75. ON THE RATE OF EJECTION OF DUST BY LONG-PERIOD COMETS

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Abstract. A model of the interplanetary dust medium that includes two subsystems, a spherical component and a flat one, has been proposed by the author as a result of analysis of radar meteor orbits. The disintegration of short-period comets is the main source of the flat dust cloud, while the disintegration of long-period comets is the main source of the spherical cloud. Assuming that dust particles fall into the Sun due to the Poynting-Robertson effect, one can estimate the intensity of ejection of dust from long-period comets necessary for the maintenance of the observed density of interplanetary dust. It has been found that the long-period comets should eject between 7×10^{14} and 2×10^{15} g of meteoric dust per year.

Analysis of the Kharkov catalogue of 12 500 orbits of meteors brighter than magnitude 7 (Lebedinets, 1968) has shown that the complex of small meteoric bodies producing radio meteors can be represented as a sum of two components: a 'spherical' component, comprising particles moving in orbits with random inclination i and more or less regular distributions of eccentricity e and perihelion distance q; and a 'flat' component, comprising particles moving in orbits with low inclinations ($i \leqslant 35°$), small semi-major axes ($a \lesssim 5$ AU), and fairly large eccentricities ($e \gtrsim 0.7$). The spherical component includes predominantly sporadic particles, while the flat component involves particles in streams. This shows that, on the average, the particles belonging to the spherical component appeared as independent celestial bodies before those of the flat component.

We have suggested that disintegration of long-period comets may be the principal source of particles in the spherical component (Lebedinets, 1968). The orbits of the particles are gradually transformed as a consequence of the Poynting-Robertson effect, and ultimately the particles fall into the Sun. As a result of the simultaneous action of these processes (disintegration of comets, transformation of particle orbits, and the particles falling into the Sun) there is a certain constant distribution of particle orbits. We have obtained formulae for this distribution with respect to a, e, and q; the distribution depends in essence only on the original distribution of injected particles by perihelion distance: $A(q_0)$.

By comparing the theoretical constant distribution with the observed distribution of orbits of the radio meteors, we have derived the function $A(q_0)$. Approximating it with a power function,

$$A(q_0) = A_0 q_0^n, \tag{1}$$

we find that the exponent n should be within the range $-1.5 \leqslant n \leqslant -1.0$.

Knowing $A(q_0)$, it is possible to evaluate the relative distribution of the spatial density of dust particles in the spherical component. Having calibrated this distribution with the results of measurements of particle densities near the orbit of the Earth,

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it is possible to evaluate the rate at which particles fall into the Sun and consequently the intensity of the process of dust ejection by the long-period comets.

Robertson (1937) obtained the following formulae for the orbital variations of an absolutely black, spherical particle due to radiative deceleration:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{\alpha(2 + 3e^2)}{a(1 - e^2)^{3/2}}, \quad \frac{\mathrm{d}e}{\mathrm{d}t} = \frac{5\alpha e}{2a^2(1 - e^2)^{1/2}}; \qquad \alpha = \frac{3E_0 r_0^2}{4R\delta c^2}, \tag{2}$$

Here E_0 is the solar constant, r_0 the distance from the Earth to the Sun, R the radius of the particle, δ its density, and c is the velocity of light.

For the case when $e \neq 0$ Wyatt and Whipple (1950) obtained the relation

$$\frac{e^{4/5}}{q(1 + e)} = \frac{e_0^{4/5}}{q_0(1 + e_0)} = \frac{1}{C}, \tag{3}$$

where q_0 and e_0 are the values of q and e at some initial epoch, and C is a constant.

From Equations (2) and (3) we have

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{5\alpha(1 - e^2)^{3/2}}{2C^2 e^{3/5}}, \tag{4}$$

and from Equations (1) and (4) we may obtain the constant distribution of orbital eccentricities of particles injected into orbits with given q_0 and e_0 (≈ 1):

$$\left(\frac{\mathrm{d}N}{\mathrm{d}e}\right)_{q_0} = -\frac{A(q_0)}{T(\mathrm{d}e/\mathrm{d}t)}, \tag{5}$$

where T is the period of revolution. This distribution refers to particles passing through perihelion in unit time.

