IMPROVED APPROXIMATIONS FOR THE AGGREGATE CLAIMS DISTRIBUTION IN THE INDIVIDUAL MODEL

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ABSTRACT

Kornya-type higher order approximations are derived for the aggregate claims distribution and for stop loss premiums in the individual model with arbitrary positive claims. Absolute error bounds and error bounds based on concentration functions are given. In the Gerber portfolio containing 31 policies, second order approximations lead to an accuracy of 3×10^{-4}, and third order approximations to 1.7×10^{-5}.

KEYWORDS

Aggregate claims distribution, compound Poisson distribution, higher order approximations.

1. INTRODUCTION AND SUMMARY

Consider a portfolio containing N policies, where for $i=1,\dots,N$ the claim amounts distribution Q_i of the individual risk i can be represented as

$$Q_i=(1-q_i)\delta_0+q_iP_i.$$

Here, δ_0 is the Dirac measure of zero, $\delta_0\{0\}=1$, and P_i is a probability measure with $P_i(0,\infty)=1$. The number $q_i\in(0,1)$ is the probability that risk i produces a claim. The distribution P_i is the conditional distribution of the claims in risk i, given that a claim occurs in risk i. We shall be concerned with approximations for the convolution

$$G=Q_1*\cdots*Q_N$$

which is the aggregate claims distribution of the portfolio in the individual model.

In this first section we shall (a) give heuristic motivations for the approximations, (b) introduce the approximations, and (c) present error bounds. In Section 2 a numerical illustration is given. All our proofs are deferred to Section 3.

(a) Assume for the moment that $N=1$, and write

$$Q_1=Q=(1-q)\delta_0+qP$$

and g for the characteristic function of P. The characteristic function of Q is given by

$$1-q+qg=\exp(\log(1-q+qg)).$$

The kth-order approximations H_k for Q suggested by KORNYA (1983) are derived as follows: Expand the right-hand side of the equation

$$\log(1-q+qg)=\log(1+qg/(1-q))-\log(1+q/(1-q))$$

in powers of $q/(1-q)$. This yields the following approximation for the characteristic function h of Q:

$$h_k=\exp\left(\sum_{j=1}^{k}(-1)^{j+1}(1/j)(q/(1-q))^j(g^j-1)\right).$$

Whenever q is small, h_k will be a good approximation for h. The approximation H_k has characteristic function h_k, an hence H_k will be a good approximation for Q according to the continuity theorem for characteristic functions (see LOÈVE 1977, p. 204).

We consider slightly different kth-order approximations H_k^* which are derived as follows: Expand the right-hand side of the equation

$$\log(1-q+qg)=\log(1+q(g-1))$$

in powers of q. We then obtain the following approximation for h:

$$h_k^*=\exp\left(\sum_{j=1}^{k}(-1)^{j+1}(1/j)q^j(g-1)^j\right).$$

The approximation H_k^* has characteristic function h_k^*.

For arbitrary $N>1$ the approximations H_k and H_k^* for G are constructed as follows. Let $H_k(i)$ and $H_k^*(i)$ be the approximations for Q_i, $i=1,\dots,N$. Then

$$H_k=H_k(1)*\cdots*H_k(N)$$

and

$$H_k^*=H_k^*(1)*\cdots*H_k^*(N)$$

respectively.

(b) We introduce first the compound Poisson distribution

$$H=\sum(\lambda^n/n!)\,e^{-\lambda}P_0^{*n}$$

with Poisson parameter

$$\lambda=q_1+\cdots+q_N$$

and claim amount distribution

$$P_0=q_1/\lambda P_1+\cdots+q_N/\lambda P_N.$$

In the collective risk theory model, H is the aggregate claims distribution of the portfolio.

Consider next the compound Poisson distribution

$$H_1=\sum(\lambda')^n(n!)^{-1}\,e^{-\lambda'}(P_0')^{*n}$$

with Poisson parameter

$$\lambda' = q_1/(1-q_1) + \cdots + q_N/(1-q_N)$$

and claim amount distribution

$$P'_0 = q_1/((1-q_1)\lambda')P_1 + \cdots + q_N/((1-q_N)\lambda')P_N.$$

The approximation H_1 for G is always on the safe side in the sense that for all real t

$$G(t,\infty) \leq H_1(t,\infty). \tag{1}$$

In order to define Kornya's approximations H_k for G it is convenient to extend the concept of compound Poisson distributions to finite signed measures M, i.e. to countably additive set functions M satisfying

