

# SOME PRELIMINARY OBSERVATIONS ON THE PLASTICITY OF GREENLAND GLACIERS

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**SUMMARY.** A series of deformation measurements have been selected for preliminary studies on the plasticity of Greenland glacier ice. The measurements to be reported on were obtained in the Red Rock and TUTO tunnels in north-west Greenland. Both tunnels were excavated during the summer of 1955 with some additional work done during the summer of 1956. Deformation measurements made up to the end of the 1956 summer season, therefore, are of limited reliability, but certain trends appearing in these data seem worth reporting. The topics discussed are (1) the shearing of an initially vertical peg system at Red Rock, (2) the deformation of core holes at TUTO, and (3) tunnel closure at both sites. These data are analysed on the basis of laminar flow of the Nye type and certain conclusions are derived.

**RÉSUMÉ.** Une série de mesures de déformation a été choisie pour servir de base à des études préliminaires sur la plasticité de la glace des glaciers du Groenland. Ces mesures ont été faites dans les tunnels de Red Rock et de TUTO dans le Nord-Ouest du Groenland. Ces deux tunnels avaient été creusés au cours de l'été 1955 et certains travaux supplémentaires avaient été exécutés au cours de l'été 1956. Pour cette raison les mesures de déformation faites avant la fin de la saison d'été 1956 n'ont qu'une validité assez relative mais certaines tendances indicatives semblaient dignes d'être mises en évidence. Les sujets traités sont (1) le cisaillement d'un système de chevilles originalement verticales à Red Rock, (2) la déformation des trous de carottage à TUTO et (3) la contraction des deux tunnels. On analyse ces données en se basant sur l'écoulement laminaire du type de Nye et on en tire certaines conclusions.

**ZUSAMMENFASSUNG.** Zum Vorstudium der Plastizität von Grönland-Gletschereis wurde eine Serie von Deformationsmessungen gewählt. Die Messungen, über die berichtet wird, wurden in den Red Rock und TUTO Tunnels in Nordwest-Grönland vorgenommen. Beide Tunnels wurden im Sommer 1955 ausgeschachtet und einige zusätzliche Arbeit wurde im Sommer 1956 geleistet. Deformationsmessungen, die bis zum Ende des Sommers 1956 gemacht worden waren, sind daher von begrenzter Zuverlässigkeit, es scheint jedoch der Mühe wert zu sein, gewisse Richtlinien, die aus diesen Daten hervorgehen, zu berichten. Folgendes wird besprochen: (1) die Scherung eines anfangs vertikalen Pegelsystems im Red Rock, (2) die Deformation von Bohrlöchern im TUTO und (3) die Stollenwände an beiden Seiten. Diese Daten werden auf der Grundlage des Laminar-Bewegung vom Typus Nye behandelt, und es werden gewisse Schlussfolgerungen gezogen.

## INTRODUCTION

Several deformation measurements have been selected for preliminary studies on the plasticity of Greenland glacier ice. The measurements reported here were obtained in the Red Rock and TUTO tunnels in north-west Greenland. Both tunnels were excavated during the summer of 1955 with some additional work done during the summer of 1956. Deformation measurements made up to the end of the 1956 summer season, therefore, are of limited reliability, but certain trends appearing in these data seem worth reporting.

## EXPERIMENTAL

Fig. 1 is a map of the area under study. A section view of the Red Rock tunnel is shown in Fig. 2. Measurements were made on the horizontal deformation of a vertical series of pegs placed in the pit wall. Eight pegs, 8 in. (0.2 m.) apart, were placed by Hilty of the Ohio State group 50 days before the horizontal deformations were determined (Goldthwait<sup>1</sup>). The bottom peg was 8 in. above the apparent floor of the glacier. Measurements were made by Hilty on the tunnel closure as measured by the rate of approach of wooden pegs frozen into the tunnel wall.