Using Kepler's third law and Equation (3) we may recast Equation (5) as

$$\left(\frac{\mathrm{d}N}{\mathrm{d}e}\right)_{q_0} = \frac{2\sqrt{2} q_0^{1/2} A(q_0)}{5\alpha e^{3/5}}. \tag{6}$$

The partial density of particles at heliocentric distance r that have the particular q_0 is then

$$f(r, q_0) = \frac{1}{4\pi r^2} \int_{e_1}^{e_2} \frac{2}{\mathrm{d}r/\mathrm{d}t} \left(\frac{\mathrm{d}N}{\mathrm{d}e}\right)_{q_0} \mathrm{d}e. \tag{7}$$

The radial component $\mathrm{d}r/\mathrm{d}t$ of velocity depends on e, r, and q_0. The lower limit e_1 of integration is found from the condition $q_1 = r$ and Equation (3):

$$\frac{e_1^{4/5}}{1 - e_1} = \frac{r}{2q_0}. \tag{8}$$

When $q_0 \leq r$ the upper limit e_2 of integration is found from

$$e_2 = \frac{K - q_0}{K + q_0}, \tag{9}$$

K being the distance from the Sun to the boundary of the solar system; on the other hand, if $q_0 > r$, the upper limit is found with $q_2 = r$ and Equation (3):

$$\frac{e_2^{4/5}}{1 + e_2} = \frac{r}{2q_0}. \tag{10}$$

The radial component of the orbital velocity of the particle is found from the equations of elliptical motion:

$$r = a(1 - e\cos E), \qquad 2\pi t/T = E - e\sin E, \tag{11}$$

where E is the eccentric anomaly and t is measured from perihelion. It follows that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{2\sqrt{2}\pi q_0^{1/2} e^{7/5}}{r(1 - e^2)^{1/2}} \sqrt{1 - \frac{1}{e^2}\left[1 - \frac{r(1 - e^2)}{2q_0 e^{4/5}}\right]^2}. \tag{12}$$

From Equations (7) and (12) we have

$$f(r, q_0) = \frac{A(q_0)}{10\pi^2 \alpha r} \int_{e_1}^{e_2} \frac{(1 - e^2)^{1/2} e^{-2}\,\mathrm{d}e}{\sqrt{1 - \dfrac{1}{e^2}\left[1 - \dfrac{r(1 - e^2)}{2q_0 e^{4/5}}\right]^2}}. \tag{13}$$

Integrating over q_0, we find the total spatial density of particles injected into orbits with any q_0:

$$\begin{aligned} f(r) = \frac{1}{10\pi^2\alpha}\Bigg\{ &\frac{1}{r}\int_{0.01}^{r} A(q_0)\,\mathrm{d}q_0 \int_{e_1}^{(K - q_0)/(K + q_0)} \frac{(1 - e^2)^{1/2} e^{-2}\,\mathrm{d}e}{\sqrt{1 - e^{-2}[1 - r(1 - e^2)/2q_0 e^{4/5}]^2}} \\ &+ \frac{1}{r}\int_{r}^{q_{02}} A(q_0)\,\mathrm{d}q_0 \int_{e_1}^{e_2} \frac{(1 - e^2)^{1/2} e^{-2}\,\mathrm{d}e}{\sqrt{1 - e^{-2}[1 - r(1 - e^2)/2q_0 e^{4/5}]^2}} \Bigg\} \end{aligned} \tag{14}$$

For the particular power distribution of initial perihelion distances given by Equation (1) we obtain

$$\begin{aligned} f(r) = \frac{A}{10\pi^2\alpha}\Bigg\{ &\frac{1}{r}\int_{0.01}^{r} q_0^n\,\mathrm{d}q_0 \int_{e_1}^{(K - q_0)/(K + q_0)} \frac{(1 - e^2)^{1/2} e^{-2}\,\mathrm{d}e}{\sqrt{1 - e^{-2}[1 - r(1 - e^2)/2q_0 e^{4/5}]^2}} \\ &+ \frac{1}{r}\int_{r}^{q_{02}} q_0^n\,\mathrm{d}q_0 \int_{e_1}^{e_2} \frac{(1 - e^2) e^{-2}\,\mathrm{d}e}{\sqrt{1 - e^{-2}[1 - r(1 - e^2)/2q_0 e^{4/5}]^2}} \Bigg\} \\ = \frac{A_0 F(r, K, n)}{10\pi^2\alpha}, & \end{aligned} \tag{15}$$

where the expression in braces is denoted by $F(r, K, n)$. In Equations (14) and (15) the lower limit of integration over q_0 is taken to be 0.01 AU, since at distances of less than this the equilibrium temperature of particles is higher than the temperature of evaporation of stone and iron meteorites; the upper limit is the maximum value q_{02}, which can be found from the condition that when $r > q_{02}$ the equilibrium temperature of a cometary nucleus is very low, and there is practically no ejection of dust.

