8] Other papers at this meeting.
[9] Stoeckly, R. and Greenberg, J. M.: unpublished.
[10] Van de Hulst, H. C.: 1946, *Rech. Astron. Obs. Utrecht XI*, Part 1.
Greenberg, J. M.: 1960, *J. Appl. Phys.* **31**, 82.
[11] Van de Hulst, H. C.: 1957, *Light Scattering by Small Particles*, Wiley, New York (section 11.2).


## Discussion

*Wickramasinghe:* Would you not expect the position of the absorption peak in relatively large icy particles ($r \sim 10^{-5}$ cm) to be rather strongly size dependent?

*Greenberg:* With the size distribution I have used I do not anticipate a large size effect.

*Bless:* Could you elaborate on your remark concerning the relation between visual polarization and UV extinction?

*Greenberg:* For dielectric grains the extinction curves depend very significantly on orientation. As a consequence of the fact that elongated particles seen end-on act in the longer wavelength region like larger particles seen sideways, the extinction in the UV produced perpendicular to magnetic fields drops more (or rises less steeply) in the UV than the extinction produced along magnetic fields.

*Van de Hulst:* The polarization in the UV by conducting particles will drop more rapidly than that produced by dielectric particles. A comparison between the polarization in the 2500–3000 Å region with that in the visual will be an important observational criterion between metallic and dielectric particles.

*Wickramasinghe:* We may be rather naïve to think in terms of a single type of grain in reflection nebulae or even in the general interstellar medium. It could be that we have a mixture of graphite, silicates, ices and whatever else might form.

*Bless:* OAO filter photometry has been obtained for several reflection nebulae, including the Merope nebula, and also for the background at several hundred points in the sky. This may give some scattering data helpful in determining the nature of interstellar particles.