At TUTO (Fig. 3), closure was measured by the approach of wooden pegs installed in 1955 by Grosvenor and Rausch<sup>2</sup> and the approach of 2-in.-long (5 cm.) steel pegs placed in 1956 and measured over a 16-day period. Also, at TUTO, core holes that were circular in 1955 had become elliptical by 1956 and measurements were made on these.



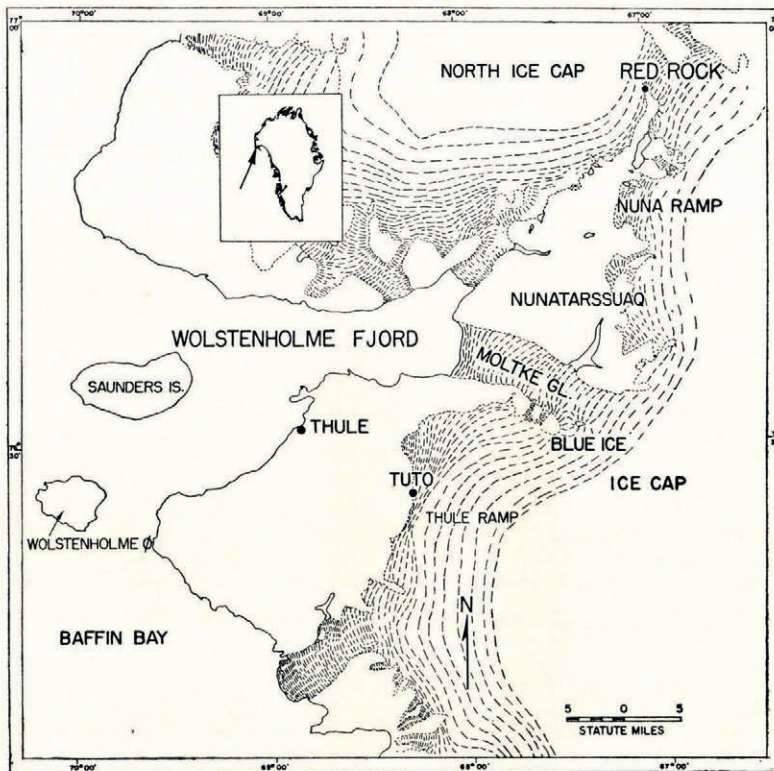


Fig. 1. Detail map of Thule area

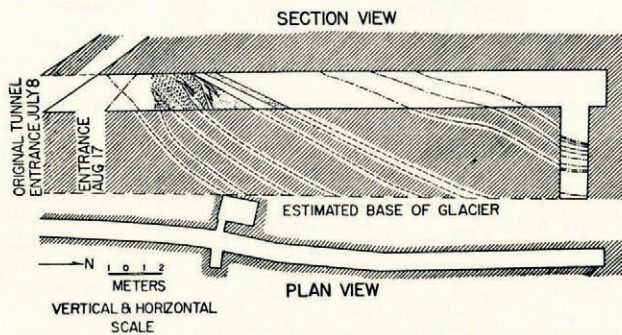


Fig. 2. Plan and longitudinal section of Red Rock ice tunnel for 1955

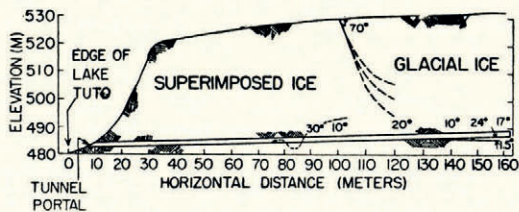


Fig. 3. TUTO tunnel for 1955. Dips of dirt bands are shown

## ANALYSIS

*The shear of glaciers.* Consider an element lying a vertical distance  $\zeta$  below the surface of a glacier with a small surface slope  $\alpha$  (Fig. 4). The vertical stress,  $\sigma$ , is

$$\sigma = \rho g \zeta, \quad (1)$$

and the maximum shear stress,  $\tau$ , for a Nye-type laminar flow is

$$\tau = \rho g \zeta \sin \alpha \cos \alpha \quad (2)$$

where  $\rho$  is the density of the ice and  $g$  is the acceleration due to gravity. Nye<sup>3</sup> proposed a shear law, based on Glen's<sup>4</sup> experimental results, in which the shear strain-rate in the direction of maximum shear stress,  $\dot{\gamma}$ , is

$$\dot{\gamma} = 2 \left( \frac{\tau}{A} \right)^n, \quad (3)$$

where  $n$  and  $A$  are constants equal to  $3.07$  and  $4.89 \times 10^8$ , respectively, and all quantities are in centimeter-gram-second units.

