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New semi-analytical calculation of lunar, solar and planetary perturbations in motion of Earth satellites

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Abstract. We suggest an advanced algorithm for semi-analytical calculation of orbital perturbations of Earth artificial satellites caused by the gravity attraction of the "3rd-bodies" (the Moon, the Sun, major planets). A new accurate analytical series for the relevant perturbation function is developed. It is obtained through a careful spectral analysis of the long-term DE406 planetary/lunar ephemerides and valid over 2000 years, 1000-3000. The series is used in the author's semi-analytical model of satellite motion. The results of the motion prediction of several Earth satellites obtained by means of the semi-analytical model and a numerical integration method are compared.

Keywords. Satellite Motion; Semi-Analytical Model; Perturbations; Sun; Moon; Major Planets

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1. Introduction

When predicting the motion of planetary (both natural and artificial) satellites by an analytical/semi-analytical integration method the first task is to represent all perturbation functions by exact analytical expressions or accurate approximating series. In the present study we deal with the perturbation function caused by the gravity attraction of the "3rd-bodies" (the Moon, the Sun, major planets). In many practical analytical models of satellite motion (e.g., Ivanov *et al.* 1988; Kolyuka *et al.* 1991) the relevant perturbation function is derived from some available analytical theories of the "3rd-bodies' motion. However, presently the accuracy of such analytical theories does not match the precision of the current numerical ephemerides of the Moon and major planets. Thus, the development of new analytical series that accurately approximate the perturbation function caused by the "3rd-bodies" is an actual task.

Here we present a new series for the "3rd-bodies" perturbation function for Earth satellites and provide some examples of its use in the author's semi-analytical model of satellite motion.

2. The perturbation function caused by the "3rd-bodies"

Following (e.g., Giacagalia 1974; Emelyanov 1980) the perturbation function caused by the gravity attraction of the "3rd-bodies", R, can be expanded as follows

$$R = \sum_{l=2}^{\infty} \sum_{m=0}^{l} \sum_{p=0}^{l} \sum_{q=-\infty}^{\infty} \left(\frac{a}{R_s}\right)^l \bar{F}_{lmp}(i) X^{l,l-2p}_{l-2p+q}(e) \left[\bar{A}_{lm} \cos\psi_{lmpq} + \bar{B}_{lm} \sin\psi_{lmpq}\right],$$

where a, e, i, Ω, π, λ are Keplerian elements of the satellite orbit, $\bar{F}_{lmp}(i)$ is an inclination function, $X^{l,l-2p}_{l-2p+q}(e)$ is a Hansen coefficient (a function of the eccentricity of a satellite orbit), R_s is a scaling factor that is assumed to be 43,000 km in our study in order to have the relation $a{R_s}^{-1} < 1$ true for the majority of Earth satellites,

$$\psi_{lmpq} = (l-2p+q)\lambda - q\pi + (m+2p-l)\Omega\,,$$

$$\bar{A}_{lm} = \begin{cases} \bar{C}_{lm}\,, & \text{if } l-m \text{ is even} \\ -\bar{S}_{lm}\,, & \text{if } l-m \text{ is odd}\,, \end{cases} \qquad \bar{B}_{lm} = \begin{cases} \bar{S}_{lm}\,, & \text{if } l-m \text{ is even} \\ \bar{C}_{lm}\,, & \text{if } l-m \text{ is odd}\,, \end{cases}$$

$$\bar{C}_{lm}(t) = \frac{1}{2l+1}\sum_j \frac{\mu_j}{R_s}\left(\frac{R_s}{r_j(t)}\right)^{l+1} \bar{P}_{lm}\left(\sin\delta_j(t)\right)\cos m\alpha_j(t)\,,$$

$$\bar{S}_{lm}(t) = \frac{1}{2l+1}\sum_j \frac{\mu_j}{R_s}\left(\frac{R_s}{r_j(t)}\right)^{l+1} \bar{P}_{lm}\left(\sin\delta_j(t)\right)\sin m\alpha_j(t)\,,$$

and μ_j, r_j, α_j, δ_j are the gravitational parameter, geocentric distance, right ascension and declination of the j^{th} perturbing body at epoch t, resp.; $\bar{P}_{lm}$ is an associated Legendre function.

3. Development of $\bar{C}_{lm}$, $\bar{S}_{lm}$ to approximating series

Step 1: Numerical values for the $\bar{C}_{lm}(t)$, $\bar{S}_{lm}(t)$ coefficients were tabulated with a one day's step over the 2000 years' time interval centered at epoch J2000. The Moon, the Sun, Mercury, Venus, Mars, Jupiter and Saturn were taken as the attracting bodies. The long-term numerical ephemerides DE406 (Standish 1998) were used as the source of the planetary/lunar positions.

Step 2: The tabulated values for the coefficients have been developed to approximating series by using a modified spectral analysis method (Kudryavtsev 2004, 2007). A feature of this method is that amplitudes and frequencies of the series' terms are obtained in the form of time polynomials. It allows us to perform the spectral analysis over a long interval of time, up to several thousand years. In this way the terms of close frequencies are better separated and the series have a higher accuracy.

