A Review of the Collective Theory of Risk by CARL PHILIPSON, (Suppl. to *Astin. Bulletin*, Vol. 5)

It was in 1930 that Harald Cramér dubbed Filip Lundberg's technique for evaluating the risks run by an insurance company *collective risk theory*. It differed essentially from the earlier *individual risk theory* by viewing the claims as a stream of random events in continuous time. The year 1909 had seen the publication of two pathbreaking papers: that by Lundberg was based on the hypotheses that the lengths of interclaim intervals are distributed exponentially and that independent individual claim sizes have an arbitrary time-invariant distribution*; and that by Erlang was based on the assumptions that the intervals between telephone calls are exponentially distributed and that holding times are constant. Here were the beginnings of the theory of (purely) discontinuous Markov processes just nine years after Bachelier's doctoral thesis had initiated the theory of (continuous) diffusion processes in his study of stock market prices.

Carl Philipson's 24-page review of collective risk theory with a further 17 pages listing about 365 references is a stimulating addition to the literature. However, it has its drawbacks.

The author is a first class mathematician whose *forte* has been the generalization of various assumptions used in work based on that of Lundberg himself or of Hans Ammeter who, in effect, replaced the exponential stream of claims by an interrelated Pareto stream. These generalizations have not been followed up by other writers—though they should be. It is not surprising that Philipson's commentary is mathematically more difficult than the majority of the papers he is reviewing. Results that are quite familiar are expressed in impenetrable specialist phraseology and notation.

After a summary introduction to his mathematical notation Philipson describes the "risk process" and discusses its relations with stochastic processes in general. However, he fails to mention its special relationship with single-server queueing processes even though he includes a reference to Prabhu's 1961 paper where this was pointed out. Sections follow relating the basic assumptions for the risk process and their extensions but the brilliant monographs by Beneš (1963) and Takács (1967) which provide valuable insight into the possibility of such generalizations, are not cited. The author then reviews the particular forms that have been used for the distribution of individual claims and discusses in some detail the distribution function of aggregate claims during an interval $[0, t]$. The probability that a company's risk reserve, augmented by premiums received and depleted by claims paid, becomes negative ("ruin") before the end of the period is then considered as well as the asymptotic results for $t \to \infty$. Finally Philipson refers to studies made on reinsurance not all of them based on *collective*, as opposed to individual, theory.

Apart from the criticisms we have mentioned we found this review of the subject very comprehensive and well documented. No one should undertake to write an article on collective risk theory without first consulting it.

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* Lundberg's papers and monographs are not easy to understand, whatever the language. His 1903 doctoral thesis is often credited with important results that we have been unable to find there.

A serious defect of the references cited in the second part of the monograph is that the papers on "related subjects" have not been collected together. The author admits to 26 items that "do not concern contributions to the risk" but this does not include nine books by Blanc-Lapierre & Fortet, Cramér (3), Gumbel, Haight, Harris, Lévy and Wald, respectively. Readers familiar with these names, and those who care to check them against Philipson's list, will see what a rag-bag of titles has been collected together on the periphery of risk theory. On the other hand we can perhaps forgive the author for including a few articles on recovery of the eyes after dazzlement since he himself has written on this subject.

Indeed we find it a disadvantage that Philipson has not relegated his selected articles on non-collective risk theory to a separate list. His cited works on individual risk theory, for example, include two of the 12 included in the transactions of the sixth actuarial congress but none of those presented to the ninth congress. Berger's two textbooks with their valuable summaries of the "individual" viewpoint are not even mentioned. And when it comes to the application of utility theory to problems of reinsurance the author's eclecticism is again apparent: for example, only seven of Borch's pre-1968 papers are quoted whereas at least double that number were published.

The foregoing criticisms of Philipson's bibliographical list may be summed up by saying that it is too long to be a list of articles on *collective* risk theory, too short to be a list of articles on risk theory, and haphazard in its choice of textbooks on mathematics and statistics helpful in the study of collective risk theory. Its greatest lacuna in this area is the lack of references to queueing theory. It is, or should be, well known that the probability of not being ruined during an interval $[0, t]$ given an initial reserve of w is maematically equivalent to the probability that the waiting time of a customer arriving at a single-server at time t after the server first became available does not exceed w.

HILARY L. SEAL

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