$$\sup_A |M(A)| < \infty.$$

Define the n-fold convolution M^{*n} of M by

$$M^{*0} = \delta_0,\ M^{*(n+1)}(A) = \int\int 1_A(x+y)M^{*n}(dy)M(dx)$$

and define the signed Poisson measure with Poisson parameter $\lambda \in \mathbb{R}$ and signed claim amount measure M_0 by

$$M = e^{-\lambda} \sum_{n=0}^{\infty} \lambda^n/(n!)M_0^{*n}.$$

For $i = 1, \ldots, N$ and $j = 1, 2, \ldots$ define

$$c_{ij} = (-1)^{j+1}(1/j)(q_i/(1-q_i))^j$$

and for $k = 1, 2, \ldots,$ let

$$\lambda_k = \sum_{i=1}^{N} \sum_{j=1}^{k} c_{ij}$$

$$R_k = \sum_{i=1}^{N} \sum_{j=1}^{k} (c_{ij}/\lambda_k)P_i^{*j}.$$

Write H_k for the signed compound Poisson measure with Poisson parameter λ_k and signed claim amount measure R_k. Notice that H_1 is the compound Poisson distribution defined earlier. For arbitrary $k \geq 1$ the signed measures are normed, $H_k(\mathbb{R}) = 1$, but H_k can be negative, $H_k(A) < 0$ for some sets A.

For $k = 1, 2, \ldots$ the approximations H_k^* are defined as follows. Let

$$U_k = \sum_{j=1}^{k} (-1)^{j+1}(1/j) \sum_{i=1}^{N} q_i^j(P_i - \delta_0)^{*j}.$$

The signed measure U_k can uniquely be represented by

$$U_k = \lambda_k^*(R_k^* - \delta_0)$$

with $\lambda_k^* \in \mathbb{R}$ and R_k^* a signed normed measure with $R_k^*\{0\} = 0$. Let H_k^* be the signed compound Poisson measure with Poisson parameter λ_k^* and signed claim amount measure R_k^*. Then

$$H_1^* = H$$

$$\lambda_2^* = \sum_{i=1}^{N} (q_i + q_i^2/2)$$

$$\lambda_3^* = \sum_{i=1}^{N} (q_i + q_i^2/2 + q_i^3/3)$$

$$R_2^* = (1/\lambda_2^*) \sum_{i=1}^{N} \{(q_i + q_i^2)P_i - (1/2)q_i^2 P_i^{*2}\}$$

$$R_3^* = (1/\lambda_3^*) \sum_{i=1}^{N} \{(q_i + q_i^2 + q_i^3)P_i - (q_i^2/2 + q_i^3)P_i^{*2} + (1/3)q_i^3 P_i^{*3}\}.$$

Notice that the computation of H_k and H_k^* can be done using fast Fourier methods (see BERTRAM 1981) or the recursion algorithm (see PANJER 1981). The characteristic functions of H_k and H_k^* equal

$$\exp\left(\sum_{i=1}^{N} \sum_{j=1}^{k} (-1)^{j+1}(1/j)(q_i/(1-q_i))^j (g_i^j - 1)\right)$$

and

$$\exp\left(\sum_{i=1}^{N} \sum_{j=1}^{k} (-1)^{j+1}(1/j)q_i^j (g_i - 1)^j\right)$$

respectively, where g_i is the characteristic function of P_i. These characteristic functions can easily be computed, and hence fast Fourier methods work.

Assume now that for some fixed positive h, the distributions P_i are concentrated on the positive integral multiples of h, i.e.

$$P_i\{h, 2h, 3h, \ldots\} = 1, \qquad i = 1, \ldots, N.$$

Then for non-negative integral p we have the recursions

$$(p+1)H_k\{h(p+1)\} = \lambda_k \sum_{r=1}^{p+1} rR_k\{hr\}H_k\{h(p+1-r)\}$$

and

$$(p+1)H_k^*\{h(p+1)\} = \lambda_k^* \sum_{r=1}^{p+1} rR_k^*\{hr\}H_k^*\{h(p+1-r)\}$$

and the initial values

$$H_k\{0\} = \exp(-\lambda_k) \quad \text{and} \quad H_k^*\{0\} = \exp(-\lambda_k^*).$$

(c) In contrast to classical higher order approximations for G such as the normal power method or Edgeworth-expansions, theoretical error bounds can easily be derived for the approximations H_k and H_k^*. Well known error bounds for the case $k=1$ are

