And generally since

$$\{a+(n-1)d\}r^{n-1}=[\{a+(n-2)d\}r^{n-2}+dr^{n-2}]r$$

we see that each segment on OP is got from the preceding segment by adding to it the appropriate segment of the G.P. D_1D_2, D_2D_3,... and multiplying the sum by r.

Sum to affinity.

Only D_1, O, P_1, H, D, T, P need be entered in the figure.

Now $$DO=\frac{d}{1-r} \text{ and } DP_1=a+\frac{d}{1-r}$$

$$\therefore \quad OP=\frac{DP_1}{1-r}=\frac{a}{1-r}+\frac{d}{(1-r)^2}.$$

Again OP is finite when DO is finite, that is when $|r|<1$. We thus have a visual proof of the limit theorem:
if $|r|<1$

$$nr^n \to 0 \text{ when } n\to\infty.$$

The cases d or r negative require no modified construction or proof, as the above are quite general if the sign convention be applied.

Exactly analogous extensions apply to the constructions of Mr R. M. Milne (§ 291) and Mr F. J. W. Whipple (§ 292) in the *Mathematical Gazette*, 1909–11, p. 138.

G. D. C. STOKES.

Note on Rational Right-Angled Triangles whose Legs are consecutive Whole Numbers.—Having given the sides of a rational right-angled triangle, to find from *them* the sides of other rational right-angled triangles.

Put a, b, c for the sides of the given right-angled triangle; then, of course,

$$a^2+b^2=c^2 \qquad \text{.................................(1)}$$

Let $x+a$, $x+b$, and $2x-c$ denote the sides of the triangle sought; then

$$(x+a)^2+(x+b)^2=(2x-c)^2 \qquad \text{......................(2)}$$

Expanding and reducing, we get from (2)

$$x=a+b+2c,$$

remembering that $a^2+b^2=c^2$.

Hence
$$x+a=2a+b+2c,$$
$$x+b=a+2b+2c,$$
and
$$2x-c=2a+2b+3c$$
are the sides of the triangle sought; for
$$(2a+b+2c)^2+(a+2b+2c)^2=(2a+2b+3c)^2 \quad\ldots\ldots\ldots\ldots(3)$$
identically when $a^2+b^2=c^2$.

If we put $a+1=b$ in (3) we get
$$(3a+2c+1)^2+(3a+2c+2)^2=(4a+3c+2)^2 \quad\ldots\ldots\ldots\ldots(4),$$
which is an identity when
$$a^2+(a+1)^2=c^2.$$

Take $a=3$; then $a+1=4$, $c=5$, and (4) gives

20, 21, 29

for the *second* right-angled triangle whose legs differ by unity.

Take $a=20$; then $a+1=21$, $c=29$, and we have from (4)

119, 120, 169

for the *third* triangle of this kiad.

Take $a=119$; then $a+1=120$, $c=169$, and (4) gives

696, 697, 985

for the *fourth* triangle whose legs are consecutive whole numbers.

Take $a=696$; then $a+1=697$, $c=985$, and we get from (4)

4059, 4060, 5741

for the *fifth* of such triangles. And so on.

And *generally*, if a_n, a_n+1, c_n be the sides of the n^{th} right-angled triangle whose legs are consecutive whole numbers, then the sides of the next or $(n+1)^{\text{th}}$ triangle are
$$3a_n+2c_n+1,$$
$$3a_n+2c_n+2.$$
$$4a_n+3c_n+2.$$

See *Mathematical Magazine*, Vol. II., No. 12, Part 2, p. 322, for a table of the first 40 rational right-angled triangles whose legs are consecutive whole numbers. The sides of the 40th triangle are

25279618814781699610480329 63696,

25279618814781699610480329 63697.

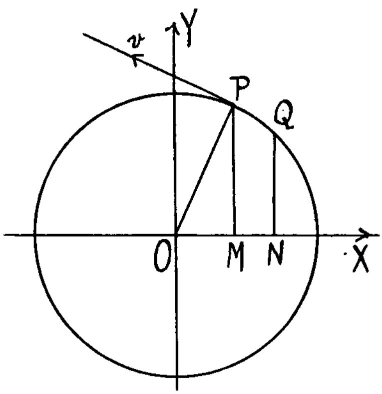
35750779779486346273940466 18865.

See *Analyst*, Vol. III., No. 2 (1876), p. 49, for the sides of the 80th triangle; also *Mathematical Visitor*, Vol I., No. 3 (1879) p. 56, and No. 5 (1880), p. 122, for sides of the 100th triangle.

ARTEMAS MARTIN.

Formula for Centrifugal Force.—The object of this note is to suggest finding the formula for centrifugal force as an exercise on space-rate of change of energy.

Let the uniform circular motion be that of a particle of mass m travelling with speed v in a circle centre O and radius r, and let the motion be regarded as resolved into two linear motions with reference to rectangular axes X′OX, Y′OY. Let MP be the ordinate of any point P on the circle.



Then

$$x\text{-component of velocity at P} = v \sin \text{XOP} = \frac{v \cdot \text{MP}}{r}.$$

Therefore

$$\text{kinetic energy of } x\text{-linear motion at P} = \tfrac{1}{2}m \cdot \frac{v^2 \cdot \text{MP}^2}{r^2}$$

Similarly, if Q be a point on the circle near to P,

$$\text{kinetic energy of } x\text{-linear motion at Q} = \tfrac{1}{2}\frac{mv^2}{r^2} \cdot \text{NQ}^2.$$