For laminar flow any layer glides with a velocity,  $U$ , which is the integral of the shear strain-rate,

$$U = U_H + \int_{\zeta}^H \dot{\gamma} d\zeta,$$

where  $U_H$  is the absolute velocity of the layer a distance  $H$  below the surface.  $H$  may be taken (but this is not necessary) as the total thickness of the glacier, in which case  $U_H$  is the velocity of slip on the glacier floor. From equations 2 and 3, the above becomes

$$U = U_H + \frac{H\dot{\gamma}_H}{n+1} \left\{ 1 - \left( \frac{\zeta}{H} \right)^{n+1} \right\}, \quad (4)$$

where  $\dot{\gamma}_H$  is the shear strain-rate at depth  $H$  from equation (3). For depths almost equal to  $H$ , equation 4 can be expanded to

$$\frac{U - U_H}{H - \zeta} = \dot{\gamma}_H \left\{ 1 - \frac{n}{2} \left( \frac{H - \zeta}{H} \right) + \frac{n(n-1)}{6} \left( \frac{H - \zeta}{H} \right)^2 - \dots \right\}. \quad (5)$$

At Red Rock a measurement over a 50-day period ending 18 August 1956 showed that the relative horizontal velocity of pegs placed in the wall of a vertical shaft increased with height above the floor. This floor was bouldery and had all the appearances of being the true glacier floor. Since the surface slope is about  $12.5^\circ$ , the observed horizontal velocities were divided by  $\cos 12.5^\circ$  to obtain the velocity in the direction of maximum shear stress. The data are given in Table I and shown in Figure 5. By extrapolating the velocity depth curve, it is seen

TABLE I. RED ROCK SHAFT

$H - \zeta$ Distance Up from Apparent Floor (cm.)	$U - U_H$ Velocity in Directions of Max. Shear Stress $\times 10^7$ (cm./sec.)	$\frac{U - U_H}{H - \zeta - 12}$ $\times 10^9$ (sec. <sup>-1</sup> )
0	0	—
20.3	1.66	20.0
40.6	5.70	19.9
61.0	9.00	18.4
81.3	13.5	19.5
101.6	16.8	18.7
122.0	19.5	17.7
142.3	22.2	17.1
162.6	24.2	16.1

that the velocity is zero at a height of about 12 cm. above the apparent floor. Perhaps these 12 cm. represent the depth of a local depression. Consistent results are obtained by assuming



that  $H-12$  cm. is the true floor. In Figure 5  $(U-U_H)/(H-\zeta-12)$  is plotted against  $H-\zeta$ . The value of  $\dot{\gamma}_{H-12}$  is the intercept at  $H-\zeta=12$  cm. and is equal to  $20.0 \times 10^{-9}$  sec.<sup>-1</sup>. Two ways to calculate  $n$  from these data are found from equations 3 and 5. The shear stress is known from the thickness ( $H=40$  m.), density ( $\rho=0.91$  g./cm.<sup>3</sup>), surface angle ( $\alpha=12.5^\circ$ ), and equation 2, and is about  $0.8 \times 10^6$  dyne/cm.<sup>2</sup>. Following Nye, the value of  $A$  in equation (3) is taken as  $4.9 \times 10^8$  c.g.s. units. Solving equation (3) for  $n$  gives  $n=2.79$ .

