ALGEBRAS STABLY EQUIVALENT TO SELFINJECTIVE ALGEBRAS WHOSE AUSLANDER-REITEN QUIVERS CONSIST ONLY OF GENERALIZED STANDARD COMPONENTS

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0. Introduction. Throughout the paper K denotes a fixed algebraically closed field. All algebras considered are finite-dimensional associative K-algebras with a unit element. Moreover, they are assumed to be basic and connected. For an algebra A we denote by $\text{mod}(A)$ the category of all finitely generated right A-modules, and $\underline{\text{mod}}(A)$ denotes the stable category of $\text{mod}(A)$, i.e. $\text{mod}(A)/\mathcal{P}$ where $\mathcal{P}$ is the two-sided ideal in $\text{mod}(A)$ of all morphisms that factorize through projective A-modules. Two algebras A and B are said to be stably equivalent if the stable categories $\underline{\text{mod}}(A)$ and $\underline{\text{mod}}(B)$ are equivalent. The study of stable equivalences of algebras has its sources in modular representation theory of finite groups. It is of importance in this theory whether two stably equivalent algebras have the same number of pairwise non-isomorphic nonprojective simple modules. Another motivation for studying stable equivalences appears in the following context. If E is a K-algebra of finite global dimension then its derived category $\mathbf{D}^b(E)$ is equivalent to the stable category $\underline{\text{mod}}(\hat{E})$ of the repetitive category $\hat{E}$ of E [**15**]. Thus the problem of a classification of derived equivalent algebras leads in many cases to a classification of stably equivalent selfinjective algebras.

We are interested in algebras which are stably equivalent to representation-infinite selfinjective algebras whose Auslander–Reiten quivers consist only of generalized standard components in the sense of Skowroński [**30**]. It was announced in [**34**] that representation-infinite selfinjective algebras, whose Auslander–Reiten quivers consist only of generalized standard components, are standard algebras of polynomial growth (for the needed definitions see Section 1). Since there is given a classification of such algebras in [**32**] we can use it to describe algebras stably equivalent to representation-infinite selfinjective algebras whose Auslander-Reiten quivers consist only of generalized standard components. The following theorem is the main result of the paper.

THEOREM. *Let B be a selfinjective representation-infinite algebra in which all components of the Auslander–Reiten quiver Γ_B are generalized standard. If C is an algebra which is stably equivalent to B then C is a standard selfinjective algebra of polynomial growth. Moreover B and C have the same number of pairwise non-isomorphic simple modules.*

Recall that the algebras stably equivalent to representation-finite selfinjective algebras were classified by Riedtmann in [**24, 25, 26, 9**]. Algebras stably equivalent to tame trivial extensions were described in [**20, 23, 22**]. In both cases there was linked a tilting-cotilting equivalence of some factor algebras to any stable equivalence. In our case the situation is a little bit different. Any stable equivalence of B and C can be lifted to a stable equivalence of Galois coverings $\tilde{B}$, $\tilde{C}$ of B and C, respectively. Furthermore the

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lifted stable equivalence of $\tilde{B}$ and $\tilde{C}$ is induced by a tilting-cotilting equivalence of some subcategories in $\tilde{B}$ and $\tilde{C}$.

1. Preliminaries.

1.1. Let $K[X]$ be the polynomial algebra in one variable. Following Drozd [**12**] an algebra A is called *tame* if, for any dimension d, there is a finite number of $K[X]$-A-bimodules Q_i, $1 \le i \le n_d$, which are finitely generated and free as right $K[X]$-modules, and satisfy the following condition: all but a finite number of isomorphism classes of indecomposable A-modules of dimension d are of the form $K[X]/(X-\lambda)\otimes_{K[X]}Q_i$ for some $\lambda \in K$ and some i, $1 \le i \le n_d$.

Denote by $\mu_A(d)$ the least number of bimodules Q_i satisfying the above condition for d. Then A is said to be of *polynomial growth* [**31**] if there is a natural number m such that $\mu_A(d) \le d^m$.

1.2. Let R be a locally bounded K-category [**8**]. We denote by mod(R) the category of all finite-dimensional contravariant functors from R to the category of K-vector spaces. For a group G of K-linear automorphisms of R acting freely on the objects of R, R/G denotes the quotient category [**14**] whose objects are the G-orbits of the objects of R. There is a Galois covering functor $F: R \to R/G$ which assigns to each object x its G-orbit $G.x$. A locally bounded K-category R is called *simply connected* [**29**] if it is triangular (its quiver has no oriented cycles) and any Galois covering of R is trivial. A locally bounded K-category R is called *standard* if it admits a Galois covering $R' \to R$ with R' simply connected. To every algebra A we can attach the locally bounded K-category R_A whose objects are formed by a complete set E of pairwise orthogonal primitive idempotents of A, $R(e,f) = fAe$, $e, f \in E$, and the composition is induced by the multiplication in A. An algebra A is called standard if the attached locally bounded K-category R_A is standard.

1.3. For an algebra A we shall denote by Γ_A its Auslander–Reiten quiver [**5**], and by τ, τ^- the Auslander–Reiten translations DTr and TrD, respectively [**4**]. We shall not distinguish between an indecomposable module, its isomorphism class and the vertex of Γ_A corresponding to it. Moreover, we denote by Γ_A^s the stable quiver of Γ_A obtained from Γ_A by removing the τ-orbits of all indecomposable projective modules and the τ^--orbits of all indecomposable injective modules and the arrows attached to them.

1.4. A connected component $\mathscr{C}$ of the Auslander–Reiten quiver Γ_A of A is said to be *sincere* if for each indecomposable projective A-module P there is an A-module X whose isomorphism class is contained in $\mathscr{C}$ such that $\mathrm{Hom}_A(P, X) \neq 0$.

1.5. Recall from [**30**] that a component $\mathscr{C}$ of the Auslander–Reiten quiver Γ_A of an algebra A is called *generalized standard* if $\mathrm{rad}^\infty(X, Y) = 0$ for all modules X and Y from $\mathscr{C}$, where $\mathrm{rad}^\infty(\mathrm{mod}(A))$ is the intersection of all powers of the Jacobson radical rad(mod(A)) of mod(A).

1.6. Following [**13**] a component T of Γ_A (respectively, of Γ_A^s) is said to be a *tube* if T contains a cyclic path and its geometrical realization $|T|$ is homeomorphic to $S^1 \times \mathbb{R}_0^+$ where S^1 is a unit circle and $\mathbb{R}_0^+$ is the set of non-negative real numbers. For a tube T the set $S^1 \times \{0\}$ of $|T|$ called its *mouth.* A stable tube of rank $n \ge 1$ is a translation quiver of the form $\mathbb{Z}A_\infty/(\tau^n)$. The stable tubes of rank one are said to be *homogeneous.* A family $\mathscr{T} = (T_i)_{i\in I}$ of tubes in Γ_A (respectively, in Γ_A^s) is said to be *standard* if the full subcategory of mod(A) (respectively, $\underline{\mathrm{mod}}(A)$) formed by the objects of $\mathscr{T}$ is equivalent to the mesh-category $K(\mathscr{T})$ of $\mathscr{T}$ (see [**27**, Section 2]).

1.7. Recall from [**32**] that a *quasitube* T is a component in Γ_A such that its stable part T^S is a tube.

1.8. Let $\mathbf{n} = (n_1, n_2, \ldots, n_t)$ be a natural t-tuple. A family $\mathcal{T} = (T_\lambda)_{\lambda \in \mathbb{P}_1 K}$, $\mathbb{P}_1 K = K \cup \{\infty\}$, of quasitubes in Γ_A is said to be a *tubular* $\mathbb{P}_1 K$*-family of type* $\mathbf{n}$ if the following condition is satisfied: the stable part $\mathcal{T}^s$ of $\mathcal{T}$ is a disjoint union of stable tubes T^s_λ, $\lambda \in \mathbb{P}_1 K$, such that t of these tubes have ranks $n_1, n_2, \ldots, n_t$ and the remaining ones are homogeneous.

1.9. Two components $\mathcal{C}_1$, $\mathcal{C}_2$ in Γ_A are said to be orthogonal if for any $X \in \mathcal{C}_1$, $Y \in \mathcal{C}_2$, $\operatorname{Hom}_A(X, Y) = 0 = \operatorname{Hom}_A(Y, X)$.

1.10. A *cycle* in $\operatorname{mod}(A)$ is a sequence $M_0 \to M_1 \to \cdots \to M_n = M_0$ of nonzero non-isomorphisms between indecomposable A-modules. Following [**33**] A is said to be *cycle-finite* if for every cycle in $\operatorname{mod}(A)$ all of morphisms on this cycle do not belong to $\operatorname{rad}^\infty(\operatorname{mod}(A))$.

1.11. Following [**7, 17**] we shall say that a module Z in $\operatorname{mod}(A)$ is a *tilting* (respectively, *cotilting*) module if it satisfies the following conditions:

(1) $\operatorname{Ext}^2_A(Z, -) = 0$; (respectively, $\operatorname{Ext}^2_A(-, Z) = 0$);

(2) $\operatorname{Ext}^1_A(Z, Z) = 0$;

(3) the number of non-isomorphic indecomposable summands of Z equals the rank of the Grothendieck group $K_0(A)$ of A.

Two algebras A and F are said to be *tilting-cotilting equivalent* if there exist a sequence of algebras $A = A_0, A_1, \ldots, A_m, A_{m+1} = F$ and a sequence of modules Z^i_A, $0 \le i \le m$, such that $A_{i+1} = \operatorname{End}_A(Z^i)$ and Z^i is either a tilting or a cotilting module.

2. Partially directed K-categories.

2.1. Let Δ be a partially ordered set. A connected locally bounded K-category R whose Auslander–Reiten quiver consists only of generalized standard components is defined to be Δ*-directed* provided that there is a partition $\bigsqcup_{\delta \in \Delta} \mathcal{T}_\delta$ of the Auslander–Reiten quiver Γ_R onto the disjoint union of families $\mathcal{T}_\delta$ of pairwise orthogonal components such that the following conditions are satisfied:

(1) If $X \in \operatorname{add}(\mathcal{T}_{\delta_1})$, $Y \in \operatorname{add}(\mathcal{T}_{\delta_2})$ and $\operatorname{Hom}_R(X, Y) \neq 0$ then $\delta_1 \le \delta_2$.

(2) For every pair δ, $\sigma \in \Delta$ such that $\delta < \sigma$ and every pair of components $\mathcal{C} \in \mathcal{T}_\delta$, $\mathcal{D} \in \mathcal{T}_\sigma$ there exists a finite sequence $\{\delta_1, \delta_2, \ldots, \delta_r, \delta_{r+1} = \sigma\} \subset \Delta$ such that $\delta \le \delta_1 \le \cdots \le \delta_r \le \delta_{r+1}$, and there exists a sequence of modules $\{X_0, X_1, \ldots, X_r, X_{r+1}\}$ such that $X_0 \in \operatorname{add}(\mathcal{C})$, $X_i \in \operatorname{add}(\mathcal{T}_i)$, $i = 1, \ldots, r$, $X_{r+1} \in \operatorname{add}(\mathcal{D})$ and $\operatorname{Hom}_R(X_j, X_{j+1}) \neq 0$ for each $j = 0, 1, \ldots, r$.

(3) There are at most finitely many projective vertices in every $\mathcal{T}_\delta$, $\delta \in \Delta$.

A partition $\bigsqcup_{\delta \in \Delta} \mathcal{T}_\delta$ satisfying the above conditions is called Δ*-induced.*

2.2. A K-automorphism $f: R \to R$ of a Δ-directed locally bounded K-category R is said to be Δ*-induced* if there is an automorphism $f': \Delta \to \Delta$ of the partially ordered set Δ such that the induced by f equivalence $F_f: \operatorname{mod}(R) \to \operatorname{mod}(R)$ satisfies the following condition: if $X \in \operatorname{add}(\mathcal{T}_\delta)$, $\delta \in \Delta$, then $F_f(X) \in \operatorname{add}(\mathcal{T}_{f'(\delta)})$.

