THE GEODYNAMO

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Abstract. A comparison is made of the differences and similarities between the solar dynamo and the geodynamo. Special attention is paid to the energetics, the dynamics, the role of turbulence, and the α- and ω-sources in these dynamos.

1. Introduction

This meeting is focussed on dynamos operating in systems of enormous scale, a large part being devoted to the creation of galactic magnetic fields. The Sun is about the smallest body under serious consideration! The theory of the solar dynamo appears, at the moment and in a certain sense that will be developed below, to be converging to that of the Earth's dynamo. I thought therefore that I should try to persuade those who delve into the grander scales of cosmic magnetism, and who have not in the past had much desire or need to acquaint themselves with the magnetism of tiny Earth, to invest a little time thinking about the geodynamo, and perhaps as a result find useful parallels with their subjects. I shall therefore try to contrast the dynamos operating in the Sun and Earth and, since I shall be using those words so frequently, I shall usually abbreviate them by the letters 'S' and 'E'.

2. The Power and the Turbulence

There are of course many striking physical differences between E and S. Perhaps the salient difference between their dynamo mechanisms is the availability of energy. Stix (private communication) estimates that, of the $L_\odot \approx 4{\cdot}10^{26}W$ of power emitted by S, only $P_\odot^{Joule} \approx 10^{22}W$ is diverted into Joule heating; thus $P_\odot^{Joule}/L_\odot \approx 3{\cdot}10^{-5}$. Of the $4.2 \cdot 10^{13}W$ energing from E, the part emanating from E's core has been estimated (within a factor of 4 either way) to be $L_\oplus \approx 3 \cdot 10^{12}W$ (see Loper and Roberts, 1983, Table 2); today, this figure is thought to be a minimum (see e.g. Loper and Roberts, 1992). The Joule expenditure of the geodynamo depends on the (unobservable) strength of the toroidal field, $\mathbf{B}_T$, in E's core; Loper and Roberts (1983, Eq. 4.14) give $P_\oplus^{Joule} \approx 10^{15}(B_T/T)^2W$. If we take $B_T \approx 0.02T$, we obtain $P_\oplus^{Joule} \approx 4 \cdot 10^{11}W$ so that $P_\oplus^{Joule}/L_\oplus \approx 0.1$. Despite considerable uncertainties in these estimates, it is clear that E has an energy problem that S does not share. S can drive its dynamo almost incidentally as a by-product of the violent and turbulent thermal convection in the solar convection zone (SCZ), but it is very doubtful if thermal convection in E's core can be of sufficiently high thermodynamic efficiency to explain E's dynamo (Braginsky, 1963, 1964; Metchnik et al., 1974; Hewitt et al., 1975; Backus, 1975; Gubbins et al., 1979; Verhoogen, 1980). E's dynamo is therefore usually thought to be powered by compositional convection, through the release and flotation of the light components of core alloy as the heavy components freeze onto the inner core surface during the general cooling of the Earth (Braginsky 1963, 1964; see Loper and Roberts, 1983).

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It might at first sight be thought that the tightness of E's energy budget could be eased by accepting a smaller value for B_T, e.g. that $\mathbf{B}_T$ has much the same strength as the poloidal field $\mathbf{B}_p$, and that therefore the Earth is a weak field dynamo, but in fact such an assumption would merely replace one difficulty by another. A strong toroidal field assists convection in E's core by countering the constraint imposed by E's rotation. According to the classical theory of rotating fluids (e.g. Greenspan, 1968; Roberts and Soward, 1978), a body of fluid is rotating rapidly if its Ekman number, $Ek = \nu/\Omega\mathcal{L}$, and Rossby number, $Ro = \mathcal{U}/2\Omega\mathcal{L}$, are both small, where ν is kinematic viscosity, Ω is the angular velocity, and $\mathcal{U}$ and $\mathcal{L}$ are characteristic of the velocity and length scales. Taking $\nu_\oplus = 10^{-6}m^2s^{-1}$, $\Omega_\oplus = 7 \cdot 10^{-5}s^{-1}$, $\mathcal{U}_\oplus = 10^{-4}ms^{-1}$ and $\mathcal{L} \approx 3.48 \cdot 10^6 m$ (the radius of the core), we obtain $Ek_\oplus \approx 10^{-15}$ and $Ro_\oplus \approx 3 \cdot 10^{-7}$. Viscous and inertial forces are both small compared with the Coriolis force. Such a system does not convect easily. If, however, the Elsasser number, $\Lambda = \sigma\mathcal{B}^2/2\Omega\rho$, where σ is electrical conductivity, ρ is density and $\mathcal{B}$ is a typical field strength, is $O(1)$, the viscosity and Ekman number become largely irrelevant. In this case the Lorentz forces are of similar magnitude to the Coriolis force and they release the convection from the rotational constraint, i.e. convection becomes easier. If we take $\sigma_\oplus = 3 \cdot 10^5 Sm^{-1}$, $\rho_\oplus = 10^4 kg\ m^{-3}$ and $\mathcal{B} = 0.02T$, we obtain $\Lambda_\oplus \approx 100$. The inertial forces also have no substantial role to play ($Ro_\oplus \ll 1$). E is highly magnetic.

