CORRESPONDENCE.

ON THE ASSURANCE OF INVALID LIVES.

To the Editor of the Assurance Magazine.

SIR,—It is gratifying to me to find that my paper, in the October Number of the *Journal*, has elicited a discussion on the important subject of the assurance of invalid and doubtful lives; and it detracts nothing from the pleasure I experience, in perusing the three letters in your last Number, to find that the writers entertain views different from my own. It was scarcely to be expected that a scheme such as I put forward, built up, as it was of necessity, upon assumptions more or less arbitrary, would command a ready and universal acceptance. Mr. Brabrook's principal objection lies in his opinion that my method makes the payment for a contingency to depend upon the issue of the event. Such, however, is by no means the case. If the extreme admission be made—that the medical opinion, in every instance, is absolutely correct—any plan having for its object the remission of a portion of the extra premium, even under the most favourable circumstances of survivance, would, doubtless, be unsound; but if, on the other hand, we regard the medical opinion in the light in which I view it—as a perfectly conjectural estimate, entirely unsupported by statistics, and almost as likely to be wholly wrong as it is to be wholly or partially right—there is nothing antagonistic to true principle in modifying it as our knowledge of the life increases, and in rescinding it altogether when a prolonged existence has demonstrated, sufficiently for every practical purpose, that no addition ought ever to have been made.

Mr. Brabrook's remark upon the danger of departing from the present practice of requiring actual payment of every extra premium as it becomes due, will, I think, be sufficiently answered by some observations I shall have to offer in reply to Mr. Gray.

With reference to Mr. Ambrose Smith's letter, I do not know that it is necessary I should do more than point out that his first formula does not express the solution of the problem which he proposes. If S be the smaller sum to be assured, then, instead of

$$S = \frac{A.M_x - \pi N_{x-1}}{M_x - M_{x+n}}, \text{ it should be } S = \frac{\pi N_{x-1} - A.M_{x+n}}{M_x - M_{x+n}}.$$

What Mr. Smith gives as the value of S, is, in fact, the value of A — S, or the difference between the larger and smaller sums. The error probably occurred in transcribing, as I find his numerical example is properly computed by the latter formula. The question, however, which he has thus solved has no bearing upon the subject of my paper, and therefore it would serve no purpose to discuss it. The remainder of his letter touches upon points which I shall have occasion to refer to presently.

I will now reply to Mr. Gray. It appears to me that this gentleman, by advancing nothing but theoretical considerations, loses sight of all the most important elements that enter into the inquiry. It is not likely that he will agree either with the system which I have proposed, or indeed with any other which has for its object the smallest deviation from the medical estimate, however plausible the reasoning may be by which such deviation

is sought to be justified, until he entertains what I conceive to be more correct ideas of the value of medical opinion in the cases we are considering. In order to reason rightly on the subject, we must be careful to start from correct premises, by bearing in mind the very limited extent and the great uncertainty of the data on which the medical officer has to pronounce off-hand a final decision. He sees a man whom he has probably never met before, asks him a series of questions, examines him, and then has to say, at once, whether the assurance will be more than ordinarily hazardous, and if so, to fix the precise magnitude of the increased risk in terms of an addition to the age. Now, surely this is a problem which no medical man, however eminent, can presume to solve, in the absence of proper statistics, with the smallest pretence to accuracy; and therefore I take it to be an almost self-evident proposition, that an opinion formed in the way described is virtually little more than a random guess. Setting out, then, with this view, I consider that every year survived by the individual after the policy is issued tends to reduce the probability that the " guess" was correct; and although no amount of survivance, even in this case, though it should extend to the last age in the table, can be taken as absolute proof that the original estimate was erroneous, yet it must be admitted that a considerably shorter term of existence would be sufficient to reduce the probability of its correct-ness to a practical zero.

Mr. Gray appears to reason throughout as if the medical opinion were certain to be right in all cases, or, at all events, as if a very high degree of probability attached to it, and, of course, on this supposition it would be contrary to all scientific principles to allow that the greatest amount of survivance could in any way affect it; but I cannot admit his hypothesis as being at all in accordance with facts, and therefore, in my opinion, the whole of his conclusions fall entirely to the ground.

I certainly cannot see the force of Mr. Gray's suggestion, that a simple rejection of some fixed percentage of all extra premiums would be a better plan to adopt, as it would enable us to "know exactly what we were about." It would be, to say the least, a strange practice to resort to a uniform rate of reduction, without having any distinct reason to assign for doing so. In the plan which I have proposed, the reason is given for every step, so that when the result is reached we know precisely what it involves. We see that all persons to whose ages additions are made, and who die within the "expectation" period, pay, one with another, as long as they live, the full extra premium in each case fixed by the medical officer at the time they entered, and that those only who survive that period are exempted from any extra charge. An actuary is thus enabled to judge for himself as to the safety and the equitable character of the theory; but if we merely struck off a percentage, as Mr. Gray suggests, we should be left quite in the dark as to what alteration in the conditions such a deduction represented, and we should have no means of determining whether it was reasonable or otherwise.