In the present study the form of series for every coefficient is

$$\begin{aligned}\bar{C}_{lm}(t)\ \left[\text{or } \bar{S}_{lm}(t)\right] \approx \sum_k \Big\{ &\left[A^c_{k0} + A^c_{k1}t + A^c_{k2}t^2\right]\cos\omega_k(t) \\ &+ \left[A^s_{k0} + A^s_{k1}t + A^s_{k2}t^2\right]\sin\omega_k(t)\Big\}\,,\end{aligned}$$

where $\omega_k(t) = \nu_{k1}t + \nu_{k2}t^2 + \cdots + \nu_{k4}t^4$ are some linear combinations of integer multipliers of Delaunay arguments and expressions for the time polynomial part of the planetary/lunar mean orbital longitudes (Simon *et al.* 1994); A^c_{k0}, A^c_{k1}, …, A^s_{k2} and ν_{k1}, …, ν_{k4} are some constants derived in the course of the development. The minimum amplitude of terms in the series for $\bar{C}_{lm}(t)$, $\bar{S}_{lm}(t)$ was chosen to be equivalent to 10^{-6} m^{2}s^{-2} over the entire time interval (or $\sim 10^{-8}$ of the maximum values of the coefficients). The maximum degree l of terms with an amplitude above that threshold is eight. Table 1 gives the number of terms in the development of every $\bar{C}_{lm}(t)$, $\bar{S}_{lm}(t)$ coefficient. The total number of terms in the final series is 38,585.

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Table 1. The number of terms in the development of $\bar{C}_{lm}(t)$ / $\bar{S}_{lm}(t)$ coefficients.

l \ m	0	1	2	3	4	5	6	7	8
2	2268 / -	2358 / 2644	2789 / 2652						
3	1160 /-	1212 / 1200	1443 / 1370	1307 / 1354					
4	694 /-	726 / 678	855 / 777	841 / 881	768 / 749				
5	401 /-	427 / 435	489 / 450	494 / 525	510 / 492	397 / 402			
6	195 /-	206 / 215	214 / 225	236 / 253	279 / 267	262 / 269	217 / 215		
7	70 /-	73 / 70	69 / 85	95 / 101	104 / 98	104 / 105	104 / 103	80 / 80	
8	19 /-	20 / 20	24 / 26	21 / 25	34 / 31	31 / 32	28 / 28	28 / 28	24 / 24

Table 2. Comparison of satellite orbital positions calculated by semi-analytical and numerical methods. Motion model: gravity perturbations from the Moon, the Sun and major planets.

Satellite	Semi-major axis, km	The number of terms[1]	Time interval[2]	RMS deviation, m
ETALON-1	25,500	22,216	1 year	8.8
LAGEOS-1	12,300	7,544	1 year	1.5
STARLETTE	7,330	3,040	1 month	0.2

Notes: [1] The number of the “3rd-bodies” terms in the semi-analytical model; [2] Time interval where the comparison is done.

4. Semi-analytical calculation of the “3rd-bodies” effect on satellite motion

To calculate the “3rd-bodies” effect on satellite positions, an algorithm of semi-analytical integration of Lagrange motion equations (Kudryavtsev 1995, 1997, 2002) was employed. It uses the above presented development of the “3rd-bodies” perturbation function. However, the algorithm requests the right-hand parts of the motion equations to be represented by a sum of pure trigonometric functions with numerical values for amplitude, frequency and phase. Thus, the derived series for the “3rd-bodies” perturbation function were further approximated by some trigonometric series. To do it we used the fact of the relative smallness of the obtained values for high-order amplitudes $A_{k1}^{c,s}$, $A_{k2}^{c,s}$ and frequencies ν_{k2}, ν_{k3}, ν_{k4}. So that one can write

$$A_{k1}^{c,s}t \approx \sin A_{k1}^{c,s}t, \qquad A_{k2}^{c,s}t^2 \approx 2\left(1 - \cos\sqrt{A_{k2}^{c,s}}\,t\right),$$

$$\cos\omega_k(t) \approx \cos\nu_{k1}t - \nu_{k2}t^2 \sin\nu_{k1}t, \qquad \sin\omega_k(t) \approx \sin\nu_{k1}t + \nu_{k2}t^2 \cos\nu_{k1}t, \qquad \text{etc.}$$

Then the subsequent multiplications of two and more trigonometric functions produces a new trigonometric series to be further used in the right-hand sides of Lagrange motion equations. Our analyses proves that for all obtained values of $A_{k1}^{c,s}$, $A_{k2}^{c,s}$ and ν_{k2}, ν_{k3}, ν_{k4} the approximating trigonometric series keep the accuracy of the original development ($10^{-6}\,\mathrm{m}^2\mathrm{s}^{-2}$) over a time interval of up to 50 years back and forward from any preset initial epoch of the approximation.

The accuracy of the series and algorithm was estimated. First, positions of a low-attitude STARLETTE, mid-attitude LAGEOS-1 and high-attitude ETALON-1 satellite by the 14th-order Everhart numerical integration method were calculated. An exact model of the lunar, solar and planetary perturbations based on the DE405 ephemeris (Standish 1998) was used. Then the calculated satellites positions were assumed as fictitious observations and processed by the semi-analytical integration method with the use of the derived series. The algorithm takes into account the “3rd-bodies” perturbations of up to the 2nd order inclusive. The results of the comparison of the satellites positions calculated by both methods are given in Table 2.

5. Conclusions

• We developed a new semi-analytical series that represents the "3rd-bodies" perturbation function acting on Earth satellites. It is valid over 1000–3000 and has an accuracy compatible with that provided by the current numerical ephemerides of the Moon and planets.

• The series is included in our semi-analytical model of satellite motion. A comparison of satellites coordinates, obtained by using the semi-analytical model and a numerical integration method, show close results. Presently the semi-analytical model calculates the perturbations from the "3rd-bodies" of up to the 2nd order. We plan to further improve the model quality by developing and taking into account the relevant perturbations of higher orders.

Supplementary material

To view supplementary material for this article, please visit http://doi.org/10.1017/S1743921323003988

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