A NEW CHARACTERIZATION OF FINITE PRIME FIELDS

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Let $N \equiv \langle N, +, .\rangle$ be a (right) near-ring with 1 (we say N is a unitary near-ring)[1] and recall that a near-field is a unitary near-ring in which $\langle N - \{0\}, .\rangle$ is a multiplicative group. In [2], Beidelman characterizes near-fields as those unitary near-rings without non-trivial N-subgroups. We show that in the finite case this absence of non-trivial N-subgroups is equivalent to the absence of non-trivial left ideals.

LEMMA. *A finite unitary near-ring* N *is a near-field* <=> N *has no non-trivial left ideals*.

Proof. If N has no non-trivial left ideals, then for each $a \in N$, $a \neq 0$, define a map $\rho_a : N \to Na$ by $\rho_a(x) = xa$. It is easily verified that ρ_a is an N-epimorphism and $\operatorname{Ker} \rho_a = (0)$. Hence $N = Na$ and consequently N is a near-field. The converse is clear.

We now use the lemma to obtain a new characterization of finite prime fields. (This was obtained independently by Clay and Malone in [3]).

THEOREM. $N = \langle N, +, .\rangle$ *is a finite prime field* <=> N *is a finite unitary near-ring and* $\langle N, +\rangle$ *is a simple group*.

Proof. If $\langle N, +\rangle$ is a simple group then N has no non-trivial left ideals and thus N is a finite near-field. Therefore $\langle N, +\rangle$ is an abelian p-group and consequently a cyclic group. However, this implies (see [3]) that N is a commutative ring.

[1] See [1] and [2] for basic definitions relative to near-rings.

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REFERENCES

1. J. C. Beidleman, A radical for near-ring modules. Mich. Math. J. 12 (1965) 377-383.

2. J. C. Beidleman, On near-rings and near-ring modules. (Doctoral dissertation, The Pennsylvania State University, 1964.)

3. J.R. Clay and J. J. Malone, Jr., The near-rings with identities on certain finite groups. Math. Scand. 19 (1966) 146-150.

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