(2) $$\sup_A |G(A)-H(A)| \leq \sum_{i=1}^N q_i^2$$

(see GERBER 1984, p. 192, theorem 1.a) and

(3) $$\sup_A |G(A)-H_1(A)| \leq (1/2) \sum_{i=1}^N (q_i/(1-q_i))^2.$$

Smaller error bounds have been derived in HIPP (1985) for the distance between the corresponding distribution functions:

(4) $$\sup_t |G(-\infty, t)-H(-\infty, t)| \leq 5 \sum_{i=1}^N q_i^2/(1-q_i)C(P, \alpha_i).$$

Here, α_i is the mean of P_i, and $C(P, r)$ is the concentration function of the probability measure P at $r>0$,

$$C(P, r) = \sup_x P[x, x+r).$$

Finally, P is the compound Poisson distribution with Poisson parameter

$$\lambda = (1/2) \sum_{i=1}^N q_i(1-q_i)$$

and claim amount distribution

$$P_0 = \sum_{i=1}^N q_i(1-q_i)/(2\lambda)P_i.$$

The right-hand side of (4) will often be considerably smaller than the right-hand side of (2). Consider, e.g.

$$P_i\{1\}=1, \qquad q_i = cN^{-1/2}, \qquad i=1,\ldots,N$$

with a fixed constant $c \in (0, 1)$. Then P is a Poisson distribution with parameter $cN^{1/2}(1-cN^{-1/2})/2$ and hence $C(P, 1)$ is of order $N^{-1/4}$ (compare (11) in HIPP 1985). So the right-hand side of (4), with $\alpha_i = 1$, is of order $N^{-1/4}$, too, while the right-hand side of (2) equals c^2.

For the presentation of error bounds for H_k and H_k^* corresponding to (2) and (4) we need some notation. Fix $k \geq 1$, and for $i=1,\ldots,N$ define

$$\tau_i = (1/(k+1))(q_i/(1-q_i))^{k+1}(1-q_i)/(1-2q_i)$$

and

$$\sigma_i = (1/(k+1))(2q_i)^{k+1}/(1-2q_i).$$

Let

$$\tau = \sum_{i=1}^N \tau_i, \qquad \sigma = \sum_{i=1}^N \sigma_i,$$

$$\delta = \sum_{i=1}^N (e^{\tau_i}-1), \qquad \delta^* = \sum_{i=1}^N (e^{\sigma_i}-1).$$

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The error bounds corresponding to (2) are

$$\sup_A |G(A)-H_k(A)| \le e^{\tau}-1 \tag{5}$$

and

$$\sup_A |G(A)-H_k^*(A)| \le e^{\sigma}-1. \tag{6}$$

The error bounds corresponding to (4) are

$$\sup_t |G(-\infty,t)-H_1(-\infty,t)| \le 5\sum_{i=1}^{N}(e^{2\tau_i}-1)C(P,2\alpha_i) \tag{7}$$

$$\sup_t |G(-\infty,t)-H_k(-\infty,t)|(1-\delta) \le 5\sum_{i=1}^{N}(e^{2\tau_i}-1)C(P,(k+1)\alpha_i) \tag{8}$$

and

$$\sup_t |G(-\infty,t)-H_k^*(-\infty,t)|(1-\delta^*) \le 5\sum_{i=1}^{N}(e^{2\sigma_i}-1)C(P,(k+1)\alpha_i). \tag{9}$$

The probability measure P occurring in the concentration function is the compound Poisson distribution defined above. The numbers τ_i in (7) have to be defined with $k=1$. In (5)-(9) we tacitly assumed that

$$q_i < 1/2, \qquad i=1,\ldots,N.$$

Comparing (5) and (8) with (6) and (9) one might expect that the approximations H_k perform better than H_k^*. In our numerical illustration this is not true. Notice also that the mean of H_k^* and the mean of G coincide, while the mean of H_k and the mean of G are different.