I cannot help noticing one sentence in Mr. Gray's letter, which I was somewhat surprised to find written by one who is so uniformly accurate on all theoretical points: he says "it is easy to predict the consequences to an Office which should adopt Mr. Younger's scheme." In making this assertion, Mr. Gray appears quite to forget that all our tables of mortality include bad lives as well as good, and, if we except the "experience" table, even such lives as no Company would think of assuring; therefore, setting aside the

sufficiency or insufficiency of the plan I proposed, and saying nothing of the large margin added by all Offices to the premiums deduced from the tables referred to, it is perfectly obvious that no Society would be ruined, even if extra premiums of the kind we are considering were dispensed with altogether, so long only as means were taken to prevent more than an average, or the tabular proportion, of diseased lives getting admission into the Office. The same remark applies, of course, to the "experience" table, as that necessarily includes all the doubtful and diseased lives found in the books of the 17 Companies whose combined experience furnished the table. These considerations alone, therefore, are sufficient to show that Mr. Gray's opinion is not merely untenable, but that it is even *demonstrably* wrong.

His solution of the mathematical portion of my theory is interesting, as exhibiting a different method of arriving at the same conclusion; although his results, when written out in full, are more complicated than my own. I see Mr. Gray treats the present value of the extra premiums as the single payment for a *temporary* assurance, to continue only during the "expectation" period; and without noticing my arguments (p. 270) for adopting a different method, he merely states that I have "arbitrarily" used a whole-life assurance instead. It would have been more satisfactory if he had shown wherein the reasoning is defective by which I establish that a whole-term assurance ought to be used. I fully discussed the plan which Mr. Gray now proposes, and explained how its adoption would entirely destroy all consistency in the scheme, by involving a contradiction of the fundamental hypothesis. To illustrate this, suppose two men, of the same age, to enter a Society, and to have a like addition made to each of their ages; and suppose that one dies within the "expectation" period, and the other lives beyond it; would it not be a manifest contradiction, after allowing that the amount of survivance attained by the latter had fairly upset the correctness of the original estimate of his life, and entitled him to be absolved from payment of the accumulated extra premiums, if, in letting him off, we were quietly to transfer the burden to the policy of the unfortunate man who had died, and deduct from *his* assurance all the accumulated extra premiums on *both* policies? Yet this is, in effect, what Mr. Gray's plan would do.

In speaking of the age that should be used in calculating the reversionary deduction, Mr. Gray intimates that the advanced age is the proper one, and that I committed an error in using the real age. Now it is obvious that *neither* is really correct, but that some intermediate age is the true one; and finding, as I did, that the whole scheme involved, in a certain qualified sense, a remission of more than half the additions originally recommended by the medical adviser, it was evident that the real age would be nearer the truth than that which included the addition, and, therefore, my use of it in preference to the other is perfectly justifiable. It is proper I should state that I do not at all admit the correctness of the conclusion arrived at by Mr. Gray, when he says that my plan involves a rejection of 77 per cent. of the extra premiums received under the system at present adopted. I have said that, "in a certain qualified sense," I found that more than 50 per cent. was forfeited, and for this reason used the real age in my calculations. Now the "qualified sense" referred to lies in the tacit assumption that, during the "expectation" period, the mortality amongst the doubtful lives will (whether we adopt the real or the increased age) be in accordance with the tabular rate. This

