THE DEPRIORITISED APPROACH TO PRIORITISED ALGORITHMS

STEPHEN HOWE

Randomised algorithms are an effective method of attacking computationally intractable problems. A simple and fast randomised algorithm may produce results to an accuracy sufficient for many purposes, especially in the average case. In this thesis we consider average case analyses of heuristics for certain NP-hard graph optimisation problems. In particular, we consider algorithms that find dominating sets of random regular directed graphs. As well as providing an average case analysis, our results also determine new upper bounds on domination numbers of random regular directed graphs.

The algorithms for random regular directed graphs considered in this thesis are known as prioritised algorithms. Each prioritised algorithm determines a discrete random process. This discrete process may be continuously approximated using differential equations [4]. Under certain conditions, the solutions to these differential equations describe the behaviour of the prioritised algorithm. Applying such an analysis to prioritised algorithms directly is difficult. However, we are able to use prioritised algorithms to define new algorithms, called deprioritised algorithms, that can be analysed in this fashion.

Defining a deprioritised algorithm based on a given prioritised algorithm, and then analysing the deprioritised algorithm, is called the deprioritised approach. The initial theory describing the deprioritised approach was developed by Wormald [5] and has been successfully applied in many cases [1–3]. However, not all algorithms are covered by Wormald's theory: for example, algorithms for random regular directed graphs. The main contribution of this thesis is the extension of the deprioritised

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approach to a larger class of prioritised algorithms. We demonstrate the new theory by applying it to two algorithms which find dominating sets of random regular directed graphs.

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STEPHEN HOWE, Centre for Mathematics and its Applications, Mathematical Sciences Institute, Australian National University, Canberra, ACT 0200, Australia
e-mail: stephen.howe@anu.edu.au