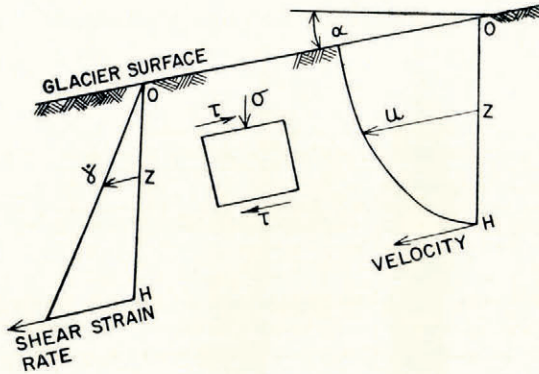


Fig. 4. Coordinate system used

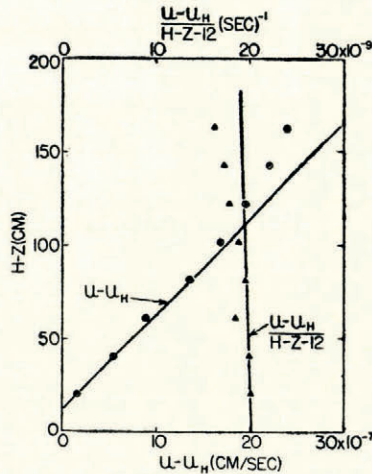


Fig. 5. Shear of peg system in Red Rock shaft

The second calculation is based on the limiting slope of  $(U-U_H)/(H-\zeta-12)$  vs  $H-\zeta$  as  $H-\zeta$  approaches 12 cm. From equation 5 this slope is  $-n\dot{\gamma}_{H-12}/2(H-12)$  and from Figure 5 equals  $-7 \times 10^{-12}$  sec.<sup>-1</sup> cm.<sup>-1</sup>. Again, using  $H=40$  m.,  $n$  becomes 2.8. These calculations for  $n$  depend on imperfectly known experimental data and the agreement between the two calculations may be less perfect than is indicated here.

The shear occurring in the TUTO tunnel can be inferred from the change in shape of horizontal core holes. These holes, drilled in the tunnel wall normal to the tunnel axis in 1955, were a few tens of centimeters deep with initial diameters of 11.1 cm. In 1956 the shape of the

holes appeared elliptical with the major axis roughly parallel to the direction of nearby dirt bands.

An analysis of this deformation can be made by considering the deformation of a circle undergoing a Nye-type laminar shear with simultaneous closure. For a circle with dimensions small compared to the ice thickness it can be seen from equation (5) that

$$\frac{dU}{dZ} = \dot{\gamma}_H, \quad (6)$$

where  $U$  is the velocity in the  $X$ -direction in the coordinate system of Figure 6. It will be sufficient to calculate the new shape considering no translation of the origin. Small deformations will be assumed. In the time,  $t$ , a point on the original circle will move from  $X_0, Z_0$  to

$$X = \frac{X_0 + \gamma Z_0}{C_1}, \quad Z = Z_0 / C_2.$$

Here

$$\gamma = \dot{\gamma}_H t \quad (7)$$

is the amount of shear in time  $t$ , and  $C_1$  and  $C_2$  are coefficients of closure in the directions of the  $X$ - and  $Z$ -axes.

The equation of the original circle is  $a^2 = X_0^2 + Z_0^2$ .

On deformation this becomes

$$a^2 = (C_1 X - C_2 \gamma Z)^2 + C_2^2 Z^2. \quad (8)$$

Transformation to the primed set of axes by a rotation,  $\theta$ , is effected by the equations

$$\begin{aligned} X &= X' \cos \theta - Z' \sin \theta \\ Z &= X' \sin \theta + Z' \cos \theta. \end{aligned} \quad (9)$$

Substituting equation (9) in equation (8), the deformed circle is expressed in the primed coordinate system as

$$\begin{aligned} a^2 &= X'^2 [C_1^2 \cos^2 \theta + C_2^2 (1 + \gamma^2) \sin^2 \theta - 2C_1 C_2 \gamma \sin \theta \cos \theta] + \\ &+ Z'^2 [C_1^2 \sin^2 \theta + C_2^2 (1 + \gamma^2) \cos^2 \theta + 2C_1 C_2 \gamma \sin \theta \cos \theta] + \\ &+ 2X'Z' [C_1 C_2 \gamma (\sin^2 \theta - \cos^2 \theta) + (C_2^2 \gamma^2 - C_1^2 + C_2^2) \sin \theta \cos \theta]. \end{aligned} \quad (10)$$