If we attempt to use the same argument for the SCZ and take $\nu \approx 10^{-4}m^2s^{-1}$ as representative of the radiative or molecular viscosity, $\Omega_\odot = 2.74 \cdot 10^{-6}s^{-1}$ (the angular velocity of the radiative core, see below), and $\mathcal{L}_\odot = 2 \cdot 10^8 m$ (the depth of the SCZ), we obtain $Ek_\odot \approx 10^{-15}$. The available power is so great, however, that it drives vigorous turbulence, and it is the associated inertial forces that break the constraint of rotation. If we base $\mathcal{U}_\odot = R_\odot\Delta\Omega \approx 600ms^{-1}$ on the difference $\Delta\Omega = \Omega^{equator} - \Omega^{poles}$ in angular velocities of equator and poles, we obtain $Ro_\odot \approx 1$. Similar magnitudes are found to be typical of the supergranulation ($Ro_\odot \approx 3$), but for the granulation $Ro_\odot \approx 300$. S, as measured by Ro, is not a rapidly rotating body. We assess its Elsasser number below, to decide whether it is highly magnetic or not.

Except near its upper and lower extremities, the energy flux in the SCZ is mainly convective. In principle, motions on all scales contribute to this process from the giant cells to the granulation. In view of the lack of observational evidence for giant cells, it seems prudent here to assume that smaller cells, having dimensions of the order of $l = 2 \cdot 10^7 m$ comparable with the scale height, are mainly responsible for the outward flux $F_r = L/4\pi r^2$, where r is distance from the solar centre. Taking a point at depth $10^8 m$ in mid-SCZ as representative, we obtain $F_{r\odot} \approx 10^8 Wm^{-2}$. Mixing length theory $\left[\text{with } \rho C_p(\nabla - \nabla_{adiabatic}) = 10^6 Jm^{-3}\right]$ gives an rms turbulent velocity of $v \approx 100ms^{-1}$. The corresponding turbulent diffusivity $D = \frac{1}{3}vl$ is approximately $10^9 m^2s^{-1}$, which greatly exceeds the kinematic viscosity assumed above. If we now insert $\nu_\odot = 10^9 m^2s^{-1}$ into our earlier definition of Ek, we obtain $Ek_\odot \approx 0.01$. Moreover, we must expect that the turbulent magnetic diffusivity will also be $O(D)$, i.e. $\eta_\odot^{turb} = 10^9 m^2s^{-1}$.

In the SCZ, the Elsasser number based on η^{turb}, i.e. $\mathcal{B}^2/2\Omega\mu\rho\eta^{turb}$ where μ is the magnetic permeability, is $145(\mathcal{B}^2/\rho)kg\ m^{-3}T^{-2}$, which works out at $60(\mathcal{B}/T)^2$