2.3. Lemma. *Let Δ be a linearly ordered set. If R is a Δ-directed locally bounded K-category then every K-automorphism $f: R \to R$ is Δ-induced.*

Proof. Assume that $f: R \to R$ is a K-automorphism of a Δ-directed locally bounded

K-category R. Then the induced equivalence $F_f:\mathrm{mod}(R)\to\mathrm{mod}(R)$ preserves indecomposability of modules and irreducibility of morphisms. Thus F_f maps connected components of Γ_R onto connected components. First we shall show that F_f yields a bijection between the set $\{\mathcal{T}_\delta\}_{\delta\in\Delta}$ of families of pairwise orthogonal components. In order to do this it is sufficient to show that if $\mathcal{C}$, $\mathcal{D}\in\mathcal{T}_\delta$, $\delta\in\Delta$, then $F_f(\mathcal{C})$, $F_f(\mathcal{D})\in\mathcal{T}_{\delta'}$. Suppose to the contrary that $F_f(\mathcal{C})\in\mathcal{T}_{\delta'}$ and $F_f(\mathcal{D})\in\mathcal{T}_{\delta''}$ with $\delta'\neq\delta''$. Without loss of generality we may assume that $\delta'<\delta''$, for Δ is linearly ordered. Consider a sequence $X_0, X_1,\dots,X_{r+1}$ of modules such that $X_0\in\mathrm{add}(F_f(\mathcal{C}))$, $X_i\in\mathrm{add}(\mathcal{T}_{\delta_i})$, $i=1,\dots,r$, $X_{r+1}\in\mathrm{add}(F_f(\mathcal{D}))$ with $\mathrm{Hom}_R(X_j,X_{j+1})\neq 0$, $j=0,1,\dots,r$ which exists by 2.1(2). Clearly $\delta'<\delta_1\leq\dots\leq\delta''$ by 2.1(1). Applying $F_{f^{-1}}$ we get that there is the sequence $F_{f^{-1}}(X_0)$, $F_{f^{-1}}(X_1),\dots,F_{f^{-1}}(X_r)$, $F_{f^{-1}}(X_{r+1})$ of modules such that $\mathrm{Hom}_R(F_{f^{-1}}(X_j),\ F_{f^{-1}}(X_{j+1}))\neq 0$, $j=0,1,\dots,r$, and $F_{f^{-1}}(X_0)\in\mathcal{C}$, $F_{f^{-1}}(X_{r+1})\in\mathcal{D}$. Then each indecomposable direct summand in $F_{f^{-1}}(X_j)\in\mathcal{T}_{\delta_{j,l}}$ with $\delta_{j,l}\geq\delta$ by 2.1(1). Since $F_{f^{-1}}(X_0)$, $F_{f^{-1}}(X_{r+1})\in\mathcal{T}_\delta$ hence all $\delta_{j,l}=\delta$ which contradicts the orthogonality of all components in $\mathcal{T}_\delta$. Consequently, if $\mathcal{C}$, $\mathcal{D}\in\mathcal{T}_\delta$ then $F_f(\mathcal{C})$, $F_f(\mathcal{D})\in\mathcal{T}_{\delta'}$. Hence F_f induces a bijection $f':\Delta\to\Delta$ given by the condition: $f'(\delta)=\sigma$ iff there is an R-module $X\in\mathrm{add}(\mathcal{T}_\delta)$ such that $F_f(X)\in\mathrm{add}(\mathcal{T}_\sigma)$. In order to finish the proof we should show that $f':\Delta\to\Delta$ is an automorphism of the partially ordered set Δ. Suppose that $\delta_1\leq\delta_2$ in Δ. Then for some $X\in\mathrm{add}(\mathcal{T}_{\delta_1})$ and some $Y\in\mathrm{add}(\mathcal{T}_{\delta_2})$ we have $F_f(X)\in\mathrm{add}(\mathcal{T}_{f'(\delta_1)})$ and $F_f(Y)\in\mathrm{add}(\mathcal{T}_{f'(\delta_2)})$. But R is Δ-directed, hence there are: a sequence $\{\sigma_1,\dots,\sigma_r,\sigma_{r+1}=\delta_2\}\subset\Delta$ with $\delta_1\leq\sigma_1$ and a sequence of R-modules $\{X_0,X_1,\dots,X_r,X_{r+1}\}$ such that $X_0\in\mathrm{add}(\mathcal{T}_{\delta_1})$, $X_i\in\mathrm{add}(\mathcal{T}_{\sigma_i})$, $i=1,\dots,r$, $X_{r+1}\in\mathrm{add}(\mathcal{T}_{\delta_2})$ with $\mathrm{Hom}_R(X_j,X_{j+1})\neq 0$, $j=0,1,\dots,r$, by 2.1(2). Since F_f is an equivalence we obtain by 2.1(1) the following sequence of inequalities: $f'(\delta_1)\leq f'(\sigma_1)\leq\dots\leq f'(\sigma_r)\leq f'(\delta_2)$. Consequently $f'(\delta_1)\leq f'(\delta_2)$ and f' is an automorphism of Δ. Thus f is Δ-induced. □

2.4. Following [**32**] a group G of K-linear automorphisms of a locally bounded K-category is said to be *admissible* if its action on the objects is free and has finitely many orbits.

2.5. Corollary. *Let Δ be a linearly ordered set. If R is a Δ-directed locally bounded K-category and G is an admissible infinite cyclic group of K-linear automorphisms of R then G consists of Δ-induced K-automorphisms.*

Proof. Clear by Lemma 2.3.

2.6. Lemma. *Let Δ be an infinite linearly ordered set. If R is a Δ-directed locally bounded K-category with infinitely many objects and G an admissible infinite cyclic group of K-linear automorphisms of R then G is a group of automorphisms of Δ.*

Proof. Under the assumptions of our lemma there is a map $(-)':\mathrm{Aut}_K(R)\to\mathrm{Aut}(\Delta)$ by Lemma 2.3, where $f':\Delta\to\Delta$ is an automorphism of Δ such that, for $f\in\mathrm{Aut}_K(R)$ and for every $X\in\mathrm{add}(\mathcal{T}_\delta)$, $F_f(X)\in\mathrm{add}(\mathcal{T}_{f'(\delta)})$. Observe that the restriction $(-)'_G$ of $(-)'$ to a subgroup G in $\mathrm{Aut}_K(R)$ is a group homomorphism $(-)'_G:G\to\mathrm{Aut}(\Delta)$. Indeed, $(\mathrm{id}_R)'=\mathrm{id}_\Delta$ by the definition of $(-)'$. If $g,f\in G$ then the induced equivalence $F_{gf}:\mathrm{mod}(R)\to\mathrm{mod}(R)$ is of the form $F_{gf}=F_gF_f$ and $(gf)'=g'f'$. Finally $(f^{-1})'=(f')^{-1}$ is clear by the definition of $(-)'$. Consequently $(-)'_G:G\to\mathrm{Aut}(\Delta)$ is a group homomorphism for every subgroup G

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of $\mathrm{Aut}_K(R)$. In order to finish the proof we should show that $(-)'_G$ is a monomorphism when G is an admissible infinite cyclic group. In this case, if $f' = \mathrm{id}_\Delta$ and $f = g^z$ for some $z \in \mathbb{Z}$, where g is a generator of G, then $(g')^z = \mathrm{id}_\Delta$ since $(-)'_G$ is a homomorphism. But the induced (by g^z) equivalence $F_{g^z} = F_g \dots F_g$ maps every family $\mathcal{T}_\delta$, $\delta \in \Delta$, onto itself. Thus F_{g^z} can map projective vertices from a family $\mathcal{T}_\delta$, for a fixed $\delta \in \Delta$, only onto projective vertices in the same family. Since R is Δ-directed there are at most finitely many projective vertices in every family $\mathcal{T}_\sigma$, $\sigma \in \Delta$, by 2.1(3). If z is the minimal natural number such that $(g')^z = \mathrm{id}_\Delta$ then $(g')^z = g' \dots g'$ and $g' \neq \mathrm{id}_\Delta$ or $z = 1$. If $z \neq 1$ then $g'(\delta_0) < \delta_0$ or $g'(\delta_0) > \delta_0$ for some $\delta_0 \in \Delta$, because Δ is linearly ordered. In the first case we obtain that $(g')^z(\delta_0) < (g')^{z-1}(\delta_0) < \dots < g'(\delta_0) < \delta_0$ which contradicts the fact that $(g')^z = \mathrm{id}_\Delta$. In the second case one gets a similar contradiction. Consequently $z = 1$. Then G is not admissible, because it has infinitely many orbits of the objects of R. This proves the lemma. □

2.7. Now we assume that there is given an infinite linearly ordered set Δ and an infinite cyclic group G acting on Δ nontrivially and nontransitively. Fix an element $\delta_0 \in \Delta$ and consider the set $\Delta \setminus G\,.\,\delta_0$ with the induced order by that in Δ. Thus we have:

2.8. Lemma. *$\Delta \setminus G\,.\,\delta_0$ is a disjoint union $\bigsqcup_{z \in Z} \Delta_z$ of linearly ordered sets Δ_z, $z \in \mathbb{Z}$, such that the following conditions are satisfied:*

(1) *If $z_1 \leq z_2$ and $\delta_1 \in \Delta_{z_1}$, $\delta_2 \in \Delta_{z_2}$ then $\delta_1 \leq \delta_2$.*

(2) *For any two $z_1, z_2 \in \mathbb{Z}$ there is an isomorphism $\Delta_{z_1} \cong \Delta_{z_2}$ of partially ordered sets.*

(3) *There is a linear order on the set of the G-orbits $\Delta' = (\Delta \setminus G\,.\,\delta_0)/G$ of $\Delta \setminus G\,.\,\delta_0$ such that $\Delta' \cong \Delta_0$.*

Proof. Under the assumptions and the notations of 2.7 consider a generator f of G. Then either $\delta_0 < f(\delta_0)$ or $f(\delta_0) < \delta_0$ since $f \neq \mathrm{id}_\Delta$ and Δ is linearly ordered. We shall consider only the case $\delta_0 < f(\delta_0)$, because the other one is similar. In the case we obtain inductively that $f^z(\delta_0) < f^{z+1}(\delta_0)$ for every integer z. We put $\Delta_z = \{\delta \in \Delta : f^z(\delta_0) < \delta < f^{z+1}(\delta_0)\}$. Since G acts on Δ nontrivially and nontransitively hence $\Delta_z \neq \emptyset$ for any integer z. Since $\Delta_z \subset \Delta$, $z \in \mathbb{Z}$, consider in Δ_z the order of Δ. Thus Δ_z is linearly ordered set obviously and $\Delta \setminus G\,.\,\delta_0 = \bigsqcup_{z \in \mathbb{Z}} \Delta_z$ which proves (1).

In order to prove (2) observe that for any two integers z_1, z_2 such that $z_1 \leq z_2$ the restriction of $f^{z_2 - z_1}$ to Δ_{z_1} yields an isomorphism of Δ_{z_1} and Δ_{z_2}.

Now consider the set Δ' of the G-orbits of $\Delta \setminus G\,.\,\delta_0$. Observe that there is a bijection $h : \Delta_0 \to \Delta'$ given by the formula $h(\delta) = G\,.\,\delta$. This bijection induces a linear order on Δ' such that $\Delta' \cong \Delta_0$ as partially ordered sets.

2.9. Let R be a Δ-directed locally bounded K-category, where Δ is a given partially ordered set. Let $\Gamma_R = \bigsqcup_{\delta \in \Delta} \mathcal{T}_\delta$ be a Δ-induced partition of Γ_R. Then a family $\mathcal{T}_{\delta_0}$, for some $\delta_0 \in \Delta$, is said to be *separating* [**28**] if for any $\delta_1, \delta_2 \in \Delta$ such that $\delta_1 \leq \delta_0 \leq \delta_2$ and any nonzero morphism $f : X \to Y$ such that $X \in \mathrm{add}(\mathcal{T}_{\delta_1})$, $Y \in \mathrm{add}(\mathcal{T}_{\delta_2})$ there are $Z \in \mathrm{add}(\mathcal{T}_{\delta_0})$, $f_1 : X \to Z$, $f_2 : Z \to Y$ with $f = f_2 f_1$. Similarly, a component $\mathcal{C}$ in $\mathcal{T}_{\delta_0}$ is called *separating* if the above $Z \in \mathrm{add}(\mathcal{C})$. Clearly, if all components of $\mathcal{T}_{\delta_0}$ are separating then $\mathcal{T}_{\delta_0}$ is separating. The converse implication is not true in general.