Finally, Kornya's approximations H_k can be used for approximate computation of the stop loss premium

$$\int (x-z)^+ G(dx)$$

in the individual model. Under the assumption

$$q_i < 1/2, \qquad i=1,\ldots,N$$

we obtain the following error bound:

$$\left|\int (x-z)^+ G(dx) - \int (x-z)^+ H_k(dx)\right| \le (e^{\tau}-1)\int (x-z)^+ G(dx) + e^{\tau}\sum_{i=1}^{N}\alpha_i (q_i/(1-q_i))^{k+1}(1-q_i)/(1-2q_i). \tag{10}$$

Notice that for fixed $N, Q_1, \ldots, Q_N$, the approximations H_k and H_k^* converge to G when k tends to infinity. The error bounds (5), (6) and (8), (9) converge to zero when k tends to infinity. Hence if an upper bound for the error is given we can choose k such that the error of approximating G by H_k (or by H_k^*) is smaller than the prescribed upper bound. The computation time which is needed for the numerical computation of H_k or H_k^*, e.g. with Panjer's recursion algorithm in the arithmetic case, is linearly increasing with k.

2. NUMERICAL ILLUSTRATION

We consider the small portfolio of GERBER (1979, p. 53, table 3). The following table shows the values $G(-\infty, x)$, $H_k(-\infty, x)$, $H_k^*(-\infty, x)$ for $k=1, 2, 3$ and $x=1, \dots, 20$.

x	G	H_1	H_2	H_3	H_1^*	H_2^*	H_3^*
1	0.238195	0.229700	0.238496	0.238183	0.246597	0.238473	0.238206
2	0.252929	0.244014	0.253249	0.252916	0.261393	0.253210	0.252940
3	0.340663	0.328876	0.341094	0.340645	0.348145	0.340851	0.340667
4	0.453846	0.438079	0.454416	0.453823	0.459370	0.453872	0.453840
5	0.564555	0.547070	0.565265	0.564526	0.569766	0.564611	0.564555
6	0.660883	0.640235	0.661712	0.660847	0.662625	0.660717	0.660869
7	0.722431	0.703134	0.723259	0.722394	0.723633	0.722303	0.722421
8	0.791453	0.770973	0.792362	0.791413	0.789060	0.791157	0.791436
9	0.846270	0.828072	0.847221	0.846230	0.843637	0.846108	0.846270
10	0.889418	0.871906	0.890284	0.889376	0.884958	0.889120	0.889402
11	0.919525	0.904912	0.920386	0.919482	0.915537	0.919389	0.919525
12	0.943054	0.930424	0.943877	0.943012	0.938845	0.942970	0.943058
13	0.961336	0.950689	0.962039	0.961299	0.957189	0.961242	0.961338
14	0.973846	0.965402	0.974490	0.973809	0.970338	0.973842	0.973853
15	0.982556	0.975869	0.983125	0.982522	0.979556	0.982596	0.982565
16	0.988468	0.983358	0.988918	0.988436	0.986061	0.988510	0.988472
17	0.992620	0.988711	0.993002	0.992594	0.990656	0.992680	0.992626
18	0.995335	0.992455	0.995640	0.995311	0.993832	0.995401	0.995339
19	0.997076	0.994992	0.997317	0.997054	0.995956	0.997142	0.997078
20	0.998193	0.996704	0.998376	0.998175	0.997370	0.998250	0.998193

For all approximations K the actual error

$$E(K) = \sup_t |G(-\infty, t) - K(-\infty, t)|$$

together with bounds (5) and (6) are shown in our next table.

K	H_1	H_2	H_3	H_1^*	H_2^*	H_3^*
$E(K)$	0.020648	0.000951	0.000043	0.008402	0.000295	0.000017
(5) or (6)	0.040015	0.001395	0.000058	0.160690	0.010060	0.000785

In this small portfolio the concentration function is quite large. Hence (7), (8), and (9) do not yield reasonable error bounds here.

3. PROOFS

Relation (1) follows from the fact that a Bernoulli random variable X with $P\{X=1\}=p$ is stochastically smaller than a Poisson random variable with parameter $p/(1-p)$.

For the proof of (3) it suffices to consider the case $N=1$ and to show that

$$\sup_A H_1(A)-Q_1(A)\leqslant (q_1/(1-q_1))^2/2.$$

This follows from $H_1\{0\}\leqslant Q_1\{0\}$ (see (1)),

$$x\,e^{-x}\leqslant x/(1+x), \qquad x=q_1/(1-q_1)>0,$$

and

$$1-e^{-x}(1+x)\leqslant x^2/2, \qquad x=q_1/(1-q_1)>0.$$

For the proof of (5)-(9) we introduce exponentials for finite signed measures M. If M has characteristic function

$$f(t)=\int e^{itx}M(dx)$$

then $\exp(M)$ is the finite signed measure with characteristic function $\exp(f(t))$. For $\exp(M)$ we have the explicit representation

