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Solutions of field equations in general relativity with spinor connection

L.A. Peters

While General Relativistic geometries have been very successful in providing cosmological and large body models, their failure to provide realistic models of fundamental particles is generally taken today as an indication that General Relativity is just not applicable in the microscopic domain - particles simply cannot be described in terms of "classical" fields.

However, by defining connection in terms of parallel displacement not of the tangent vector field, but of an underlying spinor field (and using the covering group of the Lorentz group as the structure group of connection) one finds clear geometric equivalents of both the electromagnetic field and spin. Generalized Einstein-Maxwell-Dirac equations can then be derived from a world Lagrangian. Torsion, an essential part of the associated vector connection, appears in the field equations precisely in the form demanded by the U_4 theory of Trautman, Sciarna and Kibble (see Hehl [1]). The geometry admits oppositely charged particle pairs, neutral particles, bosons and fermions of arbitrary spin.

Three particle solutions of such a geometry are presented here: two spherically symmetric charged particles, one of which has an additional magnetic monopole charge, and an axially symmetric charged particle.

The form of the metric and the electromagnetic field is determined by

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the requirement that at large distances from the souce, the solutions should melt into the appropriate classical solution, namely, the Reissner-Nordstrom or the Kerr-Newman solution.

The spinor field is represented as a tetrad of Dirac spinors. The number and type of components necessary in a particular instance is determined by global properties such as the charge or spin of the particle. The form of the tetrad components themselves is determined fairly strictly by consistency conditions within the model itself.

The resulting field equations are generally too complicated to allow an exact solution. Runge-Kutta methods are used to solve the ordinary differential equations of the spherically symmetric solutions. A "method of lines" approach has been used for the partial differential equations of the axially symmetric solution.

The particles are found to have quite different internal structures and rest masses. In each case there is no residual point charge - it has all been accounted for by a finite space charge density. The particles hold together as the self-interaction is compensated by torsion and other curvature fields. (Torsion here is a property of the spinor connection only, the metric connection is torsion free.)

The first particle is tentatively identified as a π-meson. Fundamental constants of the geometry are fixed by equating the computed mass with its known mass of $273m_e$.

The monopole solution is found to have precisely the magnetic pole strength predicted by Dirac. Two masses are admitted: the expected huge one of order 10^{20} times the fundamental mass, and one of order the fundamental mass itself (approximately that of the proton).

The third solution has many of the properties of the electron, but does not generate the required magnetic moment.

Attempts to improve these models led to an extension of the group of connection to the covering group of $O(4, 2)$. Such an extension seems to generate the right magnetic moment for the electron solution but details of the computations were left to future investigations. One interesting peculiarity of this extension is that the "pure" Maxwell field components E and B and the "material" components D and H are represented by different connection quantities.

Reference

[1] F. Hehl *et al.*, "General relativity with spin and torsion", *Rev. Mod. Phys.* **48** (1976), 393-416.

Department of Mathematics,
University of New South Wales,
PO Box 1,
Kensington,
New South Wales 2033,
Australia.