A Massive Star Odyssey, from Main Sequence to Supernova
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On the modeling of outflowing envelopes of massive evolved stars at arbitrary optical depths

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Mass loss due to outflow is one factor introducing uncertainty into our understanding of the evolution of massive stars. There is a need of a theory, that would make it possible to take into account mass loss in the process of evolutionary computations in a self-consistent way. It is currently clear that the role of outflow is extremely important for stellar evolution, but quantitative conclusions about mass-loss rates remain uncertain. The evolution of stars with masses $M \geq 15\,\mathrm{M}_\odot$ is accompanied by mass loss at rates reaching 10^{-4} - $10^{-6}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$. This strongly influences the evolution of such stars in the supergiant stage.

An equation of motion describing a spherically symmetrical, stationary outflow under the action of radiation pressure together with continuity equation reads:

$$u\frac{du}{dr} = -\frac{1}{\rho}\frac{dP_g}{dr} - \frac{\mathrm{G}M(1-\tilde{L}_{\mathrm{th}})}{r^2}, \quad \frac{\dot{M}}{4\pi} = \rho u r^2, \tag{1}$$

where $\tilde{L}_{\mathrm{th}} = L_{\mathrm{th}}(r)/L_{ed}$ and $L_{\mathrm{th}} = L_{\mathrm{th}}(r)/L_{\mathrm{ed}}$. The energy integral can be written in the form:

$$L = 4\pi\rho u r^2 \left(E + \frac{P}{\rho} - \frac{\mathrm{G}M}{r} + \frac{u^2}{2}\right) + L_{\mathrm{th}}(r). \tag{2}$$

Here L is the constant total energy flux, which is made up of the fluxes of radiative energy and of the energy of the outflowing matter. The radiation flux L_{th} can be found from the transport equation written in momentum form:

$$L_{\mathrm{th}} = -\frac{4\pi r^2 c}{\kappa\rho}\left(\frac{dP_r}{dr} - \frac{E_r\rho - 3P_r}{r}\right). \tag{3}$$

P_r is the radiation pressure and E_r is the radiation energy density. The continuity equation, expressions for the pressure and energy density, and the optical depth can be written in the form:

$$P = P_r + P_g, \quad E = E_r + E_g + \epsilon_i, \tag{4}$$

$$P_r = \frac{aT^4}{3}\left(1 - e^{-\tau}\right) + \frac{L_{\mathrm{th}}^{\infty}}{4\pi r^2 \mathrm{c}}, \quad E_r\rho = aT^4\left(1 - e^{-\tau}\right) + \frac{L_{\mathrm{th}}^{\infty}}{4\pi r^2 \mathrm{c}}, \tag{5}$$

$$P_g = \rho\mathcal{R}T \quad , \quad E_g = \frac{3}{2}\mathcal{R}T. \tag{6}$$

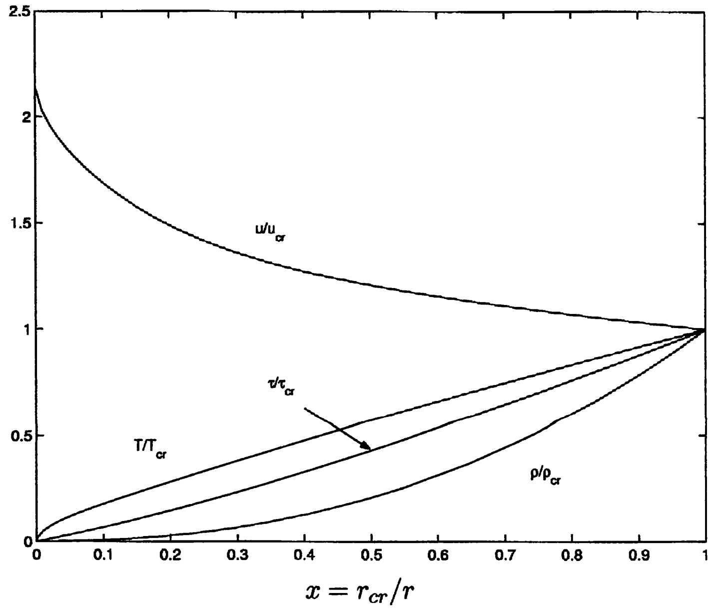

Figure 1. Distribution of velocity, density and optical depth. The sonic point is located at $x = 1$, the second critical point is located at $x = 0$. The curves are labeled with their respective ordinates.

Optical depth is determined by the following relation: $d\tau/dr = -\kappa\rho$. The obtained system of equations has two singular points: one is the sonic point and the other is situated at infinity ($T = 0, \rho = 0$). The outer boundary condition is posed at infinity:

$$T = 0, \qquad \rho \sim \frac{1}{r^2} \to 0, \qquad u \to \mathrm{const} = u_\infty. \tag{7}$$

In the vicinity of the singular points analytical expansions are adopted. It can be shown that the solution at infinity follows asymptotic behaviour:

$$T = a_\infty \sqrt{x}, \quad \rho = b_1 x^2 + b_2 x^{5/2}, \quad u \simeq u_\infty - \frac{b_2}{b_1^2}\sqrt{x}, \quad u_\infty = \frac{1}{b1}. \tag{8}$$

$$\text{an expression for } v \text{ reads:} \quad u \simeq u_\infty - \frac{b_2}{b_1^2}\sqrt{x}, \quad u_\infty = \frac{1}{b1}. \tag{9}$$

A numerical solution was found for the simplifying case of constant opacity and ionization degree. The two boundary value problem is solved using relaxation method. At vicinities of singular points a solution is represented via analytical expansions. Solution curves are depicted on the figure. This solution is for a $M = 20\,\mathrm{M}_\odot$ star, the following parameters at sonic point have been obtained: $T_{\rm cr} = 6.04\times10^2$ K, $r_{\rm cr} = 7.88\times10^{15}$, $\rho_{\rm cr} = 5.65\times 10^{-15}\,\mathrm{g\,cm^{-3}}$, $\tau_{\rm ph} \simeq 2$, $v_{\rm cr} \simeq 2.75\,\mathrm{km\,s^{-1}}$, and $\dot{M} \simeq 1.5\times10^{-2}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$.

References

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