near the base of the SCZ, $100(\mathcal{B}/T)^2$ at a depth of $10^8 m$ and $5 \cdot 10^4(\mathcal{B}/T)^2$ near the surface, at a depth of $10^6 m$, these large variations mirroring similar variations in ρ. Clearly, Lorentz forces are likely to be significant near the surface of S (and, indeed, they dominate above the photosphere!). They would be significant at all depths if $\mathcal{B} \approx 0.1T$, or greater. We return to this point later but merely note here that $\eta_\oplus^{turb} = 10^9 m^2 s^{-1}$ implies an Ω-effect magnetic Reynolds number, $R^\Omega = \Delta\Omega\mathcal{L}^2/\eta$, of $R_\odot^\Omega \approx 100$, which suggests that, if α is large enough, a mean-field $\alpha\omega$-dynamo can function throughout the SCZ, a view of the solar dynamo that generally prevailed a decade ago. Although it prevails no longer (see below), the general opinion is nevertheless that the solar dynamo is of $\alpha\omega$-type. Some corroboration is provided by results of numerical integrations of such kinematic dynamos; they create, as S does, fields that oscillate with characteristic periods of order τ_η, where $\tau_\eta = \mathcal{L}^2/\eta$ is the electromagnetic diffusion time. Using values given above, $\tau_{\eta\odot} \approx 1$ year, which is perhaps as close to the observed 22 year period as one can hope to obtain from the present rough order-of-magnitude estimates.

In the case of E, we take $\sigma_\oplus = 3 \cdot 10^5 Sm^{-1}$, so that $\eta = (\mu\sigma)^{-1}$ is $\eta_\oplus \approx 3m^2 s^{-1}$, and we obtain $R_\oplus^\Omega \approx 100$ (where we have taken $\Delta\Omega = \mathcal{U}/\mathcal{L}$). The large value $R_\oplus^\Omega$ (and the nature of core dynamics, see below) suggests that the geodynamo is also of $\alpha\omega$-type. The electromagnetic diffusion time of $E's$ core is $\tau_{\eta\oplus} \approx 10^5$ years. The frequency of reversals, although varying greatly over geological time, appears to be significantly less than (and, in some epochs, *much* less than) $\tau_{\eta\oplus}^{-1}$. If so, it must be an $\alpha\omega$-dynamo of a different, and basically steady, type from that operating in S. One idea is that meridional motions within E's core are more significant than those in S. (They are known to be small on the surface of S.) Numerical integrations of kinematic $\alpha\omega$-models usually show that a sufficiently large and suitably directed meridional motion can transform an otherwise oscillatory dynamo into a steady one. But perhaps the $\alpha\omega$-dynamo in E's core is different in another important respect: perhaps it not a turbulent dynamo at all?

Few would care to argue that the flow in $E's$ core is laminar, and indeed the spectrum of geomagnetic field variations is broad-band. Nevertheless, E has such a tight energy budget that turbulence in its core would be a luxury. Suppose that power were available to maintain as vigorous a turbulence, so that, as in the SCZ, $\eta^{molecular} \approx 10^{-11}\eta^{turb}$. Then the electromagnetic decay time, $\tau_{\eta\oplus}$, of the mean magnetic field seen on E's surface would be only of the order of a minute. Even though short period variations are screened out by $E's$ mantle, the observations suggest that $\tau_{\eta\oplus}$ is not greatly shortened by turbulence in the core. For example, palaeomagnetic studies have indicated that the time taken by the geomagnetic field to change its polarity during a reversal is of order 10^4 years. Such results strongly indicate that the total magnetic diffusivity, $\eta^T = \eta^{molecular} + \eta^{turb}$, does not much exceed $\eta^{molecular}$, and that the energy of the fluctuating fields in $E's$ core are not large compared with that of the mean field. In the classical turbulence of incompressible fluids (which of course cannot really be legitimately applied to $E's$ rapidly rotating, highly magnetic core), the rms fluctuating mean velocity, v, is $O((\epsilon\nu)^{\frac{1}{4}})$, where ϵ is the rate of viscous dissipation of energy per unit mass. If we take $P_\oplus^{Joule} = 10^{11} W$ as being representative of, or probably more significant than, the unknown dissipation by viscosity ($P_\oplus^{visc}$), we obtain $\epsilon_\oplus \approx 3 \cdot 10^{-15} m^2 s^{-3}$, and taking $\nu_\oplus = 10^{-6} m^2 s^{-1}$ as before we find $v_\oplus \approx 10^{-5} m s^{-1}$, or about a tenth of the mean velocity we assumed earlier. In the same spirit, we might take $l \approx (\eta^3/\epsilon)^{\frac{1}{4}}$, which gives $l_\oplus \approx 10^4 m$, so that $\eta_\oplus^{turb} = \frac{1}{3} v_\oplus l_\oplus \approx 0.03 m^2 s^{-1} \approx 0.01 \eta_\oplus^{molecular}$.