2.10. Lemma. *Let Δ be an infinite linearly ordered set. Let R be a Δ-directed locally bounded K-category that has a separating family $\mathcal{T}_{\delta_0}$. If G is an admissible infinite cyclic group of K-linear automorphisms of R then for any $g \in G$, $\mathcal{T}_{g'(\delta_0)}$ is a separating family.*

Proof. Under the assumptions and the notations of our lemma consider a Δ-induced partition $\Gamma_R = \bigsqcup_{\delta \in \Delta} \mathcal{T}_\delta$ and an element $g \in G$. We deduce from Lemma 2.3 that g is Δ-induced. Then there is an automorphism $g' : \Delta \to \Delta$ of the partially ordered set Δ such that the induced by g equivalence $F_g : \mathrm{mod}(R) \to \mathrm{mod}(R)$ satisfies the following implication: if $X \in \mathrm{add}(\mathcal{T}_\delta)$ then $F_g(X) \in \mathrm{add}(\mathcal{T}_{g'(\delta)})$, $\delta \in \Delta$. Now suppose that $X \in \mathrm{add}(\mathcal{T}_{\delta_1})$, $Y \in \mathrm{add}(\mathcal{T}_{\delta_2})$ and $\delta_1 \leq g'(\delta_0) \leq \delta_2$. Let $f : X \to Y$ be any nonzero morphism. Then $(g')^{-1}(\delta_1) \leq \delta_0 \leq (g')^{-1}(\delta_2)$ and $F_{g^{-1}}(X) \in \mathrm{add}(\mathcal{T}_{(g')^{-1}(\delta_1)})$, $F_{g^{-1}}(Y) \in \mathrm{add}(\mathcal{T}_{(g')^{-1}(\delta_2)})$, $F_{g^{-1}}(f) : F_{g^{-1}}(X) \to F_{g^{-1}}(Y)$. Since $\mathcal{T}_{\delta_0}$ is separating, there are $Z \in \mathrm{add}(\mathcal{T}_{\delta_0})$, $f_1 : F_{g^{-1}}(X) \to Z$, $f_2 : Z \to F_{g^{-1}}(Y)$ such that $F_{g^{-1}}(f) = f_2 f_1$ by 2.8. Therefore $F_g(Z) \in \mathrm{add}(\mathcal{T}_{g'(\delta_0)})$ and $f = F_g(f_2) F_g(f_1)$. Consequently, $\mathcal{T}_{g'(\delta_0)}$ is a separating family of components, and our proof is finished. □

3. Selfinjective standard algebras of polynomial growth.

3.1. The *repetitive category* (see [**18**]) of a locally bounded category R is the selfinjective locally bounded category $\hat{R}$ whose objects are pairs $(n, x) = x_n$, $x \in R$, $n \in \mathbb{Z}$, and $\hat{R}(x_n, y_n) = \{n\} \times R(x, y)$, $\hat{R}(x_{n+1}, y_n) = \{n\} \times DR(y, x)$, and $\hat{R}(x_p, y_q) = 0$ if $p \neq q$, $q + 1$, where DV denotes the dual space $\mathrm{Hom}_K(V, K)$.

3.2. A Euclidean algebra is a representation-infinite tilted algebra of Euclidean type having a complete slice in the preprojective component [**28**]. We shall use also tubular algebras in the sense of Ringel. For the basic definitions and results concerning Euclidean and tubular algebras we refer the reader to [**28**]. Recall that the extension type $\mathbf{n}_A$ of a Euclidean algebra A is one of the following (p, q), $1 \leq p \leq q$, $(2, 2, m)$, $m \geq 2$, $(2, 3, 3)$, $(2, 3, 4)$ or $(2, 3, 5)$. The extension type $\mathbf{n}_A$ of a tubular algebra A is one of the following $(3, 3, 3)$, $(2, 4, 4,)$, $(2, 3, 6)$ or $(2, 2, 2, 2)$. We shall call them, briefly, *types*.

3.3. The following theorem is the main result of [**32**].

THEOREM. *Let B be a standard, selfinjective K-algebra. Then B is representation-infinite of polynomial growth if and only if B is isomorphic to an algebra $\hat{A}/G$, where A is either a Euclidean or a tubular algebra and G is an admissible infinite cyclic group of K-linear automorphisms of $\hat{A}$.*

3.4. LEMMA. *Let A be a Euclidean algebra. Then $\hat{A}$ is a $\mathbb{Z}$-directed locally bounded K-category such that the $\mathbb{Z}$-induced partition $\Gamma_{\hat{A}} = \bigsqcup_{z \in \mathbb{Z}} \mathcal{T}_z$ satisfies the following conditions:*

(1) *For any even z, $\mathcal{T}_z$ consists of one connected component which contains at least one projective vertex, and whose stable Auslander–Reiten quiver is isomorphic to $\mathbb{Z}Q_A$, where Q_A is the ordinary quiver of A. Moreover, $\mathcal{T}_z$ contains only finitely many projective vertices.*

(2) *For any odd z, $\mathcal{T}_z$ is a tubular $\mathbb{P}_1K$-family of quasitubes, whose stable Auslander–Reiten quiver is a tubular $\mathbb{P}_1K$-family of type $\mathbf{n}_A$.*

(3) *Every family $\mathcal{T}_z$ is a separating family of components.*

Proof. See [**32,** 2.1].

3.5. LEMMA. *Let A be a tubular algebra. Then $\hat{A}$ is a $\mathbb{Q}$-directed locally bounded K-category such that the $\mathbb{Q}$-induced partition $\Gamma_{\hat{A}} = \bigsqcup_{q \in \mathbb{Q}} \mathcal{T}_q$ satisfies the following conditions:*

(1) *For every $q \in \mathbb{Q} \setminus \mathbb{Z}$, $\mathcal{T}_q$ is a tubular $\mathbb{P}_1K$-family of type $\mathbf{n}_A$ of stable tubes.*

(2) *For any* $q \in \mathbb{Z}$, $\mathcal{T}_q$ *is a tubular* $\mathbb{P}_1K$*-family of quasitubes which contains at least one and at most finitely many projective vertices. Moreover, the stable Auslander–Reiten quiver of* $\mathcal{T}_q$ *is a tubular* $\mathbb{P}_1K$*-family of type* $\mathbf{n}_A$.

(3) *Every family* $\mathcal{T}_q$ *is a separating family of components. Moreover, for* $q \in (\mathbb{Q}\setminus\mathbb{Z})$ *every component in* $\mathcal{T}_q$ *is a separating component.*

Proof. See [**32**, 3.1] or [**19**, Section 3] or else [**16**].

3.6. Lemma. *Let* A *be a Euclidean algebra. If* P, P' *are indecomposable projective* $\hat{A}$*-modules and* $P \in \mathcal{T}_z$, $P' \in \mathcal{T}_{z'}$, $\mathrm{Hom}_{\hat{A}}(P, P') \neq 0$, *then* $z \leq z' \leq z+4$.

Proof. The lemma is a consequence of [**1**, Proposition 2.5].

3.7. Lemma. *Let* A *be a tubular algebra. If* P, P' *are indecomposable projective* $\hat{A}$*-modules and* $P \in \mathcal{T}_q$, $P' \in \mathcal{T}_{q'}$, $\mathrm{Hom}_{\hat{A}}(P, P') \neq 0$ *then* $q \leq q' \leq q+3$.

Proof. The lemma is a consequence of [**1**, Proposition 2.5].

4. The structure of the Auslander–Reiten quivers of some standard algebras.

4.1. Let R be a locally bounded K-category and G an admissible group of K-linear automorphisms of R. Then there is a covering functor $F: R \to R/G$ induced by the action of G on R [**14**] which attaches to every object of R its G-orbit. Then F_λ denotes the induced push-down functor $F_\lambda: \mathrm{mod}(R) \to \mathrm{mod}(R/G)$ [**8, 14**]. If G is torsion-free then F_λ preserves indecomposables, Auslander–Reiten sequences, and maps projective R-modules onto projective R/G-modules, injective R-modules onto injective R/G-modules.

4.2. A locally bounded K-category R is said to be *locally support-finite* [**10, 11**] if for every indecomposable projective R-module P, the set of isomorphism classes of indecomposable projective R-modules P' such that there exists an indecomposable finite-dimensional R-module M with $\mathrm{Hom}_R(P, M) \neq 0 \neq \mathrm{Hom}_R(P', M)$ is finite. If R is locally support-finite K-category then F_λ is dense and induces a bijection between the set $(\mathrm{ind}(R)/\cong)/G$ of the G-orbits of the isomorphism classes of finite-dimensional indecomposable R-modules and the set $\mathrm{ind}(R/G)/\cong$ of the isomorphism classes of indecomposable finite-dimensional R/G-modules [**10**].

4.3. Proposition. *Let* Δ *be an infinite linearly ordered set. If* R *is a* Δ*-directed locally support-finite* K*-category which has a separating family* $\mathcal{T}_{\delta_0}$ *and* G *is an admissible infinite cyclic group of* K*-linear automorphisms of* R *then there are a family* $\mathcal{T}$ *of components in* $\Gamma_{R/G}$, *a linearly ordered set* $\mathbb{M} \neq \emptyset$ *and a partition* $\Gamma_{R/G}\setminus\mathcal{T} = \bigsqcup_{\mu\in\mathbb{M}} \mathcal{T}_\mu$ *onto a disjoint union of families of components such that the following conditions are satisfied:*

(1) *For every two different components* $\mathscr{C}$, $\mathscr{D} \in \mathcal{T}_\mu$, $\mu \in \mathbb{M}$, *if* $X \in \mathscr{C}$, $Y \in \mathscr{D}$ *and* $f: X \to Y$ *is a nonzero morphism then there are* $Z \in \mathrm{add}(\mathcal{T})$, $f_1: X \to Z$, $f_2: Z \to Y$ *such that* $f = f_2 f_1$.

(2) *If* $\mu_1 < \mu_2$ *in* $\mathbb{M}$, $Y \in \mathrm{add}(\mathcal{T}_{\mu_1})$, $X \in \mathrm{add}(\mathcal{T}_{\mu_2})$ *and* $f: X \to Y$ *is a nonzero morphism then there are* $Z \in \mathrm{add}(\mathcal{T})$, $f_1: X \to Z$, $f_2: Z \to Y$ *such that* $f = f_2 f_1$.

Moreover, if the induced action of G *on* Δ *is nontransitive and there is a separating family* $\mathcal{T}_{\delta_0'}$ *of components such that* $\delta_0 < \delta_0' < g(\delta_0)$ *for the generator* g *of* G *then*

(3) *For every two different components* $\mathscr{C}$, $\mathscr{D} \in \mathscr{T}$, *if* $X \in \mathscr{C}$, $Y \in \mathscr{D}$ *and* $f: X \to Y$ *is a nonzero morphism then there are* $Z \in \operatorname{add}(\Gamma_{R/G} \setminus \mathscr{T})$, $f_1: X \to Z$, $f_2: Z \to Y$ *such that* $f = f_2 f_1$.