This is an ellipse whose principal axes are in the direction of the coordinate system when the coefficient of the mixed term is zero or

$$\tan 2\theta = \frac{2C_1 C_2 \gamma}{C_2^2 \gamma^2 - C_1^2 + C_2^2}. \quad (11)$$

The lengths of the semi-major and semi-minor axes are, respectively, the value of  $X'$  when  $Z'$  is zero and the value of  $Z'$  when  $X'$  is zero as calculated from equation (10).

According to Nye<sup>3</sup> (equation (18)) the stresses on a tunnel wall are twice as great in the circumferential as in the longitudinal direction. The circumferential stress, which corresponds to the  $Z$ -axis stress acting on the core hole, is also twice as large as the effective stress for radial closure. The effective radial stress is, therefore, equal to the longitudinal (or  $X$ -axis) stress, and the strain-rate for core-hole closure in the  $X$ -direction, in the absence of shear, would be identical to the strain-rate of closure of the tunnel. In the  $Z$ -direction the closure rate will be assumed to be double that of the tunnel. This assumption may need some revision at a later time but is probably sufficiently accurate for the present calculations.

The core holes studied were from positions about 500 ft. (150 m.) inside the tunnel where  $H \approx 50$  m.; the surface angle,  $\alpha$ , about  $4^\circ$ ; and the tunnel closure rate about 13% per year. From equation (2), the shear stress is  $0.31 \times 10^6$  dyne/cm.<sup>2</sup> and, using  $n=2.80$  and  $A=4.89 \times 10^8$  with equations (3) and (7), the shear in 1 yr. is  $\gamma=0.07$ . Derived from the tunnel closure rate,  $C_1$  is 1.13 and  $C_2$  is taken as 1.26.

The angle of rotation can now be calculated from equation (11) and is  $\theta=16.5^\circ$ . Origin-



ally, the diameter of the core hole was 11.1 cm. Using equation (10), the lengths of the major and minor axes of the ellipse are found (Table II).

TABLE II. CALCULATED AND OBSERVED MAJOR AND MINOR AXES FOR THE TUTO TUNNEL CORE HOLES

Axis	Calculated (cm.)	Observed (cm.)
$2a'$	9.9	9.8 → 10.0
$2b'$	8.7	8.0 → 8.9

To compare the observed and calculated rotation of the core holes, it must be noted that the calculated angle was measured from the direction of maximum shear stress and, therefore,  $4^\circ$  should be subtracted from this value to obtain the rotation from the horizontal. The

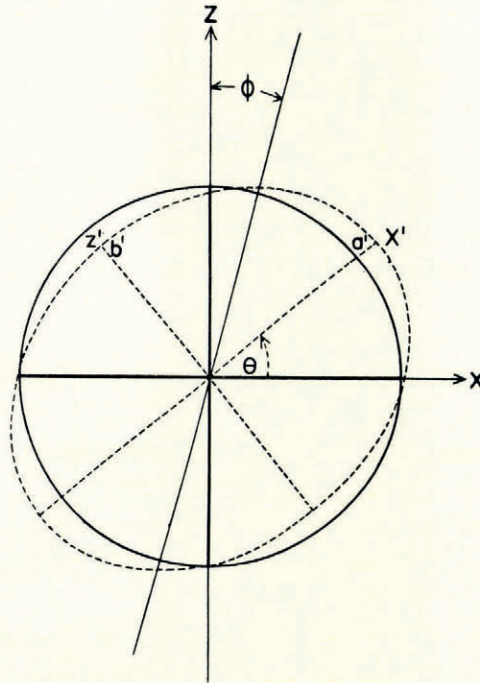


Fig. 6. Shear deformation of circular hole. Original radius is  $a$  and amount of shear is  $\tan \phi$

calculated rotation is then  $\theta = 12.5^\circ$ . This compares adequately with the observed dips, which are between  $10^\circ$  and  $17^\circ$ .