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hypothesis, upon which Mr. Gray's conclusion entirely rests, cannot surely be admitted by any one who has had the least practical experience. Without venturing too far into the domain of medical science, I may give, by way of illustration, a case which can be taken as a type of those which most frequently occur. A young man, say under thirty years of age, proposes to assure, and his life is found to be unexceptionably good, as far as a thorough medical examination can determine, but because one of his parents died at an early age of phthisis, an addition of seven years or more is made to cover the possibility of the same disease existing in the son's system, and destroying his life prematurely. Now it is perfectly obvious that the great bulk of the extra risk, if not the whole of it, will be comprised within a very limited number of years, certainly less than the 35 or 40 years during which, under my scheme, he would be held liable for the additional premium. It is, therefore, a great mistake to assume, as Mr. Gray does, in testing my system, that a true comparative result is arrived at by using the advanced age, and then supposing the tabular rate of mortality to obtain. The special hazard is not spread uniformly like a thin crust or stratum over the whole of life, but speedily reaches its maximum, and then rapidly diminishes, becoming practically evanescent before many years have expired; it must, therefore, be presumed that most, if not all, the really bad lives will drop before the long term of the " expectation " has passed away, and that, consequently, these will all pay, under my scheme, the full additional premium to the time of death. If such premiums were hereafter found to be insufficient, the error should be corrected by making larger additions in the first instance, so that those who die early may be called upon to pay more, but not by compelling those to make up the deficiency whose superior longevity has afforded the strongest presumptive evidence, that the adverse medical opinion formed with regard to their lives was a mistake altogether from the beginning. I hold it to be impossible for any one to determine how much less would be received under the system I have suggested, than if every extra premium had been regularly paid, because the difference must necessarily depend upon the mortality amongst the doubtful lives during the " expectation " period, which is an element entirely unknown. The greater this mortality is, the less will the total receipts, under my scheme, differ from what they would have been under the ordinary system. Mr. Gray objects to the principle generally of remitting, after any length of time, the addition originally made, and his evident faith in the correctness of the medical estimate is the basis of this objection; but I am quite sure that no medical officer could be found with sufficient presumption to attach a like value to his own opinion. If it were possible, in the supposititious case just given, to know beforehand that the man would certainly not die *before* completing his 63rd or 64th year, I believe I can say, without fear of contradiction, that no medical gentleman in the kingdom would hesitate for a moment to recommend the life to be taken at the ordinary rate. I here meet Mr. Gray upon his own ground, and justify the plan of waiving all extra charge after a lengthened term of survivance, by showing that it harmonises with what would unquestionably be the opinion of the medical man himself, if he could only have, at the outset, the additional knowledge of the life, which, under my system, is obtained before the question of extra premium is finally decided. If such a scheme as I have recommended be called arbitrary, how vastly more so must the present practice be which depends upon guesswork alone.

On the subject of notation, I should like to offer one or two suggestions, it being a topic on which a good deal has been said of late. Mr. Gray's remark that v could be advantageously used in the place of $(1+i)^{-1}$ is doubtless correct as far as *some* investigations are concerned; but I do not see that it is by any means universally true. In the particular case, for instance, to which he refers, suppose I had commenced by speaking of the successive accumulations of extra premiums at the end of the first, second, &c., years, as being v^{-1}, $v^{-1}+v^{-2}$, &c., it would certainly have been a most needless mystification of the simple quantities $(1+i)$, $(1+i)+(1+i)^2$, &c., which convey their meaning to the mind so much more readily, and it surely would not have been in accordance with elegance or good taste in such matters to have started with one system of notation and ended with another in an investigation which occupies only half-a-dozen lines. A much more important question is, I think, involved in the use of such symbols as $N_{x-1|t}$ and $M_{x|t}$, which I see Mr. Gray employs. It has always appeared to me that these abbreviations of $N_{x-1}-N_{x+t-1}$, and M_x-M_{x+t} are highly objectionable, more particularly when they are allowed to remain in final results intended for practical use, because they are quantities of which we have no tabulated values, and we can make no use of any formula in which they appear without first replacing them (mentally at least) by the quantities for which they stand. Moreover, they give to a formula a degree of simplicity which it does not really possess. If I had used abbreviations of this description, I might have expressed the formula—

$$\frac{1+i}{i}\left(1-\frac{l_{m+e}}{l_m}-\frac{M_m-M_{m+e}}{D_m}\right),$$

which occurs in my paper, by $\dfrac{1}{d}(\Sigma_{e|}q_{m(n)}-M_{m|e})$, but no use could possibly be made of this if a numerical result were wanted without first of all retranslating it into its original form. Upon what principle then could it be called a more simple expression?

I believe I have now fully replied to all the objections raised by Mr. Gray. The anecdote of the Laputa Tailors and their sextants I will leave for the present, not having been able in the short space of two months properly to realize the force of the simile.

I am, Sir,
Your most obedient Servant,
SAMUEL YOUNGER.

316, *Regent Street*, 28*th February*, 1863.

RULES TO BE OBSERVED IN CONVERTING THE PARTS OF ONE POUND INTO DECIMALS.

To the Editor of the Assurance Magazine.

MY DEAR SIR,—In taking the decimals of the parts of a pound by the *head-rule*, I have always followed the rule given in my *Arithmetic*, which I now repeat.

First three places.—For every pair of shillings, 100; for the odd shilling, 50; for every farthing above shillings, 1, with a unit of carriage at and after 6*d.*