$$\exp(M)=\sum_{n=0}^{\infty}(1/n!)M^{*n}.$$

Notice that for finite signed measures M_1, M_2, the signed measure $\exp(M_1+M_2)$ is the convolution of $\exp(M_1)$ and $\exp(M_2)$. In the following we shall always assume that

$$q_i<1/2, \qquad i=1,\ldots,N.$$

In this case, the set function

$$M_0=\sum_{i=1}^{N}\sum_{j=1}^{\infty}(-1)^{j+1}(1/j)(q_i/(1-q_i))^j(P_i^{*j}-\delta_0)$$

is a finite signed measure, and

$$\exp(M_0)=G.$$

For finite signed measures M we shall write $M=M^+-M^-$ for the Hahn-Jordan decomposition of M, and $|M|=M^++M^-$, $\|M\|=|M|(\mathbb{R})$.

3.1. LEMMA. *For measurable functions f and finite signed measures M, $\exp(|M|)-\delta_0$ is a (positive) measure, and*

(a) $$\left|\int f(x)(\exp(M)-\delta_0)(dx)\right|\leqslant\int |f(x)|(\exp(|M|)-\delta_0)(dx)$$

(b) $$\|\exp(M)-\delta_0\|\leqslant e^{\|M\|}-1.$$

PROOF. (a) Notice first that

$$\left|\int f(x)(\exp(M)-\delta_0)(dx)\right| \leq \int |f(x)||\exp(M)-\delta_0|(dx).$$

Hence it suffices to show that for arbitrary measurable sets A

$$|(\exp(M)-\delta_0)(A)| \leq (\exp(|M|)-\delta_0)(A).$$

This inequality is true for $A=\{0\}$, and for sets A with $0 \notin A$ we have

$$\begin{aligned}|(\exp(M)-\delta_0)(A)| &\leq \sum_{n=1}^{\infty} (1/n!)|M|^{*n}(A) \\ &= \exp(|M|)(A) = (\exp(|M|)-\delta_0)(A).\end{aligned}$$

(b) With (a) we obtain

$$\begin{aligned}\|\exp(M)-\delta_0\| \leq (\exp(|M|)-\delta_0)(\mathbb{R}) &= \sum_{n=1}^{\infty} \|M\|^n/n! \\ &= e^{\|M\|}-1.\end{aligned}$$

PROOF OF (5). Notice that $H_k = \exp(R_k)$ with

$$R_k = \sum_{i=1}^{N}\sum_{j=1}^{k} c_{ij}(P_i^{*j}-\delta_0).$$

Hence

$$G-H_k = \exp(M_0)-\exp(R_k) = G*(\delta_0-\exp(R_k-M_0)).$$

Because of

$$\|G-H_k\| \leq \|\exp(R_k-M_0)-\delta_0\|$$

we obtain with Lemma 3.1b)

$$\|G-H_k\| \leq \exp(\|M_0-R_k\|)-1.$$

Now $\|M_0-R_k\| \leq \tau$ implies the assertion.

PROOF OF (7) AND (8). For $i=1,\ldots,N$ let

$$\begin{aligned}R_k^{(i)} &= \sum_{j=1}^{k} (-1)^{j+1}(1/j)(q_i/(1-qi))^j(P_i^{*j}-\delta_0) \\ H_k^{(i)} &= \exp(R_k^{(i)}) \\ M_i &= Q_1*\cdots*Q_{i-1}*H_k^{(i+1)}*\cdots*H_k^{(N)} \\ R^{(i)} &= \sum_{j=1}^{\infty} (-1)^{j+1}(1/j)(q_i/(1-q_i))^j(P_i^{*j}-\delta_0)\end{aligned}$$

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and

$$\Delta = \max_{m=1}^{N} \sup_t |G(-\infty, t) - Q_m * M_m(-\infty, t)|.$$

Then

$$\Delta \leqslant \sum_{i=1}^{N} \sup_t \left| \int M_i(-\infty, t-s)(Q_i - H_k^{(i)})(ds) \right|.$$

For $i = 1, \ldots, N$ and $t \in \mathbb{R}$ we have

$$\begin{aligned}
&\left| \int M_i(-\infty, t-s)(Q_i - H_k^{(i)})(ds) \right| \\
&\quad = \left| \int M_i * Q_i(-\infty, t-s)(\exp(R_k^{(i)} - R^{(i)}) - \delta_0)(ds) \right| \\
&\quad = \left| \int ((M_i * Q_i - G)[t-s, t) + G[t-s, t))(\exp(R_k^{(i)} - R^{(i)}) - \delta_0)(ds) \right. \\
&\quad \leqslant \int (\Delta + G[t-s, t))(\exp(|R_k^{(i)} - R^{(i)}|) - \delta_0)(ds) = I_1 + I_2, \quad \text{say.}
\end{aligned}$$