Observed and calculated angles of rotation and lengths of axes agree only for the limited range of values of  $n$  between about 2.75 and 2.85;  $n = 2.8$  was found to give the best values for  $\theta$  and was therefore used.

The observation that the core holes appear less elliptical with depth into the tunnel wall can be explained by the fact that the stress condition becomes more nearly hydrostatic away from the tunnel wall. Consequently,  $C_2$  approaches the value of  $C_1$  and the eccentricity decreases. For  $C_2$  equal to  $C_1$  there is still an eccentricity but for the small amount of shear at TUTO it becomes almost negligible. However, since tunnel closure is accompanied by a circumferential shortening,  $C_2$  will always be greater than one. An ice-filled core hole in a tunnel which is not stretching will have  $C_1 = 1.0$  and  $C_2 = 1 +$  the percentage tunnel closure.

*Tunnel closure.* The calculations for tunnel closure are based directly on Nye's equation<sup>3</sup>

$$\dot{\epsilon} = \left( \frac{\sigma}{nA} \right)^n \quad (12)$$

where  $\dot{\epsilon}$  is the closure strain-rate and  $\sigma$  is the hydrostatic stress. Using the observed closure rates and the value of  $A$  previously used,  $n$  can be calculated. The values of  $n$  are given in Table III.

TABLE III.  $n$  CALCULATED FROM TUNNEL DATA

	Tunnel	
	Red Rock	TUTO
$H$ Thickness of ice (m.)	40	50
$\sigma$ Hydrostatic pressure (kg./cm. <sup>2</sup> )	3.6	4.5
$\dot{\epsilon}$ Measured closure rate (sec. <sup>-1</sup> )	$5.5 \times 10^{-9}$	$4.1 \times 10^{-9}$
$n$ Calculated from closure	3.15	3.25
$\alpha$ Surface angle (deg.)	12.5	4
$\tau$ Shear stress (kg./cm. <sup>2</sup> )	0.8	0.31
$n$ Calculated from shear	2.80	2.80

### CONCLUSIONS

The calculations presented here are approximate in most cases and care should be exercised in applying them over wide ranges. It should be noted that the simple treatment, used here, of adding shear motion to tunnel motion would be strictly true only for a linear flow law. Since the flow law is non-linear a certain error is to be expected.

Uncertainty in the experimental data can be expected to limit the reliability of the calculated results. However, very large changes in motion are reflected by very small changes in  $n$ . It is, therefore, unlikely that the calculated values of  $n$  are far from those that might be calculated from more precise data.

The assumption that  $A$  is a constant and that  $n$  varies is somewhat arbitrary, but is based on the knowledge that the densities, grain sizes and orientations, and temperatures of the ice from the two tunnels are similar. Actually, the ice temperature at Red Rock is about  $-11^\circ$  C. while at TUTO it is about  $-8^\circ$  C. This difference should be small insofar as plasticity is concerned.

The differences in the calculated values of  $n$  can be understood by considering that shear stresses are less than 1 kg./cm.<sup>2</sup> while the hydrostatic stresses used in the closure calculations are about 5 kg./cm.<sup>2</sup>. The use of a simple power law for the flow equation is an over-simplification. Glen's experimental data<sup>4</sup> confirm this in the low stress region where lower values of  $n$  could well be used. Jellinek and Brill<sup>5</sup> also observed a flow law changing from linear at low stresses to a higher power at stresses greater than about 1 kg./cm.<sup>2</sup>.

The use of both closure and shear data in tunnels leads to a better understanding of the flow law, for at any position the strains resulting from two stresses of considerably different magnitudes are obtained. Observations at positions of various ice thicknesses and surface slopes can lead to a more detailed flow law.

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