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Evolution equations of the multi-planetary problem with variable masses

A.B. Kosherbayeva and M.Zh Minglibayev

Al-Farabi Kazakh National University, Almaty, Kazakhstan

Abstract. We investigated the influence of the variability of the masses of planets and the parent star on the dynamic evolution of n planetary systems, considering that the masses of bodies change isotropically with different rates. The methods of canonical perturbation theory, which developed on the basis of aperiodic motion over a quasi-conical cross section and methods of computer algebra were used. $4n$ evolutionary equations were obtained in analogues of Poincare elements. As an example, the evolutionary equations of the three-planet exosystem $K2-3$ were obtained explicitly, which is a system of 12 linear non-autonomous differential equations. Further, the evolutionary equations will be investigated numerically.

Keywords. celestial mechanics, variable mass, analogues of Poincare elements, multi-planetary system, secular perturbation.

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1. Introduction

To date, there are more than 5,000 confirmed exoplanets and more than 3,800 planetary systems in the NASA (2022) database. To research exoplanetary systems in the non-stationary stage of their evolution is represented important interest.

2. Problem statment

We considered the problem of $n+1$ bodies with variable masses $m_0 = m_0(t)$ – mass of the parent star S, $m_i = m_i(t)$, – the mass of the planet P_i. The laws of mass are known and given functions of time $m_0 = m_0(t)$, $m_1 = m_1(t), \ldots,$ $m_n = m_n(t)$, $(n \geq 3)$. The masses of spherical symmetric bodies change isotropically with different rates $\dot{m}_0/m_0 \neq \dot{m}_i/m_i$, $\dot{m}_i/m_i \neq \dot{m}_j/m_j$ $i, j = 1, 2, \ldots, n$, $i \neq j$.

Differential equations of motion of n bodies with isotropically varying masses in a relative coordinate system are given in Minglibayev (2012)

$$\ddot{\vec{r}}_i = -f\frac{(m_0 + m_i)}{r_i^3}\vec{r}_i + f\sum_{j=1}^{n}{}' m_j\left(\frac{\vec{r}_j - \vec{r}_i}{r_{ij}^3} - \frac{\vec{r}_j}{r_j^3}\right), \quad (i, j = 1, 2, \ldots, n), \tag{2.1}$$

where $r_{ij} = r_{ji} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}$ – mutual distances of the center of spherical bodies, f – gravitational constant, $\vec{r}_i(x_i, y_i, z_i)$ – the radius-vector of center of the planet P_i, the sign "stroke" when summing means that $i \neq j$.

For our purposes, analogues of the second system of canonical Poincare elements are preferred

$$\Lambda_i, \quad \lambda_i, \quad \xi_i, \quad \eta_i, \quad p_i, \quad q_i, \tag{2.2}$$

which are introduced on the basis of elements of aperiodic motion over a quasi- conical cross section Minglibaev (2012)

3. Evolutionary equations of n planets with variable masses

The evolutionary equations of n planets with variable masses in dimensionless variables (2.2)-(2.5) in the non-resonant case have the form in our work Prokopenya et al. (2022)

$$\xi_i' = \sum_{s=1}^{i-1} m_s \left(\frac{\Pi_{ii}^{is}}{\Lambda_i}\eta_i + \frac{\Pi_{is}^{is}}{\sqrt{\Lambda_i\Lambda_s}}\eta_s \right) + \sum_{k=i+1}^{n} m_k \left(\frac{\Pi_{kk}^{ik}}{\Lambda_i}\eta_i + \frac{\Pi_{ik}^{ik}}{\sqrt{\Lambda_i\Lambda_k}}\eta_k \right) - \frac{3\gamma_i''}{2\gamma_i}\frac{\Lambda_i^3}{\mu_{i0}^2}\eta_i, \quad (3.1)$$

$$\eta_i' = -\sum_{s=1}^{i-1} m_s \left(\frac{\Pi_{ii}^{is}}{\Lambda_i}\xi_i + \frac{\Pi_{is}^{is}}{\sqrt{\Lambda_i\Lambda_s}}\xi_s \right) - \sum_{k=i+1}^{n} m_k \left(\frac{\Pi_{kk}^{ik}}{\Lambda_i}\xi_i + \frac{\Pi_{ik}^{ik}}{\sqrt{\Lambda_i\Lambda_k}}\xi_k \right) + \frac{3\gamma_i''}{2\gamma_i}\frac{\Lambda_i^3}{\mu_{i0}^2}\xi_i, \quad (3.2)$$

$$p_i' = -\sum_{s=1}^{i-1} m_s B_1^{is} \left(\frac{q_i}{4\Lambda_i} - \frac{q_s}{4\sqrt{\Lambda_i\Lambda_s}} \right) - \sum_{k=i+1}^{n} m_k B_1^{ik} \left(\frac{q_i}{4\Lambda_i} - \frac{q_k}{4\sqrt{\Lambda_i\Lambda_k}} \right), \quad (3.3)$$

$$q_i' = \sum_{s=1}^{i-1} m_s B_1^{is} \left(\frac{p_i}{4\Lambda_i} - \frac{p_s}{4\sqrt{\Lambda_i\Lambda_s}} \right) + \sum_{k=i+1}^{n} m_k B_1^{ik} \left(\frac{p_i}{4\Lambda_i} - \frac{p_k}{4\sqrt{\Lambda_i\Lambda_k}} \right). \quad (3.4)$$

At the same time, the expressions Π_{ii}^{is}, Π_{is}^{is}, Π_{kk}^{ik}, Π_{ik}^{ik} in equations (3.1) -(3.4) and the Laplace coefficients retain their form, but they are already dimensionless quantities. All notations are given in the article Prokopenya et al. (2022).

4. The evolutionary equations of the three-planet exosystem $K2-3$ in explicit form

As an example, the case of $n=3$ was considered. The evolutionary equations (3.1)–(3.4) for the $K2-3$ exosystem are described by a system of 12 linear non-autonomous differential equations, which are obtained explicitly. The resulting system splits into two subsystems for eccentric and oblique elements. The resulting equations of secular perturbations are difficult, so they will be investigated numerically.

5. Conclusion

The evolutionary equations of a multi-planetary problem with isotropically varying masses at different rates in analogues of osculating Poincare elements were obtained. These evolutionary equations can be used for any n planetary problem with variable masses.The evolutionary equations for the $K2-3$ exosystem were written explicitly. The obtained evolutionary equations will be investigated numerically.

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