Proof. Let R be a Δ-directed locally support-finite K-category, where Δ is an infinite linearly ordered set. Assume that G is an admissible infinite cyclic group of K-linear automorphisms of R. Let $\Gamma_R = \bigsqcup_{\delta \in \Delta} \mathscr{T}_\delta$ be a Δ-induced partition of Γ_R, and $\mathscr{T}_{\delta_0}$ a separating family of components, $\delta_0 \in \Delta$. Denote by C the quotient category R/G. Then $F: R \to C$ denotes the covering functor induced by the action of G on R. $F_\lambda: \operatorname{mod}(R) \to \operatorname{mod}(C)$ is the induced push-down functor which is dense by 4.2, because R is locally support-finite. Put $\mathscr{T} = F_\lambda(T_{\delta_0})$. We deduce from Lemma 2.8 that there is a linearly ordered set $\mathbb{M} = \Delta'$ such that $\Gamma_C \setminus \mathscr{T}$ has a partition $\bigsqcup_{\mu \in \mathbb{M}} \mathscr{T}_\mu$ onto a disjoint union of families of components, where $\mathscr{T}_\mu = F_\lambda(\mathscr{T}_\delta)$, $\delta \in \Delta$ is such an element that $G.\delta = \mu$. Thus for any two different components $\mathscr{C}$, $\mathscr{D}$ from $F_\lambda(\mathscr{T}_\delta)$ if $X \in \mathscr{C}$, $Y \in \mathscr{D}$ and $f: X \to Y$ is a nonzero morphism then there are components $\mathscr{C}' \in \mathscr{T}_{\delta_1}$ and $\mathscr{D}' \in \mathscr{T}_{\delta_2}$ with $\delta_1, \delta_2 \in G.\delta$ and there are R-modules $X' \in C'$, $Y' \in \mathscr{D}'$ with a nonzero morphism $f': X' \to Y'$ such that $F_\lambda(X') = X$, $F_\lambda(Y') = Y$ and $F_\lambda(f') = f$, for F_λ is dense. But R is Δ-directed and $\{\mathscr{T}_\delta\}_{\delta \in \Delta}$ are the families of the Δ-induced partition of Γ_R hence $\delta_1 < \delta_2$ by 2.1. Since δ_1, $\delta_2 \in G.\delta$ hence there is $\delta_0' \in G.\delta_0$ such that $\delta_1 < \delta_0' < \delta_2$. Since $\mathscr{T}_{\delta_0}$ is a separating family hence we deduce from Lemma 2.10 that $\mathscr{T}_{\delta_0'}$ is a separating family. Then there are $Z' \in \operatorname{add}(\mathscr{T}_{\delta_0'})$, $f_1': X' \to Z'$, $f_2': Z' \to Y'$ such that $f' = f_2' f_1'$ by 2.9. Clearly $F_\lambda(\mathscr{T}_{\delta_0'}) = \mathscr{T}$. Thus $F_\lambda(Z') = Z \in \operatorname{add}(\mathscr{T})$ and $f = f_2 f_1$, where $F_\lambda(f_1') = f_1: X \to Z$ and $F_\lambda(f_2') = f_2: Z \to Y$. Consequently condition (1) is proved.

Now assume that $\mu_1 < \mu_2$ in $\mathbb{M}$, $Y \in \operatorname{add}(\mathscr{T}_{\mu_1})$, $X \in \operatorname{add}(\mathscr{T}_{\mu_2})$ and $f: X \to Y$ is a nonzero morphism. Then there are δ_1, $\delta_2 \in \Delta$ such that $G.\delta_1 = \mu_1$, $G.\delta_2 = \mu_2$. Moreover, there are $X' \in \operatorname{add}(\mathscr{T}_{\delta_2})$, $Y' \in \operatorname{add}(\mathscr{T}_{\delta_1})$ and $f': X' \to Y'$ such that $F_\lambda(X') = X$, $F_\lambda(Y') = Y$ and $F_\lambda(f') = f$. Since R is Δ-directed and Δ is linearly ordered hence $\delta_2 < \delta_1$, because $\mu_1 < \mu_2$. If there is no $\delta_0' \in G.\delta_0$ such that $\delta_2 < \delta_0' < \delta_1$ then $\mu_2 < \mu_1$ by the definition of the order in Δ' (see the proof of Lemma 2.8), which contradicts the assumption that $\mu_1 < \mu_2$. Thus there is $\delta_0' \in G.\delta_0$ such that $\delta_2 < \delta_0' < \delta_1$. Therefore there are $Z' \in \operatorname{add}(\mathscr{T}_{\delta_0'})$, $f_1': X' \to Z'$, $f_2': Z' \to Y'$ such that $f' = f_2' f_1'$ for $\mathscr{T}_{\delta_0'}$ is separating by Lemma 2.10. Then there are $Z = F_\lambda(Z') \in \mathscr{T}$, $F_\lambda(f_1') = f_1: X \to Z$, $F_\lambda(f_2') = f_2: Z \to Y$ such that $f = f_2 f_1$ which proves condition (2).

The proof of (3) is similar to that of (1), since we can use conditions (1), (2) for the family $F_\lambda(\mathscr{T}_{\delta_0'})$. We leave the details to the reader. □

4.4. If G is an admissible infinite cyclic group of K-linear automorphisms of a locally support-finite K-category R then the quotient category R/G is a finite-dimensional K-algebra. For a finite-dimensional K-algebra H a family $\mathscr{T}$ of components in the Auslander–Reiten quiver Γ_H is defined to be *weakly separating* if there is a non-empty linearly ordered set $\mathbb{M}$ and a partition $\Gamma_H \setminus \mathscr{T} = \bigsqcup_{\mu \in \mathbb{M}} \mathscr{T}_\mu$ of $\Gamma_H \setminus \mathscr{T}$ onto a disjoint union of families $\mathscr{T}_\mu$ of components such that the conditions (1)–(3) of Proposition 4.3 are satisfied. The set $\mathbb{M}$ will be called $\mathscr{T}$*-induced.*

4.5. THEOREM. *If B is a standard selfinjective representation-infinite K-algebra of polynomial growth then there is a weakly separating family $\mathscr{T}$ of components in Γ_B with a $\mathscr{T}$-induced set $\mathbb{M}$ of one of the following forms:*

$\Delta_{2i+1} = \{1, 2, \ldots, 2i+1\}$, $i = 0, 1, 2, \ldots$, *with the order as in* $\mathbb{N}$,

$\Delta_{(0,i)} = (0, i) \cap \mathbb{Q}$, $i = 1, 2, \ldots$, *with the order as in* $\mathbb{Q}$.

Moreover, every family $\mathcal{T}_\mu$ *of the partition* $\Gamma_B \setminus \mathcal{T} = \bigsqcup_{\mu \in \mathbb{M}} \mathcal{T}_\mu$ *is also a weakly separating family of components in* Γ_B *with a* $\mathcal{T}_\mu$*-induced set which is isomorphic to* $\mathbb{M}$.

Proof. Let B be a standard selfinjective representation-infinite K-algebra of polynomial growth. Then $B \cong \hat{A}/G$, where A is either a Euclidean or a tubular K-algebra and G is an admissible infinite cyclic group of K-linear automorphisms of $\hat{A}$ by Theorem 3.3. If A is a Euclidean K-algebra then $\hat{A}$ is a $\mathbb{Z}$-directed locally support-finite K-category which has a separating family $\mathcal{T}_z$, $z \in \mathbb{Z}$, by Lemma 3.4. Thus B has a weakly separating family $\mathcal{T}$ of components in Γ_B by Proposition 4.3. Moreover, the $\mathcal{T}$-induced set $\mathbb{M}$ is of the form $\Delta_{2i+1} = \{1, 2, \ldots, 2i+1\}$. $i = 0, 1, 2, \ldots$ by its construction in the proof of Proposition 4.3 and by Lemma 3.4, 2.8, 2.10. If A is a tubular K-algebra then $\hat{A}$ is a $\mathbb{Q}$-directed locally support-finite K-category which has a separating family $\mathcal{T}_q$, $q \in \mathbb{Q}$, by Lemma 3.5. Therefore B has a weakly separating family $\mathcal{T}$ of components in Γ_B by Proposition 4.3. Moreover, the $\mathcal{T}$-induced set $\mathbb{M}$ is of the form $\Delta_{(0,i)} = (0, i) \cap \mathbb{Q}$, $i = 1, 2, 3, \ldots$, by its construction in the proof of Proposition 4.3 and by Lemma 3.5, 2.8, 2.10. The last statement of the theorem is clear. $\square$

5. Selfinjective algebras with Auslander–Reiten quivers all of whose components are generalized standard.

5.1. Throughout this section we shall assume that B is a standard selfinjective representation-infinite K-algebra of polynomial growth. Sometimes we shall use in the notations of Theorem 4.5 the convention that the family $\mathcal{T}$ has the index either $2i+2$ if $\mathbb{M} = \Delta_{2i+1}$ or i if $\mathbb{M} = \Delta_{(0,i)}$.

5.2. Let $\operatorname{ind}(B)$ denotes the full subcategory in $\operatorname{mod}(B)$ formed by the indecomposable B-modules. For a nonzero non-isomorphism $f: X \to Y$ we define its $\mathbb{M}$*-length* $l_{\mathbb{M}}(f)$ as follows: if $f = F_\lambda(\hat{f}): F_\lambda(\hat{X}) \to F_\lambda(\hat{Y})$, where $F_\lambda: \operatorname{mod}(\hat{A}) \to \operatorname{mod}(B)$ is the push-down functor, then $\hat{X} \in \mathcal{T}_{\delta_1}$, $\hat{Y} \in \mathcal{T}_{\delta_2}$ and we put $l_{\mathbb{M}}(f) = \delta_2 - \delta_1$. Furthermore, for a nonzero non-isomorphism $h: X \to Y$ in $\operatorname{mod}(B)$ we have a decomposition $h: X_1 \oplus \ldots \oplus X_n \to Y_1 \oplus \ldots \oplus Y_m$, where X_l, $l = 1, \ldots, n$, Y_j, $j = 1, \ldots, m$, are indecomposable and $h = (h_{lj})$ with $h_{lj}: X_l \to Y_j$. Then we define the $\mathbb{M}$-length of h as $l_{\mathbb{M}}(h) = \max_{l,j}\{l_{\mathbb{M}}(h_{lj})\}$. Moreover for any isomorphism f we put $l_{\mathbb{M}}(f) = 0$. It is easy to verify that the above definition does not depend on the choice of pullings-up $\hat{X}$, $\hat{Y}$, and so $\mathbb{M}$-length of a morphism is well-defined. For the algebra B we define its $\mathbb{M}$*-spread* $\operatorname{spr}_{\mathbb{M}}(B)$ as $\sup_{0 \neq f}\{l_{\mathbb{M}}(f)\}$.

5.3. Lemma. *If* $0 \neq f: X \to Y$ *is a morphism in* $\operatorname{mod}(B)$ *then there is a morphism* $h: P \to P'$ *between indecomposable projective* B*-modules* P, P' *such that* $l_{\mathbb{M}}(f) \leq l_{\mathbb{M}}(h)$.

Proof. In order to prove the lemma consider the composed morphism $h_1 = w_Y f p_X$, where $p_X: P(X) \to X$ is a projective covering morphism and $w_Y: Y \to I(Y)$ is an embedding of Y into its injective hull. Then $l_{\mathbb{M}}(h_1) \geq l_{\mathbb{M}}(f)$. If we decompose $h_1: P(X) \to I(Y)$ then there are an indecomposable direct summand P in $P(X)$, an indecomposable direct summand P' in $I(Y)$ and a nonzero morphism $h: P \to P'$ such that $l_{\mathbb{M}}(h) = l_{\mathbb{M}}(h_1) \geq l_M(f)$ by 5.2. $\square$

5.4. If G is an admissible infinite cyclic group of K-linear automorphisms of $\hat{A}$ and g is a generator of G then g is a Δ-induced automorphism by Corollary 2.5, where $\Delta = \mathbb{Q}$ or

$\Delta = \mathbb{Z}$. Thus there is an automorphism $g' : \Delta \to \Delta$ associated to g as in 2.2. Then a *Δ-stroke* of g is defined to be the natural number $\mathrm{st}_\Delta(g) = |g'(\delta) - \delta| > 0$ for any $\delta \in \Delta$. Clearly $\mathrm{st}_\Delta(g)$ is independent of the choice of $\delta \in \Delta$.

5.5. Proposition. *Let A be either a Euclidean or a tubular algebra. Let G be an admissible infinite cyclic group of K-linear automorphisms of $\hat{A}$, g a generator of G and $B \cong \hat{A}/G$. Then the following conditions are equivalent:*

(1) *All components in Γ_B are generalized standard.*

(2) *One of the following conditions holds:*

(2a) $\mathrm{st}_\Delta(g) > \mathrm{spr}_{\mathbb{M}}(B)$.