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From the radio observations of meteors we have found (Lebedinets, 1971) that in the vicinity of the Earth's orbit (i.e., at $r=1$ AU) one cubic kilometre contains on the average 10^{-5} particles with masses greater than $M=2\times10^{-4}$ g, and of these 6×10^{-6} particles belong to the spherical component. Adopting a power distribution of particles through mass with exponent $S=2$, we find the integral density of the spherical component at $r=1$ AU to be

$$D_0 = 1.2 \times 10^{-9}\ \mathrm{g\ km^{-3}} = 4 \times 10^{15}\ \mathrm{g\ AU^{-3}}. \tag{16}$$

The differential density of particles with mass M is

$$f(1, M) = \left|\frac{\mathrm{d}D(1, M)}{\mathrm{d}M}\right| = \frac{D_0}{M^2}. \tag{17}$$

From Equations (15) and (17) we may find the unknown quantity A_0:

$$A_0 = \frac{10\pi^2 \alpha D_0}{M^2 F(1, K, n)}. \tag{18}$$

Substituting for the values of E_0, r_0 and c in Equation (2) we have

$$\alpha = \frac{3.55 \times 10^{-8}}{R\delta} = \frac{3.55 \times 10^{-8}}{(3\delta^2/4\pi)^{1/3} M^{1/3}} = \frac{C(\delta)}{M^{1/3}}\ \mathrm{AU^2\ yr^{-1}}, \tag{19}$$

where M is expressed in grams and δ in g cm^{-3}. With $\delta=3.5$ g cm^{-3}, $C(\delta)=2.48\times10^{-8}$. Hence

$$A_0 = \frac{10\pi^2 D_0 C(\delta)}{M^{7/3} F(1, K, n)}. \tag{20}$$

From Equations (1) and (20) we may obtain the total number N_Σ of particles which should be ejected by long-period comets in unit time, and their total mass M_Σ:

$$\begin{aligned} N_\Sigma(n, M_1, M_2) &= \int_{0.01}^{5} q_0^n\, \mathrm{d}q_0 \int_{M_1}^{M_2} A_0\, \mathrm{d}M \\ &= \frac{15\pi^2 D_0 C(\delta)}{2F(1, K, n)}\, G(n) \left(\frac{1}{M_1^{4/3}} - \frac{1}{M_2^{4/3}}\right), \end{aligned} \tag{21}$$

$$\begin{aligned} M_\Sigma(n, M_1, M_2) &= \int_{0.01}^{5} q_0^n\, \mathrm{d}q_0 \int_{M_1}^{M_2} A_0 M\, \mathrm{d}M \\ &= \frac{30\pi^2 D_0 C(\delta)}{F(1, K, n)}\, G(n) \left(\frac{1}{M_1^{1/3}} - \frac{1}{M_2^{1/3}}\right), \end{aligned} \tag{22}$$

where

$$G(n) = \begin{cases} -\dfrac{1}{n+1}(5^{n+1} - 0.01^{n+1}) & n \neq -1 \\ 6.2 & n = -1. \end{cases} \tag{23}$$

Here M_1 and M_2 are the minimum and maximum particle masses.

Numerical integration of Equation (15) and the computation of the function $F(r, K, n)$ for different values of K and n were performed on a Minsk-22 computer. It was found that when $K > 1000$, $F(r, K, n)$ is practically independent of K. For the limiting values of $n = -1.0$ and -1.5 we have $F(1, \infty, -1.0) = 5.47$ and $F(1, \infty, -1.5) = 6.06$. With $M_1 = 10^{-13}$ g and $M_2 = 10^{13}$ g, $\delta = 3.5$ g cm^{-3}, Equations (22) and (23) give

$$\text{for } n = -1.0, \qquad M_\Sigma = 7 \times 10^{14} \text{ g yr}^{-1} \tag{24}$$

$$\text{for } n = -1.5, \qquad M_\Sigma = 2 \times 10^{15} \text{ g yr}^{-1}.$$

It follows that the mass of dust ejected by all the long-period comets in one year is comparable to that of a single comet nucleus. During the lifetime of the solar system (5×10^9 yr) the total mass ejected would have been some 3×10^{23} to 10^{24} g, this constituting approximately one ten-thousandth part of the mass of the Earth.

References

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Discussion

F. L. Whipple: For masses greater than 10^{-5} g I find that erosion by smaller particles and collisions with comparable particles are more important than the Poynting-Robertson effect in eliminating the particles. For the zodiacal cloud I find that 10^7 g s^{-1}, or 3×10^{14} g yr^{-1} must be supplied by short-period comets to maintain the cloud. The spherical cloud does not appear to be very significant.