Perhaps these strands, taken together, indicate that the irregularities of motion in $E's$ core are not caused by "macho" turbulence of the classical cascading type, such as arises in the SCZ.

Braginsky and Meytlis (1990) have developed a theory of turbulence that is tailor-made to fit conditions in $E's$ core. They argue that the geomagnetic field is maintained by motions of a scale comparable with the core radius. These motions are much affected by Coriolis forces and so create the required α-effect (see §2). Highly anisotropic turbulence exists, but is on too weak, and on too small, a scale to contribute significantly to either α or η; it is fed directly by the unstable density stratification in the core, rather than by cascade of energy from the macroscales. This "wimp" turbulence greatly enhances the effective diffusivities of heat and of the light component of composition for which the molecular values are $\kappa^{heat} \approx 10^{-5}\eta$, $\kappa^{composition} \approx 10^{-10}\eta$, respectively. Although the turbulent diffusion of these quantities are highly anisotropic, their greatest values are, according to Braginsky and Meytlis, of order η. Their estimates of v are of the same order and smaller than the value of $U_\oplus$ adopted above.

3. Large-scale and Small-scale Instabilities

Another marked difference between E and S arises from their very different compressibilities. The SCZ spans 9 density scale-heights, but the bottom of $E's$ fluid core is only 10% denser than its surface. Thus convection in $E's$ core is commonly considered in a Boussinesq approximation ($\nabla \cdot \mathbf{V} = 0$ and $\rho \approx \rho_c =$ constant), while that in S is studied using the anelastic approximation [$\nabla \cdot (\rho_0 \mathbf{V}) = 0$ where ρ_0 is the isentropic, hydrostatic reference state]. Magnetic buoyancy, which is very significant in the SCZ is irrelevant in $E's$ core.

The geomagnetic field is most probably generated by large-scale convective instabilities, that are much affected by Lorentz forces and rotation, and the acronym MAC, standing for '$\underline{M}$agnetic', '$\underline{A}$rchimedean' (i.e. buoyant) and '$\underline{C}$oriolis', is often employed to describe them. This emphasizes that these forces are key ingredients in their dynamics, along with pressure forces but not the viscous and inertial forces ($EK_\oplus \ll 1$ and $Ro_\oplus \ll 1$, see above). The basic state is taken to be zonal:

$$\mathbf{V}_0 = V_0(s,z)\mathbf{1}_\phi, \quad \mathbf{B} = B_0(s,z)\mathbf{1}_\phi, \quad C_0 = C_0(s,z), \tag{1}$$

where $C = \delta\rho/\rho_0 = (\rho - \rho_0)/\rho_0$ is the fractional density excess, creating buoyancy, $\mathbf{1}_q$ is the unit vector in the direction of q-increasing, and (s,ϕ,z) are cylindrical coordinates, with $\mathbf{1}_z = \boldsymbol{\Omega}/\Omega$; $\underline{V}_0$, B_0 and C_0 may depend on t but, since they apparently vary slowly on the τ_η time scale in the case of E, they are usually taken to be constant. The α-effect is created by finite amplitude waves riding on the basic state (1). The characteristic time scale of these waves is the so-called "slow time scale": $\tau_s = 2\Omega\mu\rho_0/\mathcal{B}^2 = \tau_\eta/\Lambda$, which in the case of E is reminiscent of the observed time scale ($10^2 - 10^3$ years) of the geomagnetic secular variation. The waves and instabilities are usually analysed in the *magnetostrophic approximation*, in which

$$2\rho_c \boldsymbol{\Omega} \times \mathbf{V} = -\nabla p + \mathbf{J} \times \mathbf{B} + \rho_0 C \mathbf{g}_e, \tag{2}$$