(2b) $\mathrm{st}_\Delta(g) = \mathrm{spr}_{\mathbb{M}}(B)$ *and for every indecomposable projective B-Module P it holds that P and the injective hull $I(\mathrm{top}(P))$ of $\mathrm{top}(P)$ do not belong to the same component in Γ_B.*

Proof. First we shall show that condition (2) implies condition (1). Assume that $\mathrm{st}_\Delta(g) > \mathrm{spr}_{\mathbb{M}}(B)$. Suppose to the contrary that there is a component $\mathscr{C}$ in Γ_B which is not generalized standard. Then there are X, $Y \in \mathscr{C}$ and $0 \neq f : X \to Y$ such that $f \in \mathrm{rad}^\infty(\mathrm{mod}(B))$. Since $\hat{A}$ is locally support-finite, there are $\hat{X}$, $\hat{Y} \in \mathrm{ind}(\hat{A})$, $0 \neq \hat{f} : \hat{X} \to \hat{Y}$ with $\hat{X} \in \mathscr{T}_{\delta_1}$, $\hat{Y} \in \mathscr{T}_{\delta_2}$ such that $F_\lambda(\hat{f}) = f$, $F_\lambda(\hat{X}) = X$, $F_\lambda(\hat{Y}) = Y$. But all components in $\Gamma_{\hat{A}}$ are generalized standard hence $\delta_1 \neq \delta_2$. We deduce from Corollary 2.5 that g is Δ-induced, so there is an automorphism $g' : \Delta \to \Delta$ such that if $Z \in \mathscr{T}_\delta$ then $F_g(Z) \in \mathscr{T}_{g'(\delta)}$, where F_g is an automorphism of $\mathrm{mod}(\hat{A})$ induced by g. Since X, $Y \in \mathscr{C}$ hence there is a natural number n such that either $(g')^n(\delta_1) = \delta_2$ or $(g')^{-n}(\delta_1) = \delta_2$. Then $\mathrm{st}_\Delta(g) \leq |\delta_1 - \delta_2|$ and clearly $l_{\mathbb{M}}(f) = |\delta_1 - \delta_2| \leq \mathrm{spr}_{\mathbb{M}}(B)$. Consequently $\mathrm{st}_\Delta(g) \leq \mathrm{spr}_{\mathbb{M}}(B)$ which contradicts our assumption. Thus every component in Γ_B is generalized standard.

Now assume that $\mathrm{st}_\Delta(g) = \mathrm{spr}_{\mathbb{M}}(B)$ and for every indecomposable projective B-module P it holds that P and $I(\mathrm{top}(P))$ do not belong to the same component. Again suppose to the contrary that there is a component $\mathscr{C}$ in Γ_B which is not generalized standard. Hence there are X, $Y \in \mathscr{C}$ and $0 \neq f : X \to Y$ such that $f \in \mathrm{rad}^\infty(\mathrm{mod}(B))$. Then by Lemma 5.3 there are indecomposable projective B-modules P, P' and $0 \neq h : P \to P'$ such that $h \in \mathrm{rad}^\infty(\mathrm{mod}(B))$. Clearly we can choose P as an indecomposable direct summand in $P(X)$ and P' as $I(\mathrm{top}(P))$. Then we can consider the following composed morphism $w\pi fp$, where $p : P \to X$ is induced by a covering morphism $P(X) \to X$, $\pi : Y \to Y/\mathrm{rad}(\mathrm{im}(f))$ is an epimorphism and $w : Y/\mathrm{rad}(\mathrm{im}(f)) \to P'$ is induced by an embedding $Y/\mathrm{rad}(\mathrm{im}(f)) \to I(Y/\mathrm{rad}(\mathrm{im}(f)))$. Then $0 \neq w\pi fp \in \mathrm{rad}^\infty(\mathrm{mod}(B))$. Since $\mathrm{st}_\Delta(g) = \mathrm{spr}_{\mathbb{M}}(B)$ hence P and P' belong to the same component which contradicts our assumption. Consequently, every component in Γ_B is generalized standard.

Now assume that every component in Γ_B is generalized standard. First we shall show that $\mathrm{st}_\Delta(g) \geq \mathrm{spr}_{\mathbb{M}}(B)$. Suppose to the contrary that $\mathrm{st}_\Delta(g) < \mathrm{spr}_{\mathbb{M}}(B)$. Then there is $0 \neq f : P \to P'$ between indecomposable projective B-modules such that $l_{\mathbb{M}}(f) > \mathrm{st}_\Delta(g)$ by Lemma 5.3. Since $\hat{A}$ is locally support-finite hence there is $0 \neq \hat{f} : \hat{P} \to \hat{P}'$ in $\mathrm{mod}(\hat{A})$, where $\hat{P} \in \mathscr{T}_{\delta_1}$, $\hat{P}' \in \mathscr{T}_{\delta_2}$ and $F_\lambda(\hat{P}) = P$, $F_\lambda(\hat{P}') = P'$, $F_\lambda(\hat{f}) = f$, $|\delta_1 - \delta_2| > \mathrm{st}_\Delta(g)$. But $\hat{A}$ is Δ-directed, so $\delta_2 > \delta_1$ and $\delta_1 < g'(\delta_1) < \delta_2$ or $\delta_1 < (g')^{-1}(\delta_1) < \delta_2$, because Δ is linearly ordered. We shall consider only the first case since the other one is similar. If A is tubular then there exists $\delta_0 \in \mathbb{Q}$ such that $\delta_1 < \delta_0 < g'(\delta_1)$ and $g'(\delta_1) < g'(\delta_0) < \delta_2$. Clearly $\hat{f}$ factorizes through a module $\hat{W}_1$ from $\mathrm{add}(\mathscr{T}_{\delta_0})$ and through a module $\hat{W}_2$ from $\mathrm{add}(\mathscr{T}_{g'(\delta_0)})$ by Lemma 3.5. We can choose $\hat{W}_1 \in \mathrm{add}(\mathscr{C})$ and $\hat{W}_2 \in \mathrm{add}(F_g(\mathscr{C}))$, where $\mathscr{C}$ is a component from $\mathcal{T}_{\delta_0}$ by [**16**, Section 2], because we can choose $\delta_0 \in (\mathbb{Q}\backslash\mathbb{Z})$. Thus $\hat{f} = \hat{f}_3\hat{f}_2\hat{f}_1$, where $\hat{f}_1 : \hat{P} \to \hat{W}_1$, $\hat{f}_2 : \hat{W}_1 \to \hat{W}_2$, $\hat{f}_3 : \hat{W}_2 \to \hat{P}'$. Therefore $F_\lambda(\hat{f}_2) : F_\lambda(\hat{W}_1) \to F_\lambda(\hat{W}_2)$ is a morphism from $\mathrm{rad}^\infty(\mathrm{mod}(B))$ and $F_\lambda(\hat{W}_1)$, $F_\lambda(\hat{W}_2) \in \mathrm{add}(F_\lambda(\mathscr{C}))$ which contradicts generalized standardness of the components in Γ_B. If A is Euclidean then $\mathrm{st}_\Delta(g) > 2$ by Lemma 3.6 and we obtain similarly that $F_\lambda(\mathcal{T}_z)$, z even, is not a generalized standard component in Γ_B by Lemma 3.4. Consequently we have proved that $\mathrm{st}_\Delta(g) \geq \mathrm{spr}_{\mathbb{M}}(B)$. In order to finish our proof we should only show that if $\mathrm{st}_\Delta(g) = \mathrm{spr}_{\mathbb{M}}(B)$ then P and $I(\mathrm{top}(P))$ do not belong to the same component, where P is any indecomposable projective B-module. Suppose to the contrary that there is an indecomposable projective B-module P such that P, $I(\mathrm{top}(P)) \in \mathscr{C}$. Then the composed morphism $wp \neq 0$, where $p : P \to \mathrm{top}(P)$, $w : \mathrm{top}(P) \to I(\mathrm{top}(P))$ and $wp \in \mathrm{rad}^\infty(\mathrm{mod}(B))$. This contradicts the assumption that all components in Γ_B are generalized standard. Thus the proposition follows. □

5.6. Corollary. *If B is a selfinjective representation-infinite K-algebra with all components in Γ_B generalized standard then the following conditions are satisfied:*

(1) *There is a partition $\Gamma_B = \bigsqcup_{v\in N} \mathcal{T}_v$ such that for every $v \in N$, $\mathcal{T}_v$ is a weakly separating family of components and $N\backslash\{v\}$ is isomorphic to Δ_{2i+1}, $i \geq 1$ or to $\Delta_{(0,i)}$, $i \geq 3$.*

(2) $\mathrm{spr}_{N\backslash\{v\}}(B) = \begin{cases} 4 & \text{if} \quad N\backslash\{v\} \cong \Delta_{2i+1} \\ 3 & \text{if} \quad N\backslash\{v\} \cong \Delta_{(0,i)} \end{cases}$

Proof. Since any selfinjective representation-infinite K-algebra B with all components in Γ_B generalized standard is a standard algebra of polynomial growth by [**34**] hence the corollary is clear by Theorem 4.5, Proposition 5.5 and Lemma 3.6, 3.7, 5.3. □

5.7. Under the assumptions and the notations of 5.6 we have the following.

Corollary. *If $0 \neq f : X \to Y$ is a morphism such that its coset modulo $\mathscr{P}(X, Y)\underline{f} \neq 0$ then*

$$l_{N\backslash\{v\}}(f) \leq \begin{cases} 2 & \text{if} \quad N\backslash\{v\} = \Delta_{2i+1}, i \geq 1, \\ 1\frac{1}{2} & \text{if} \quad N\backslash\{v\} = \Delta_{(0,i)}, i \geq 1. \end{cases}$$

Proof. Observe that by [**21**] we have the following fact. For every nonprojective indecomposable B-module X there is $0 \neq \underline{p} : \tau^-\Omega X \to X$ such that for any $0 \neq \underline{f} : Y \to X$ there is $\underline{h} : \tau^-\Omega X \to Y$ with $\underline{p} = \underline{f}\underline{h}$, where Ω is the Heller's loop-space functor. Thus $l_{n\backslash\{v\}}(f)$ is maximal, where $\underline{f} \neq 0$, iff f acts from $\tau^-\Omega X$ into X. Then for any hereditary algebra H a simple verification shows that $l_{N\backslash\{v\}}(f) \leq 2$. For canonical tubular algebra E it follows from [**23**, Lemma 1.6] that $l_{N\backslash\{v\}}(f) \leq 1\frac{1}{2}$. Since for a Euclidean algebra A we have $\underline{\mathrm{mod}}(\hat{A})$ is equivalent to $\underline{\mathrm{mod}}(\hat{H})$ and for a tubular algebra A we have $\underline{\mathrm{mod}}(\hat{A})$ is equivalent to $\underline{\mathrm{mod}}(\hat{E})$, the corollary follows. □

6. Components of algebras stably equivalent to algebras whose components are generalized standard.

6.1. Throughout this section we assume that B is a selfinjective representation-infinite K-algebra such that all components in Γ_B are generalized standard. Moreover, we assume that C is an algebra which is stably equivalent to B and $\Phi:\underline{\mathrm{mod}}(B)\to\underline{\mathrm{mod}}(C)$ is a fixed equivalence. We shall denote by Φ^{-1} a quasi-inverse of Φ. It is well-known and easy to prove that C is also selfinjective.