Lemma 3.1b) yields $I_1 \leqslant \Delta(e^{\tau_i} - 1)$. In order to compute an upper bound for I_2 we fix a positive r and notice that for $s \geqslant 0$

$$G[t-s, t) \leqslant (1 + s/r) C(G, r).$$

Furthermore,

$$\begin{aligned}
\int s(\exp(|R_k^{(i)} - R^{(i)}|) - \delta_0)(ds)| &= e^{\|R^{(i)} - R^{(i)}\|} \int s|R_k^{(i)} - R^{(i)}|(ds) \\
&\leqslant e^{\tau_i}(k+1)\tau_i \alpha_i.
\end{aligned}$$

This implies

$$I_2 \leqslant (e^{\tau_i} - 1 + e^{\tau_i}(k+1)\tau_i \alpha_i / r) C(G, r).$$

With $r = (k+1)\alpha_i$ we obtain

$$I_2 \leqslant (e^{2\tau_i} - 1) C(G, (k+1)\alpha_i)$$

and hence

$$\Delta(1 - \delta) \leqslant \sum_{i=1}^{N} (e^{2\tau_i} - 1) C(G, (k+1)\alpha_i).$$

As in Hipp (1985, p. 231, (24)), we obtain the following upper bound for $C(G, r)$:

$$C(G, r) \leqslant (\pi^2/2) C(P, r).$$

Hence

$$\Delta(1-\delta)\leq(\pi^2/2)\sum_{i=1}^{N}(e^{2\tau_i}-1)C(P,(k-1)\alpha_i)$$

which proves (8).

If $k=1$, then for $i=1,\ldots,N$ the signed measures $M_i * Q_i$ are (positive) measures, and therefore the above mentioned methods can be applied to derive an upper bound for $C(M_i * Q_i, r)$. We obtain with $r=2\alpha_i$

$$\begin{aligned}
&\left|\int M_i * Q_i(-\infty, t-s)(\exp(R_1^{(i)}-R^{(i)})-\delta_0)(ds)\right| \\
&=\left|\int M_i * Q_i[t-s, t)(\exp(R_1^{(i)}-R^{(i)})-\delta_0)(ds)\right| \\
&\leq C(M_i * Q_i, r)\int(1+s/r)(\exp(|R_1^{(i)}-R^{(i)}|)-\delta_0)(ds) \\
&\leq(\pi^2/2)C(P,2\alpha_i)(e^{\tau_i}-1+e^{\tau_i}\alpha_i 2\tau_i/r) \\
&\leq(\pi^2/2)C(P,2\alpha_i)(e^{2\tau_i}-1).
\end{aligned}$$

This proves (7).

The proofs for (6) and (9) are modifications of the above proofs.

PROOF OF (10). For arbitrary x, z and positive y we have

$$(x+y-z)^+\leq(x-z)^++y.$$

This implies

$$\begin{aligned}
\left|\int(x-z)^+(G-H_k)(dx)\right| &= \left|\int(x+y-z)^+G(dx)(\exp(R_k-M_0)-\delta_0)(dy)\right| \\
&\leq\int(x-z)^+G(dx)\|\exp(R_k-M_0)-\delta_0\| \\
&\quad+\int y|\exp(R_k-M_0)-\delta_0|(dy) \\
&=I_3+I_4,\quad\text{say.}
\end{aligned}$$

Lemma 3.1 yields the following bounds:

$$I_3\leq(e^{\tau}-1)\int(x-z)^+G(dx)$$

$$I_4\leq\int y\exp(|R_k-M_0|)(dy)\leq e^{\tau}\int y|R_k-M_0|(dy).$$

From

$$\int y|R_k - M_0|(dy) \leq \sum_{j=k+1}^{\infty} (1/j) \sum_{i=1}^{N} (q_i/(1-q_i))^j \int yP_i^{*j}(dy)$$

$$= \sum_{i=1}^{N} \alpha_i (q_i/(1-q_i))^{k+1}(1-q_i)/(1-2q_i)$$

we obtain the asserted inequality (10).

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