

BOERNER, H., *Representations of Groups with Special Consideration for the Needs of Modern Physics*, translated by P. G. MURPHY in co-operation with J. MAYER-KALKSCHMIDT and P. CARR (North Holland Publishing Co., Amsterdam, 1962), xii+325 pp., 80s.

Algebraists who find the German language difficult or irksome to read will welcome this translation of Professor Boerner's fine book which was first published in Springer's "yellow back" series in 1955. The translators have given a competent and accurate rendering and only very occasionally is one aware of the construction of the German sentence which lies behind the translation. The pitfalls confronting the translator are many and the reader will smile at rather than criticise the few into which the present translators have stumbled. For instance the Bibliography refers to the book by M. Hameresh (1962) as one of the older books on the subject. Again, *Gruppenkeim* is translated in the text as *local Lie group* but in the index as *group embryo*. The phrase "restrict things to the subgroup" which occurs in the two branching theorems is unpleasant and this lapse is not attributable to the German version. This instance is not, however, typical of the translation. Many of the footnotes in the German edition appear to the reviewer to have been inserted by way of explanation at the proof stage. It is a pity that the opportunity was not taken of incorporating these in the text of the English edition. Apart from four pages on Freudenthal's treatment of the spin representations of the rotation group there is no additional material in the English version though in a few places the argument has been modified.

This book is the best presentation of the theory of matrix representations of groups to date and it assumes very little previous knowledge on the part of the reader. Preliminary chapters deal with the relevant portions of the theory of groups and matrices. The general representation theory is followed by a detailed treatment of the symmetric group, the full linear group, the real linear group, the unimodular group, the real unimodular group, the unitary group, the unimodular unitary group, the orthogonal group, the rotation group and the Lorentz group.

These are the groups of greatest interest to physicists whose requirements have been specially considered in the construction of the book. The volume does not, however, include actual applications of the theory to physical problems. The pure mathematician, on the other hand, will regret that no account is given of the theory of modular representations though the author can justly claim that this theory lies outside the scope of his book.

The printing of the text is excellent, but the same cannot be said of some of the diagrams, in particular figures 11, 12 and 15, which appear to have been reduced excessively from the drawings. No actual misprints were observed but a few letters appear to have dropped out of the type here and there.

The book will form a valuable addition to the algebraist's library.

D. E. RUTHERFORD

GOLDBERG, S. I., *Curvature and Homology* (Academic Press, 1962), xvii+315 pp., 68s.

This book is a graduate text and the reader is assumed to have some knowledge of Riemannian geometry, Lie groups, and the elements of analytic and algebraic topology. It is essentially a monograph which collects together relevant material available elsewhere in books, in collections of lecture notes and in scattered research papers. The contents are indicated by the headings of the chapters and appendices: I. Riemannian manifolds; II. Topology of differentiable manifolds; III. Curvature and homology of Riemannian manifolds; IV. Compact Lie groups; V. Complex manifolds; VI. Curvature and homology of Kaehler manifolds; VII. Groups of transformations of Kaehler and almost-Kaehler manifolds; (A) de Rham's theorems; (B) The cup product; (C) The Hodge existence theorem; (D) Partition of unity.

Each chapter ends with exercises which test the understanding of the text and indicate extensions of the theory. A very substantial number of topics are dealt with, and the treatment of some is, perhaps necessarily, somewhat sketchy. Some readers would no doubt prefer a more intrinsic treatment of parts of the book, especially Chapter IV; moreover, the book contains a number of inaccuracies. However, the author has succeeded in giving the reader a good commentary on the subject as a whole, and a useful list of references to original sources.

T. J. WILLMORE

LANG, S., *Introduction to Differentiable Manifolds* (John Wiley & Sons, 1962), vii+126 pp., 53s.

The purpose of this book is to fill the gap which exists in the literature dealing with that branch of mathematics which borders on differential topology, differential geometry and differential equations. The essential contribution of this book is to show that nothing is lost in clarity of exposition if a manifold is defined by means of charts of Banach or Hilbert Spaces rather than finite dimensional spaces. In fact, the claim is that there is a positive gain from the indiscriminate use of local coordinates  $x_1, \dots, x_n$  and their differentials  $dx_1, \dots, dx_n$ . Moreover such a treatment is necessary when dealing with infinite-dimensional spaces, and there is every indication that the systematic introduction of infinite-dimensional topological spaces will have successful results in the theory of differentiable manifolds.

Chapter I gives a brief resumé of differential calculus, following the viewpoint of Dieudonné's *Foundations of Modern Analysis*, Chapter VIII. Chapter II describes manifolds by means of charts of Banach spaces. Chapter III describes vector-bundles, and exact sequences of bundles. Chapter IV, on "Vector fields and differential equations", collects a number of results which make use of the notion of differential equations and solutions of differential equations. In particular, there is an interesting account of "sprays". Chapter V is about differential forms, exterior differentiations and the Poincaré Lemma. Chapter VI gives a proof of a generalisation of the Frobenius Existence Theorem. Chapter VII, about riemannian metrics, shows how a riemannian metric determines a spray and hence geodesics. In this chapter use is made of the standard spectral theorem for (bounded) symmetric operators, and a proof of this theorem is given in Appendix I. Although his treatment avoids the use of local coordinates, the author recognises that in the finite-dimensional case, they constitute an effective computational tool. In Appendix II he interprets differential forms, sprays and the riemannian spray in terms of these local coordinates.

I think that few readers will find the book easy reading, even though it is largely self-contained. There is little doubt, however, that this is an important contribution to the literature, and that it will have an important influence on workers in the fields of differential topology, differential geometry and differential equations.

T. J. WILLMORE

WILLIAMSON, J. H., *Lebesgue Integration* (Holt, Rinehart and Winston, London, 1962), viii+117 pp., 26s.

Although this book was conceived by its author as an introduction to more advanced texts on measure and integration, it is not aimed, as for example is J. C. Burkill's Cambridge tract on the subject, towards readers who may have no wish to plumb the depths of the theory of real functions; it is a book on Lebesgue integration for those who have an interest in functional analysis, and in this field it is undoubtedly a good book. The treatment is in the general setting of  $n$ -dimensional Euclidean space.