6.2. Lemma. *Let X, Y be two nonprojective indecomposable C-modules which belong to a component $\mathscr{C}$ in Γ_C. If $f \in \mathrm{rad}^\infty(X, Y)$ is a nonzero morphism then $\underline{f}=0$.*

Proof. Under the assumptions and the notations of the lemma suppose to the contrary that there is $f \in \mathrm{rad}^\infty(\underline{\mathrm{mod}}(C))$ with $\underline{f}\neq 0$. Then there is $0\neq \underline{d}:X_1\to Y_1$ such that $\Phi(X_1)=X$, $\Phi(Y_1)=Y$ and $\Phi(\underline{d})=\underline{f}$. Since the stable Auslander–Reiten quiver is invariant under taking stable equivalences by [**6**] hence there is a component $\mathscr{C}_1$ in Γ_B such that X_1, $Y_1 \in \mathscr{C}_1$. Observe that there is such $d':X_1\to Y_1$ that $\Phi(\underline{d'})=\underline{f}$ and $d' \in \mathrm{rad}^\infty(\mathrm{mod}(B))$. Indeed, if $f \in \mathrm{rad}^\infty(\mathrm{mod}(C))$ then $f=f'_sh_{-s}h_{-s+1}\dots h_{-1}h_0h_1\dots h_{r-1}h_rf''_r$, where $h_j \in \mathrm{rad}(\mathrm{mod}(C))$, so it is neither a split epimorphism nor a split monomorphism, $j=-s$, $-s+1,\dots,-1$, 0, $1,\dots,r$. Moreover, for arbitrary large natural s or r, f has a decomposition of the above form. Thus we can choose $d'=d'_st_{-s}t_{-s+1}\dots t_{-1}t_0t_1\dots t_{r-1}t_rd''_r$ such that $\Phi(\underline{d'_s})=\underline{f'_s}$, $\Phi(\underline{d''_r})=\underline{f''_r}$, $(\Phi\underline{t_j})=\underline{h_j}$, $j=-s,\dots,-1$, 0, $1,\dots,r$. But passing from a decomposition $f'_sh_{-s}\dots h_{-1}h_0h_1\dots h_rf''_r$ to $f'_{s+1}h_{-s-1}h_{-s}\dots h_{-1}h_0h_1\dots h_rh_{r+1}f''_{r+1}$ we decompose $f'_s=f'_{s+1}h_{-s+1}$ and $f''_r=h_{r+1}f''_{r+1}$ in the following way. If $f'_s:\oplus_{i=1}^n Z_i\to Y$ with all Z_i indecomposable, $i=1,\dots,n$, then for every $i=1,\dots,n$ there is an Auslander–Reiten sequence of the form $0\to Z_i\xrightarrow{l_i}R_i\to\tau^{-1}(Z_i)\to 0$ by [**4**]. Clearly $f'_s=(f'_{s,1},\dots,f'_{s,n})$, where $f'_{s,i}:Z_i\to Y$. Then we know from [**4**] that there is a morphism $f'_{s+1,i}:R_i\to Y$ such that $f'_{s,i}=f'_{s+1,i}l_i$. Thus putting

$$f'_{s+1}=(f'_{s+1,1},\dots,f'_{s+1,n}) \text{ and } h_{-s+1}=\begin{pmatrix} l_1 & 0 & 0 & \dots & 0\\ 0 & l_2 & 0 & \dots & 0\\ & \cdot & \cdot & \cdot & \cdot\\ 0 & 0 & \dots & 0 & l_n\end{pmatrix}$$

we obtain a decomposition $f'_s=f'_{s+1}h_{-s-1}$ with $h_{-s-1}\in\mathrm{rad}(\mathrm{mod}(C))$. Dually one decomposes $f''_r=h_{r+1}f''_{r+1}$. Thus for arbitrary large natural s or r, d' has a decomposition of the above form. Therefore $d' \in \mathrm{rad}^\infty(\mathrm{mod}(B))$ and $\underline{d'}=\underline{d}$ since $\Phi(\underline{d'})=\Phi(\underline{d})=\underline{f}$. Since $\underline{d'}\neq 0$ hence $d'\neq 0$, and so we get a contradiction to our assumption on generalized standardness of all components in Γ_B, which finishes the proof.

6.3. Lemma. *If $0\neq f:X\to Y$ is such a morphism that $f\in\mathrm{rad}^\infty(\mathrm{mod}(C))$ and X, $Y\in\mathrm{add}(\mathscr{C})$ are without projective direct summands, where $\mathscr{C}$ is a component in Γ_C, then f factorizes through a module $W\in\mathrm{add}(\Gamma_C\setminus\mathscr{C})$.*

Proof. Assume that $0\neq f:X\to Y$ with X, $Y\in\mathrm{add}(\mathscr{C})$ without projective direct summands, $f\in\mathrm{rad}^\infty(\mathrm{mod}(C))$, where $\mathscr{C}$ is a component in Γ_C. Since $f\in\mathrm{rad}^\infty(\mathrm{mod}(C))$ there is a decomposition $f=f'_sh_{-s}\dots h_{-1}h_0h_1\dots h_rf''_r$, where $h_j\in\mathrm{rad}(\mathrm{mod}(C))$, $j=-s,\dots,-1,0,1,\dots,r$. Moreover, for arbitrary large s or r, f has a decomposition of this form.

Passing from $f=f'_sh_{-s}\dots h_{-1}h_0h_1\dots h_rf''_r$ to $f=f'_{s+1}h_{-s-1}\dots h_{-1}h_0h_1\dots h_rf''_r$ we decompose $f'_s=f'_{s+1}h_{-s-1}$ as in the proof of Lemma 6.2. Passing from $f=f'_sh_{-s}\dots h_{-1}h_0h_1\dots h_rf''_r$ to $f=f'_sh_{-s}\dots h_{-1}h_0h_1\dots h_{r+1}f''_{r+1}$ we decompose $f''_r=h_{r+1}f''_{r+1}$ as in the proof of Lemma 6.2.

Let $h_j: W_j \to W_{j-1}$, $j = -s+1, \dots, -1, 0, 1, \dots, r$. Clearly we may assume that all W_j are in add($\mathscr{C}$). Indeed, if $W_{j_0} \in \mathrm{add}(\Gamma_C \setminus \mathscr{C})$ then f factorizes through W_{j_0} and the required condition (in the lemma) holds. If W_{j_0} has a decomposition $W'_{j_0} \oplus W''_{j_0}$ with $W'_{j_0} \in \mathrm{add}(\mathscr{C})$ and $W''_{j_0} \in \mathrm{add}(\Gamma_C \setminus \mathscr{C})$ then we can consider the morphism $f'_s h_{-s} \dots h_{j_0-1} h'_{j_0} h'_{j_0+1} h_{j_0+2} \dots h_r f''_r$, where $h'_{j_0+1}: W_{j_0+1} \to W'_{j_0}$, $h_{j_0+1} = (h'_{j_0+1}, \; h''_{j_0+1})$, $h''_{j_0+1}: W_{j_0+1} \to W''_{j_0}$; $h_{j_0} = \begin{pmatrix} h'_{j_0} \\ h''_{j_0} \end{pmatrix} : W'_{j_0} \oplus W''_{j_0} \to W_{j_0-1}$. In the same way we can assume that $\mathrm{im}(f'_s h_{-s} \dots h_j) \in \mathrm{add}(\mathscr{C})$, $j = -s, \dots, 0, \dots, r$.

Now consider the case when r can be arbitrarily large. Then there is $j_0 \in \mathbb{N}$ such that all h_j are epimorphisms for $j \geq j_0$ by the construction of decompositions of f given in the proof of Lemma 6.2. If $\underline{h_{j_0} \dots h_r f''_r} \neq 0$ then $\Phi^{-1}(\underline{h_{j_0} \dots h_r f''_r}) = \Phi^{-1}(\underline{h_{j_0}}) \dots \Phi^{-1}(\underline{h_r}) \Phi^{-1}(\underline{f''_r}) \neq 0$. Thus we can choose h'_j in $\mathrm{rad}(\underline{\mathrm{mod}(B)})$ such that $\underline{h'_j} = \Phi^{-1}(\underline{h_j})$, $j = j_0, \dots, r$, because stable equivalences preserve irreducible morphisms. Hence there is a morphism h in mod(B) such that $\underline{h} = \Phi^{-1}(\underline{h_{j_0} \dots h_r f''_r})$ and $h = h'_{j_0} \dots h'_r \bar{f}''_r$, where $\underline{\bar{f}''_r} = \Phi^{-1}(\underline{f''_r})$. Consequently, $h \in \mathrm{rad}^\infty(\underline{\mathrm{mod}(B)})$ and $h: \Phi^{-1}(X) \to \Phi^{-1}(W_{j_0})$. This means that the component $\Phi^{-1}(\mathscr{C})$ in Γ_B is not generalized standard which contradicts our assumption made in 6.1.

If $\underline{h_{j_0} \dots h_r f''_r} = 0$ then $h_{j_0} \dots h_r f''_r$ factorizes through the projective cover P of W_{j_0}. The same reasoning as in the first part of the proof shows that we may consider the largest direct summand P' in P with $P' \in \mathrm{add}(\mathscr{C})$. Then it is clear that the induced morphism $p: P' \to W_{j_0}$ has a decomposition $p = h_{j_0} \dots h_r p''$ for some $p'': P \to W_{r+1}$ by projectivity of P since h_j are epimorphisms for $j \geq j_0$. Thus $p \in \mathrm{rad}^\infty(\mathrm{mod}(C))$. Furthermore, p factorizes through $P'/\mathrm{soc}(P')$, because $p = p_1 \pi$ for an irreducible morphism $\pi: P' \to P'/\mathrm{soc}(P')$. It is obvious that $p_1 \in \mathrm{rad}^\infty(\mathrm{mod}(C)$. Moreover $\underline{p_1} \neq 0$, because the canonical epimorphism $q: P'/\mathrm{soc}(P') \to \mathrm{top}(P')$ factorizes through p_1 hence $q = lp_1$ for some $l: W_{j_0} \to \mathrm{top}(P')$. But $\underline{q} \neq 0$, because q is an epimorphism. Hence $\underline{p_1} \neq 0$. Thus as in the case $\underline{h_{j_0} \dots h_r f''_r} \neq 0$ we get a contradiction to generalized standardness of the component $\Phi^{-1}(\mathscr{C})$.

In the case when s can be arbitrarily large dual arguments apply and the lemma follows. □

6.4. Lemma. *Let $\mathscr{C}$ be a component in Γ_C. If $X, Y \in \mathscr{C}$ and $0 \neq f: X \to Y$ such that $f \in \mathrm{rad}^\infty(\mathrm{mod}(C))$ then there are $X_1, Y_1 \in \mathscr{C}$, $W \in \mathrm{add}(\Gamma_C \setminus \mathscr{C})$ and $h_1: X_1 \to W$, $h_2: W \to Y_1$ such that $\underline{h_1} \neq 0$ and $\underline{h_2} \neq 0$.*

Proof. Let $\mathscr{C}$ be a component in Γ_C. If $X, Y \in \mathscr{C}$ and $0 \neq f: X \to Y$ such that $f \in \mathrm{rad}^\infty(\mathrm{mod}(C))$ then there is a module $W_1 \in \mathrm{add}(\Gamma_C \setminus C)$ and there are $f_1: X \to W_1$, $f_2: W_1 \to Y$ such that $f = f_2 f_1$ by Lemma 6.3. If there is a direct summand V in $\mathrm{im}(f)$ with $V \in \mathrm{add}(\Gamma_C \setminus \mathscr{C})$ then the required condition holds for $W = V$, $X_1 = X$, $Y_1 = Y$, $h_1: X \to V$ an epimorphism and $h_2: V \to Y$ a monomorphism. If $\mathrm{im}(f) \in \mathrm{add}(\mathscr{C})$ then consider $\mathrm{im}(f_1)$. If there is a direct summand V_1 in $\mathrm{im}(f_1)$ such that $V_1 \in \mathrm{add}(\Gamma_C \setminus \mathscr{C})$ then there is an epimorphism $t: \mathrm{im}(f_1) \to \mathrm{im}(f)$ and take $t_1: V_1 \to Z_1$ as a restriction of t to V_1 composed with a projection from $\mathrm{im}(f)$ onto an indecomposable direct summand in $\mathrm{im}(f)$ such that $pt_1 \neq 0$, where $p: Z_1 \to S$ is a projection and S is a simple C-module. Thus the composed morphism $t = t_1 t_2$ has the properties $\underline{t} \neq 0$, $t \in \mathrm{rad}^\infty(\mathrm{mod}(C))$, where $t_2: X \to V_1$ is an epimorphism. This contradicts Lemma 6.2. Hence $\mathrm{im}(f_1) \in \mathrm{add}(\mathscr{C})$. Applying the usual duality D to $\mathrm{im}(f_2)$ we obtain similarly that $\mathrm{im}(f_2) \in \mathrm{add}(\mathscr{C})$. Since $f \neq 0$ there is an indecomposable direct summand X_1 in $\mathrm{im}(f_1)$ and there is an indecomposable direct summand Y_1 in $\mathrm{im}(f_2)$ such that $qv \neq 0$, where $v: X_1 \to W_1$ is a monomorphism induced by f_1 and $q: W_1 \to Y_1$ is an epimorphism induced by f_2. Consequently the required condition holds for X_1, Y_1, $W = W_1$, $h_1 = v$, $h_2 = q$ and the proof is finished. □