where $\mathbf{J} = \nabla \times \mathbf{B}/\mu_0$ is the electric current density, and $\mathbf{g}_e$ is the effective acceleration due to gravity (with centrifugal forces included). Braginsky (1967) developed an elegant theory of small amplitude MAC waves and instabilities using a formalism devised by Frieman and Rotenberg (1960). This theory ignored all forms of dissipation, i.e. it treated the core as an ideal fluid; the resulting α was identically zero. This unrealism was removed by Braginsky and Roberts (1975) who generalized the analysis to include a finite magnetic resistivity and obtained nonzero α. Braginsky (1980) used these techniques to study both ideal and resistive instabilities. It is not difficult to restore inertial forces to the Frieman-Rotenberg theory; nor is it hard, at a price, to generalise the Braginsky-Roberts approach to include all the turbulent diffusivities. The large variation in ρ across the SCZ creates very severe complications however, even in the anelastic approximation, and there was until recently little incentive for applying these methods to S.

For some as yet unfathomed reason, the solar internal rotation, $\Omega(r, \theta)$ is approximately independent of r throughout most of the SCZ while, in contrast, the radiative zone (RZ) rotates as a solid body (although, strictly speaking, we are not entitled to make such a strong statement; helioseismology is as yet unable to determine Ω reliably for $r < 0.4R_\odot$, approximately). There must, therefore be very large radial gradients in $\Omega(r, \theta)$ in a layer near the base of the SCZ, and this has in fact been confirmed helioseismologically, although the thickness, δ of the layer has not been resolved - probably it is at most 0.3 pressure scale heights. We will here take it to be $\delta = 10^7 m$, and following Spiegel and Zahn (1992) we shall call it the *tachocline*. The first question one must answer about the tachocline is why it exists at all. Why does the shear not spread out into the SCZ and into the RZ? Presumably it does not spread into the SCZ because the Reynolds stresses that create the r-independent Ω in the SCZ are too strong (see Rüdiger 1989), and return the upward flux of angular momentum to the tachocline. In the so-called overshoot region, however, where the turbulent intensity begins to diminish with depth, the turbulent stresses become ineffective and cannot obliterate the radial shear in Ω.

The question of why Ω does not spread into the RZ is much harder to answer. The very fact, that the RZ rotates at a uniform angular velocity, $\boldsymbol{\Omega}_c$, intermediate between the values of $\Omega(0)$ and $\Omega\left(\frac{1}{2}\pi\right)$ just above the tachocline, strongly indicates that there is an effective coupling mechanism between the bulk of the SCZ and the bulk of the RZ. But the RZ is fluid, and such stresses should engender in it not a solid body rotation but a differential shear V_0; in fact, we may expect that Ω should be constant on each member of a family of axisymmetric surfaces, but would differ from one surface to the next. Because of the complications of self-gravitation, we shall not attempt a full analysis here, but try to get a little feeling for the situation. Suppose that the differential motion in the RZ is small and transform to a reference frame rotating with angular velocity $\boldsymbol{\Omega}_c$. Then we may write

$$2\rho_0 \boldsymbol{\Omega}_c \times \mathbf{V} = -\nabla p + \rho_0 \mathbf{g}_e. \tag{3}$$

If, for simplicity, we take $\rho_0 = \rho_0(r)$ and $\mathbf{g}_e = g_e(r)\mathbf{r}/r$, we easily see that $\rho_0 V_\phi$

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is constants on cylinders, i.e. that $\rho_0(r)\Omega(s,z) = F(s)$ for some F. In particular, Ω cannot be constant. The real situation is more complicated, but this gives the flavour: the solution again depends on an arbitrary function F that cannot be determined without the addition of more physics. The fact that a differential shear is not observed in acoustic sounding of the RZ suggests that there must be a very effective mechanism linking the different surfaces together, perhaps a magnetic field, as suggested by Spruit (1990). Such a field, even if only of order $10^{-4}T$, effectively couples the surfaces together, so giving the RZ an effective rigidity that causes it to respond to the applied stresses as a solid body. It is hard to believe that the RZ does not contain *some* (primordial) magnetic field of at least this size; see Mestel and Weiss (1987). An alternative possibility is that differential rotation in the radially stratified RZ would set up anisotropic, horizontally-dominated turbulence that would enforce uniform rotation on spherical surfaces. It would then follow from (3) that Ω is constant in the RZ; see Zahn (1993) and also Spiegel and Zahn (1992), where a detailed theory of the tachocline is developed. They argue that the observations are compatible only with a model in which the horizontal viscous transport is dominant, despite the thinness of the tachocline. Their theory requires pole-equator temperature differences to be present in the tachocline (see below).