6.5. Lemma. *If $\mathscr{C}$ is a component in Γ_B such that there are $X, Y \in \mathscr{C}$, an indecomposable $W \in \mathrm{add}(\Gamma_B \setminus \mathscr{C})$ and $0 \neq \underline{h_1}: X \to W$, $0 \neq \underline{h_2}: W \to Y$ then $\mathscr{C}$ does not contain a projective vertex.*

Proof. Let $\mathscr{C}$ be a component in Γ_B such that there are $X, Y \in \mathscr{C}$, an indecomposable $W \in \mathrm{add}(\Gamma_B \setminus \mathscr{C})$ and $0 \neq \underline{h_1}: X \to W$, $0 \neq \underline{h_2}: W \to Y$. Suppose to the contrary that $\mathscr{C}$ contains a projective vertex. We know from Corollary 5.6 that $\Gamma_B = \bigsqcup_{v \in N} \mathscr{T}_v$, where for every $v \in N$, $\mathscr{T}_v$ is a weakly separating family of components. Moreover, for every $v \in N$ we have that $N \setminus \{v\}$ is isomorphic to Δ_{2i+1}, $i \geq 1$, or $\Delta_{(0,i)}$, $i \geq 3$. Let $\mathscr{C} \in \mathscr{T}_{v_0}$, $v_0 \in N$. Let $W \in \mathscr{T}_{v_1}$. If $N \setminus \{v_0\} \cong \Delta_{(0,i)}$, $i \geq 3$, then $i = 3$. Indeed, if $i > 3$ then either $l_N(h_1) > 1\frac{1}{2}$ or $l_N(h_2) > 1\frac{1}{2}$ which contradicts Corollary 5.7. If $i = 3$ then $l_N(h_1) = l_N(h_2) = 1\frac{1}{2}$, because $X, Y \in \mathscr{C}$ and $W \in \Gamma_B \setminus \mathscr{C}$. But we know from [**6, 21**] and Corollary 5.7 that there is $0 \neq \underline{t}: \tau^{-}\Omega W \to W$ such that $l_N(t) = 1\frac{1}{2}$. We infer by [**6**; Proposition 4.1] that the simple functor from $\underline{\mathrm{mod}}(B)$ to $\mathrm{mod}(K)$ concentrated in $\tau^{-1}\Omega(W)$ is the socle of $\underline{\mathrm{Hom}}_B(-, W)$. Thus there is $\underline{r}: \tau^{-1}\Omega(W) \to X$ such that $\underline{h_1}\underline{r} = \underline{t}$. Consequently, $\tau^{-1}\Omega(W) \in \mathscr{C}$. Dual arguments show that $\tau\Omega^{-1}(W) \in \mathscr{C}$.

For any indecomposable projective B-module R we have the following Auslander–Reiten sequence (see [**5**])

$$0 \to \mathrm{rad}(R) \to R \oplus \mathrm{rad}(R)/\mathrm{soc}(R) \to R/\mathrm{soc}(R) \to 0$$

because B is selfinjective, and so every projective module is projective-injective and its top is simple as well as its socle. Thus $\tau\Omega^{-1}(\mathrm{soc}(R)) \cong \tau(R/\mathrm{soc}(R)) \cong \mathrm{rad}(R)$.

If there is a projective vertex P in $\mathscr{C}$ then $\Omega^{-1}(\mathrm{rad}(P))$ belongs to the same component as W. But $\Omega^{-1}(\mathrm{rad}(P)) \cong \mathrm{top}(P)$. If we consider an injective envelope $I(\mathrm{top}(P))$ of the simple B-module $\mathrm{top}(P)$ then $I(\mathrm{top}(P))$ is indecomposable projective-injective and $\mathrm{soc}(I(\mathrm{top}(P))) \cong \mathrm{top}(P)$. Then $\Omega^{-1}(\mathrm{top}(P)) \cong I(\mathrm{top}(P))/\mathrm{soc}(I(\mathrm{top}(P)))$ and $\tau\Omega^{-1}(\mathrm{top}(P)) \cong \mathrm{rad}(I(\mathrm{top}(P)))$ by the above Auslander–Reiten sequence. Since $\tau\Omega^{-1}(W) \in \mathscr{C}$ hence $\mathrm{rad}(I(\mathrm{top}(P))) \cong \tau\Omega^{-1}(\mathrm{top}(P)) \in \mathscr{C}$ which contradicts Proposition 5.5(2b) for all components in Γ_B are generalized standard. The proof in the case when $N \setminus \{v_0\} \cong \Delta_{2i+1}$, $i \geq 1$, is similar and we leave the details to the reader. □

6.6. Lemma. *If $\mathscr{C}$ is a component in Γ_C which is not generalized standard, then $\mathscr{C}$ contains a projective vertex or a simple one.*

Proof. Assume that $\mathscr{C}$ is a component in Γ_C which is not generalized standard. Then there are $X, Y \in \mathscr{C}$ and $0 \neq f: X \to Y$ such that $f \in \mathrm{rad}^{\infty}(\mathrm{mod}(C))$. Clearly we may assume that X and Y are not projective. Indeed, if X is projective then $f = f_2 f_1$, where $f_1: X \to X/\mathrm{soc}(X)$ is an epimorphism and $f_2: X/\mathrm{soc}(X) \to Y$. Moreover, we deduce from Lemma 6.3 that f factorizes through a module $W \in \mathrm{add}(\Gamma_C \setminus \mathscr{C})$. Thus f_2 factorizes through the same W, and so $f_2 \in \mathrm{rad}^{\infty}(\mathrm{mod}(C))$. Consequently we may consider $X/\mathrm{soc}\, X$ instead of X and f_2 instead of f. Dual arguments show that Y can be chosen nonprojective. We obtain from Lemma 6.2 that $\underline{f} = 0$. If $\mathrm{im}(f) \in \mathrm{add}(\Gamma_C \setminus \mathscr{C})$ then $f = h_2 h_1$, where $h_1: X \to \mathrm{im}(f)$ is an epimorphism and $h_2: \mathrm{im}(f) \to Y$ is a monomorphism. Thus $\underline{h_1}$, $\underline{h_2} \neq 0$ by [**27**]. If $\Gamma_B = \bigcup_{v \in N} \mathscr{T}_v$ and $N \setminus \{v_0\}$ is isomorphic to $\Delta_{(0,i)}$, then $i = 3$ and $l_N(h_1) \leq 1\frac{1}{2}$, $l_N(h_2) \leq 1\frac{1}{2}$ by Corollary 5.7. But we have an epimorphism $p: P(\mathrm{im}(f)) \to \mathrm{im}(f)$.

Therefore there is a morphism $t:P(\mathrm{im}(f))/\mathrm{soc}(P(\mathrm{im}(f)))\to X$ such that th_1 is an epimorphism. Thus $\underline{th_1}\neq 0$ and if $P(\mathrm{im}(f))/\mathrm{soc}(P(\mathrm{im}(f)))$ does not belong to $\mathrm{add}(\mathscr{C})$ then $l_N(th_1)>1\frac{1}{2}$ which contradicts Corollary 5.7. If $N\setminus\{v_0\}$ is isomorphic to Δ_{2i+1}, then $i=1$ and $l_N(h_1)\leq 2$, $l_N(h_2)\leq 2$. Repeating the above arguments we get that $P(\mathrm{im}(f))\in\mathrm{add}(\mathscr{C})$.

If there is no direct summand Z in $\mathrm{im}(f)$ such that $Z\in\mathrm{add}(\Gamma_C\setminus\mathscr{C})$ then we infer by Lemma 6.4 that $f=h_2h_1$, where $h_1:X\to W$, $h_2:W\to Y$ and $W\in\mathrm{add}(\Gamma_C\setminus\mathscr{C})$. Thus there is a submodule W_1 in W such that there is an epimorphism $t_1:X\to W_1$ and there is an epimorphism $t_2:W_1\to\mathrm{im}(f)$. If $W_1\in\mathrm{add}(\mathscr{C})$ then consider the module $W_2=\mathrm{im}(h_2)$. There are monomorphisms $v_1:\mathrm{im}(f)\to W_2$ and $v_2:W_2\to Y$. Furthermore there is an epimorphism $q:W\to W_2$ such that $h_2h_1=v_2qh_1$. Since v_2v_1 is a monomorphism there is no nonzero direct summand W_2' in W_2 such that $W_2'\in\mathrm{add}(\Gamma_C\setminus\mathscr{C})$, because we get a contradiction to Lemma 6.2 otherwise. Consequently we can consider qw instead of f, where $w:W_1\to W$ is a monomorphism. Thus, applying dual arguments to those in the case $\mathrm{im}(f)\in\mathrm{add}(\Gamma_C\setminus\mathscr{C})$, one obtains that $\mathscr{C}$ contains a simple module. In order to finish our proof we should consider the case when there is a direct summand W_1' in W_1 such that $W_1'\in\mathrm{add}(\Gamma_C\setminus\mathscr{C})$. But in that case we get a contradiction to Lemma 6.2, because t_2t_1 is an epimorphism. □

6.7. Proposition. *One of the following conditions holds:*

(1) *All components in Γ_C are generalized standard.*

(2) *There is a partition $\Gamma_B=\bigsqcup_{v\in N}\mathscr{T}_v$ such that for every $v_0\in N$, $N\setminus\{v_0\}$ is isomorphic to $\Delta_{(0,3)}$.*

Proof. Fix a partition $\Gamma_B=\bigsqcup_{v\in N}\mathscr{T}_v$ described in Corollary 5.6. Assume for the proof of the proposition that $N\setminus\{v_0\}$ is isomorphic either to $\Delta_{(0,i)}$, $i>3$, or to Δ_{2i+1}, $i\geq 1$. Suppose to the contrary that there is a component $\mathscr{C}$ in Γ_C which is not generalized standard. Then there are $X,Y\in\mathscr{C}$, $0\neq f:X\to Y$ such that $f\in\mathrm{rad}^\infty(\mathrm{mod}(C))$. From Lemma 6.2 we have $\underline{f}=0$. Furthermore we know from Lemma 6.3 that there is $W\in\mathrm{add}(\Gamma_C\setminus\mathscr{C})$ and there are $f_1:X\to W$, $f_2:W\to Y$ such that $f=f_2f_1$. Using Lemma 6.4 we can choose f_1, f_2, W in such a way that $\underline{f_1}\neq 0\neq\underline{f_2}$. Therefore it is obvious by Corollary 5.7 that $\Gamma_B=\bigsqcup_{v\in N}\mathscr{T}_v$ with $N\setminus\{v_0\}$ isomorphic to Δ_3, because we assumed that it is not isomorphic to $\Delta_{(0,3)}$. But for $N\setminus\{v_0\}$ isomorphic to Δ_3 we deduce from Lemma 3.4 that there is $v_1\in\Delta_3$ such that $\mathscr{T}_{v_1}$ consists of one component which contains projective vertices. Thus, if P is a projective vertex in $\mathscr{T}_{v_1}$ and $I(\mathrm{top}(P))$ is the injective envelope of $\mathrm{top}(P)$ then $P,I(\mathrm{top}(P))\in\mathscr{T}_{v_1}$ by 3.1 which contradicts (by Proposition 5.5) the assumption that all components in Γ_B are generalized standard. Consequently the proposition is proved. □

7. Selfinjective algebras of type $\Delta_{(0,3)}$.

7.1. Throughout this section we shall assume that B is a representation-infinite selfinjective algebra such that all components in Γ_B are generalized standard, and there is a partition $\Gamma_B=\bigsqcup_{v\in N}\mathscr{T}_v$, where $N\setminus\{v_0\}$ is isomorphic to $\Delta_{(0,3)}$, $v_0\in N$. Moreover C is stably equivalent to B and $\Phi:\underline{\mathrm{mod}}(B)\to\underline{\mathrm{mod}}(C)$ is a fixed equivalence.

7.2. Lemma. *Γ_C contains a sincere tube which is generalized standard.*

Proof. We deduce from Lemma 6.6 that all components, which do not contain a projective module or a simple one, are generalized standard. If we carry over the partition $\Gamma_B=\bigsqcup_{v\in N}\mathscr{T}_v$ via Φ then we have $\Gamma_C=\bigsqcup_{v\in N}\mathscr{T}_v'$. Without loss of generality we may assume that $N = [0,3) \cap \mathbb{Q}$. Since C is a finite-dimensional algebra there are only finitely many rational numbers $q \in [0,3)$ such that $\mathcal{T}'_q$ contains projective or simple modules. Thus we can choose a rational number $q_0 \in [0,3)$ such that $q_0 \notin \{0, \frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}\}$ and all components in $\mathcal{T}'_{q_0}$ are generalized standard. But we have that for every simple C-module S either a projection $p: P(S)/\mathrm{soc}(P(S)) \to S$ or an injection $w: S \to \mathrm{rad}(I(S))$ factorizes through a module from $\mathrm{add}(\mathcal{T}'_{q_0})$. But $\underline{p} \neq 0$ and $\underline{w} \neq 0$ hence for $\underline{\pi} = \Phi^{-1}(\underline{p})$, $\underline{u} = \Phi^{-1}(\underline{w})$ it holds that either π or u factorizes through a module from $\mathrm{add}(\mathscr{C})$, where $\mathscr{C}$ ranges over all components in $\mathcal{T}_{q_0}$ by Lemma 3.5(1). Therefore either p or w factorizes through a module from $\mathrm{add}(\mathscr{C}')$, where $\mathscr{C}'$ ranges over all components in $\mathcal{T}'_{q_0}$. Consequently every component in $\mathcal{T}'_{q_0}$ is a generalized standard sincere tube and the lemma follows. □

7.3. Fix a generalized standard sincere tube T in Γ_C. Denotes by $\mathbb{I}$ the annihilator $\mathrm{ann}_C(T)$ of T in C, which is the intersection of the annihilators of all modules from T. Clearly $\mathbb{I}$ is nonzero, because $\mathrm{soc}(C) \subset \mathbb{I}$. Moreover $\mathbb{I} \subset \mathrm{rad}(C)$ hence T is a sincere tube in Γ_F, where $F = C/\mathbb{I}$.

7.4. Lemma. *F is cycle-finite.*

Proof. Repeat the arguments from the proof of [**23**, Lemma 2.9].

7.5. Lemma. *C is a representation-infinite selfinjective standard algebra of polynomial growth.*

Proof. It is clear that C is representation-infinite selfinjective. The final part of the lemma can be deduced from a result of Skowroński and Yamagata [**35**] which says that if a selfinjective algebra C has a generalized standard sincere stable tube T such that $F = C/\mathrm{ann}_C(T)$ is cycle-finite then C is standard of polynomial growth. Thus our lemma follows by Lemma 7.2 and Lemma 7.4. □

8. Proof of the main result.

8.1. Let B be a selfinjective representation-infinite algebra such that all components of Γ_B are generalized standard. Let C be an algebra which is stably equivalent to B and let $\Phi: \underline{\mathrm{mod}}(B) \to \underline{\mathrm{mod}}(C)$ be a fixed equivalence.

8.2. Lemma. *C is a representation-infinite selfinjective standard algebra of polynomial growth.*

Proof. We know from Proposition 6.7 and Lemma 7.5 that all components of Γ_C are generalized standard or C is a representation-infinite selfinjective standard algebra of polynomial growth. But in the case when all components of Γ_C are generalized standard it follows from [**34**, Theorem 3.13] that C is representation-infinite selfinjective standard algebra of polynomial growth. □

8.3. Lemma. *Let Δ be an infinite linearly ordered set and let R be a Δ-directed locally bounded K-category. Let G be an admissible infinite cyclic group of K-linear automorphisms of R such that for every nonprojective $X \in \mathcal{T}_{\delta_0}$, $\delta_o \in \Delta$, and every indecomposable Y if $\underline{\mathrm{Hom}}_R(X,Y) \neq 0$ then $Y \in \mathcal{T}_{\delta_1}$ with $\delta_0 \leq \delta_1 < g'(\delta_0)$, where $\Gamma_R = \bigsqcup_{\delta \in \Delta} \mathcal{T}_\delta$ is a Δ-induced partition of Γ_R and g is a generator of G which satisfies $\delta < g'(\delta)$, $\delta \in \Delta$. If $F_\lambda: \mathrm{mod}(R) \to \mathrm{mod}(R/G)$ is an induced by the action of G on R push-down functor and $\underline{\mathrm{Hom}}_R(U,V) \neq 0$, $U, V \in \mathrm{ind(R)}$, then $\underline{\mathrm{Hom}}_R(U,V) \cong \underline{\mathrm{Hom}}_{R/G}(F_\lambda(U), F_\lambda(V))$ as K-linear spaces.*

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Proof. Under the assumptions and the notations of the lemma observe that if $f: U \to V$ is such a morphism in mod(R) that $\underline{f} \neq 0$ then $F_\lambda(f): F_\lambda(U) \to F_\lambda(V)$ satisfies $\underline{F_\lambda(f)} \neq 0$. Indeed, F_λ preserves projective modules and factorizations of morphisms through projective modules. Let $U, V \in \operatorname{ind}(R)$ and $U \in \mathcal{T}_{\delta_0}$, $\delta_0 \in \Delta$. If $\underline{\operatorname{Hom}}_R(U, V) \neq 0$ then $V \in \mathcal{T}_{\delta_1}$ with $\delta_0 \leq \delta_1 < g'(\delta_0)$ by the assumption. But the G-orbit of V intersects with $\bigsqcup_{\delta \in [\delta_0, g'(\delta_0)]} \mathcal{T}_\delta$ in exactly one point V. Thus the lemma follows. □

8.4. Let Δ be an infinite linearly ordered set. An admissible infinite cyclic group G of K-linear automorphisms of a Δ-directed locally bounded K-category R is called *stably admissible* if it satisfies the following condition: for every Δ-induced partition of Γ_R and for every nonprojective $X \in \mathcal{T}_{\delta_0}$ and every nonprojective indecomposable R-module Y if $\underline{\operatorname{Hom}}_R(X, Y) \neq 0$ then $Y \in \mathcal{T}_{\delta_1}$ with $\delta_0 \leq \delta_1 < g'(\delta_0)$, where g is a generator of G such that $g'(\delta) > \delta$, $\delta \in \Delta$.

8.5. Proposition. *Let Δ be an infinite linearly ordered set. Let R_1, R_2 be two selfinjective Δ-directed locally bounded K-categories. Let G_i be a stably admissible group of K-linear automorphisms of R_i, $i = 1, 2$. If R_1/G_1 is stably equivalent to R_2/G_2 then R_1 is stably equivalent to R_2.*

Proof. Under the assumptions and the notations of the proposition assume that $\Phi: \underline{\operatorname{mod}}(R_1/G_1) \to \underline{\operatorname{mod}}(R_2/G_2)$ is a fixed equivalence. Let $F_{i,\lambda}: \operatorname{mod}(R_i) \to \operatorname{mod}(R_i/G_i)$, $i = 1, 2$ be an induced by the action of G_i on R_i push-down functor. We shall construct an equivalence $\Psi: \underline{\operatorname{mod}}(R_1) \to \underline{\operatorname{mod}}(R_2)$ as follows. Since Φ preserves the stable Auslander–Reiten quiver $\Gamma^s_{R_1/G_1}$ and $F_{i,\lambda}$ maps components of Γ_{R_i} onto components of Γ_{R_i/G_i}, $i = 1, 2$, hence we can carry over the partition of Γ_{R_1} onto the partition of Γ_{R_2}. Thus we may assume that if nonprojective $X, Y \in \mathcal{T}_{\delta_1} \subset \Gamma_{R_1}$ then there is $\delta_2 \in \Delta$ such that there are nonprojective $X', Y' \in \mathcal{T}'_{\delta_2} \subset \Gamma_{R_2}$ such that $\Phi(F_{1,\lambda}(X)) = F_{2,\lambda}(X')$ and $\Phi(F_{1,\lambda}(Y)) = F_{2,\lambda}(Y')$). Fix a family $\mathcal{T}_{\delta_0}$ in Γ_{R_1}, $\delta_0 \in \Delta$. Choose a family $\mathcal{T}'_{\delta_1}$ in Γ_{R_2} in such a way that for every nonprojective $X \in \mathcal{T}_{\delta_0}$ there is $Y \in \mathcal{T}'_{\delta_1}$ satisfying $\Phi(F_{1,\lambda}(X)) = F_{2,\lambda}(Y)$. It is possible to do this by the above choice of the partition $\bigsqcup_{\delta \in \Delta} \mathcal{T}'_\delta$ of Γ_{R_2}. Then for every $X \in \mathcal{T}_{\delta_0}$ we put $\Psi(X) = Y$ with $Y \in \mathcal{T}'_{\delta_1}$ as above. If g_i, $i = 1, 2$ is a generator of G_i satisfying the condition in 8.4 and $g'_i: \Delta \to \Delta$ is an automorphism induced by g_i, $i = 1, 2$, then clearly $g'_1 = g'_2$ by the choice of the partition of Γ_{R_2}. Therefore for any $z \in \mathbb{Z}$ if $X \in \mathcal{T}_{(g'_1)^z(\delta_0)}$ then we can find the only $Y \in \mathcal{T}'_{(g'_2)^z(\delta_2)}$ such that $\Phi(F_{1,\lambda}(X)) = F_{2,\lambda}(Y)$. We put $\Psi(X) = Y$ again. Moreover, for every $\delta \in [\delta_0, g'_1(\delta_0)]$ there is exactly one $\sigma \in [\delta_1, g'_2(\delta_1)]$ such that for every nonprojective $X \in \mathcal{T}_\delta$ there is exactly one $Y \in \mathcal{T}'_\sigma$ with $\Phi(F_{1,\lambda}(X)) = F_{2,\lambda}(Y)$. Again for $X \in \mathcal{T}_\delta$ we put $\Psi(X) = Y$, where $Y \in \mathcal{T}'_\sigma$ and $\Phi(F_{1,\lambda}(X)) = F_{2,\lambda}(Y)$. Now we can prolongate Ψ for the objects of the shifted families $\mathcal{T}_{(g'_1)^z(\delta)}$, $z \in \mathbb{Z}$, $\delta \in [\delta_0, g'_1(\delta_0)]$. In this way we have defined Ψ for the indecomposable objects. Furthermore we prolongate Ψ for the objects of $\underline{\operatorname{mod}}(R_1)$ additively. Moreover, for every $0 \neq \underline{f}: X \to Y$ in $\underline{\operatorname{mod}}(R_1)$ we define $\Psi(\underline{f}) = \underline{h}: \Psi(X) \to \Psi(Y)$, where $\Phi \underline{F_{1,\lambda}(f)}) = \underline{F_{1,\lambda}(h)}$. It is easy to verify that Ψ is a well-defined functor. Clearly Ψ is dense by its definition. Finally Ψ is fully faithful by Lemma 8.3 and the proposition follows. □

Proof of Theorem. Let B be a representation-infinite selfinjective algebra such that all components of Γ_B are generalized standard. Then B is standard of polynomial growth by [**34**, Theorem 3.13] and so $B \cong \hat{A}/G$ by Theorem 3.3, where A is Euclidean or tubular and G is an admissible infinite cyclic group of K-linear automorphisms of $\hat{A}$. If C is stably equivalent to B then C is also a representation-infinite selfinjective standard algebra of polynomial growth by Lemma 8.2. Thus $C \cong \hat{A}_1/G_1$, where A_1 is Euclidean or tubular and G_1 is an admissible infinite cyclic group of K-linear automorphisms of $\hat{A}_1$. It is clear that A is Euclidean (respectively, tubular) iff A_1 is Euclidean (respectively, tubular). It is obvious by Corollary 5.7 that G, G_1 are stably admissible. Then we deduce from Proposition 8.5 that $\hat{A}$ and $\hat{A}_1$ are stably equivalent. Thus A, A_1 are tilt-cotilting equivalent algebras by **[3]** and they have the same number of pairwise non-isomorphic simple modules by **[17]**. Furthermore, if g is a generator of G and g_1 is a generator of G_1 then $\mathrm{st}_\Delta(g) = \mathrm{st}_\Delta(g_1)$. Consequently, B and C have the same number of pairwise non-isomorphic simple modules which finishes the proof. □

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