It is perhaps "lucky" for the existence of the solar dynamo that the tachocline is somehow maintained! The large shears within it engender a strong Ω-effect that winds up the poloidal field to produce a toroidal field whose maximum strength may be as great as 1T (or even larger). To explain the progression of magnetic activity towards the equator, it is necessary to invoke an α that is predominantly negative (positive) deep in the northern (southern) hemisphere of the SCZ. Fortunately, numerical simulations have indicated that the convection in the SCZ is in the form of concentrated downdraughts, the return upward flow being slower and extending over broader regions; see Glatzmeier, 1985; Nordlund et al., 1992. As the downdraughts approach the base of the SCZ and the overshoot layer they spread out and, tending to conserve their angular momentum as they do so, they acquire downward (upward) vorticity, i.e. the helicity deep in the northern (southern) hemisphere of the SCZ is therefore predominantly positive (negative), leading to the required sign for α. (There is still, however, a difficulty: the phase relationship between B_r and B_ϕ in the solar cycle is not convincingly of the type that would be expected with this sign of α; see Stix, 1976; Yoshimura, 1976.)

A field of 1T in the tachocline would correspond (see above) to an Elsasser number of order 60, at least comparable with that of E. The tachocline is therefore, like $E's$ core, likely to be a highly magnetic system, and may be subject to MAC instabilities. The main differences between the instabilities of $E's$ core and of the tachocline are (first) that the magnetic field B_0 in (1) is strongly time dependent; it waxes and wanes in strength according to the phase of magnetic activity. The time scale of the instability sought must therefore be of the same order, $\tau_{\eta\odot}$. Second, the instability of the tachocline is apparently not *directly* powered by thermal/compositional buoyancy (as for $E's$ core), but by magnetic buoyancy. *Indirectly*, however, the power is again supplied by thermal buoyancy, since it is the convective motions in the bulk of the SCZ that maintain the $\Omega(\theta)$ required to create the large radial shears in the tachocline. Because δ is so small, the variation in ρ_0

across the tachocline is small, and suddenly the whole Braginsky method and its developments become viable and interesting. And, moreover, the analogy between the dynamos operating in E and S then becomes closer. Both may rely for their α-effect on large-scale instabilities, of the whole fluid core in the case of E, and of the tachocline in the case of S (see Schmitt 1984, 1985, 1987; Prautzsch, 1993). Schmitt calls these instabilities 'magnetostrophic'. They are in fact closely related to what Acheson called 'field gradient instabilities', which he initially studied in the geodynamo context, but which, with the solar context in mind, he also analysed for a compressible medium (Acheson, 1978), when they are usually spoken of as being driven by "magnetic buoyancy". They are particular forms of MAC wave instability, or perhaps one should say 'MC wave instability', since they are driven magnetically rather than thermally. What we are describing here is the recent partial convergence of solar dynamo theory and geomagnetic dynamo theory, alluded to in §1. It is pertinent to remark that there have been very many studies of instabilities in $E's$ core, notably by Acheson and Fearn, and that a local method of analysis has proved to be illuminating; see for example the review by Fearn (1989) where many of these references may be found. Such methods have been less used in solar physics [but see Gilman and Cadez (1970), Cadez (1974), Acheson (1978), Hughes (1985)]. Recent work in the solar context has tended to focus on the magnetic instabilities associated with B_0 alone in (1), but at least some geomagnetic studies have investigated the stabilizing effect of the velocity shear associated with V_0 in (1). The existence of pole-equator temperature differences in the tachocline reminds one of Gilman's model of the solar dynamo as an MHD Rossby wave (e.g. see Gilman 1969a,b); one must wonder whether that old but interesting approach could be adapted to modern requirements, and make a contribution to understanding the tachocline dynamo.

It is clear from these considerations why the prevailing opinion today is that the solar dynamo is an $\alpha\omega$-dynamo driven by sources that occupy a comparatively shallow layer near the base of the SCZ, and which we may call 'the solar dynamo layer' (SDL). There are two main scenarios, in both of which the ω-source is confined to the tachocline. In scenario A, the one described above, α is supplied by instabilities of the distributions of **B** and **V** within the tachocline, i.e. both the α- and ω-sources are confined to the tachocline, which therefore *is* the SDL (although, strictly speaking, the currents created in the SDL will leak upwards into the SCZ, and the motions in the main SCZ may then induce further fields, i.e. secondary dynamo action of a non-self-excited character may exist in the SCZ). In the other scenario (B), the α-source is provided by turbulence within, and/or above (Parker 1992), the tachocline, i.e. the SDL is more extensive than the tachocline but includes it. It is not clear in either scenarios what the effective value of η^T is in the SDL. We might expect $\eta_{\odot}^{turb}$ to be less than the $10^9 m^2 s^{-1}$ assumed earlier for the SCZ, for two reasons. First, as noted above, one must expect a diminution in turbulent intensity as the RZ is approached; second, the presence of an intense magnetic field will tend to suppress the turbulence and so decrease $\eta_{\odot}^{turb}$ by perhaps two orders of magnitude (Parker, 1992).

Let us now consider scenario B. There are two main possibilities: B_1, the α- and ω-sources are separated; B_2, they are both present in the tachocline. Scenario B_1 is one that Parker (1992) has explored in a "surface dynamo wave" model. Although

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he modestly calls this model a "pedagogical device", it illustrates well the effects of separated sources and of a magnetically-quenched $\eta_\odot^{turb}$. The history of scenario B_2 goes back 25 years. Although ρ is almost constant within the SDL, the rms turbulent velocity v varies considerably. This is an ingredient for a potent mechanism for α-creation, as the seminal paper of Steenbeck et al. (1967) demonstrated. In the limit of small microscale Reynolds numbers, they obtained an expression of the form

$$\alpha = -\frac{l^3 v}{\eta}\boldsymbol{\Omega}\cdot\boldsymbol{\nabla}\left[\ln(\rho_0 v^2)\right], \tag{4}$$

where η stands for $\eta^{molecular}$. Later, Krause and Rädler (1980, Ch. 9) gave the general expression in the first order smoothing approximation, and in the limit of large microscale magnetic Reynolds number (and small Strouhal number). This has the form

$$\alpha = -\frac{l^2}{v}\boldsymbol{\Omega}\cdot\boldsymbol{\nabla}v, \tag{5}$$

(where we have ignored spatial variations in ρ_0). Since v decreases across the SDL from its full value at the top (i.e. its value in the bulk of the SCZ) to effectively zero at the top of the RZ, $|\boldsymbol{\nabla}v| = O(v/\mathcal{L}_{SDL})$, where $\mathcal{L}_{SDL}$ is the thickness of the SDL. Taking also $l = \mathcal{L}_{SDL}$, we obtain from (5), $|\alpha| \approx \mathcal{L}_{SDL}\Delta\Omega \approx 20ms^{-1}$. As a result, the α-effect Reynolds number, $R^\alpha = \alpha\mathcal{L}/\eta^T$, for the SDL is $R^\alpha_{SDL} \approx 4$. This suggests that, in addition to a possible α-effect, from the instabilities (scenario A), there may be an appreciable α-effect from the turbulence in the SDL.

4. Conclusions

It is evident that, although some features of the geodynamo and solar dynamo mechanisms are still unclear, many more seem to have been understood, at least qualitatively. It is perhaps unwise for someone with my geophysical background to speculate on the way that solar dynamo theory is likely to develop, but the question of whether scenario A or scenario B will prevail is so interesting that it is hard to resist stating a preference. I am impressed by the orderliness with which sunspots follow Hale's polarity rules, despite the violent turbulence elsewhere on the solar surface. It suggests a similar orderliness in the tachocline, and therefore a dynamo that operates, as in the Earth, through large-scale instabilities of a well ordered MHD state (scenario A). The 1T field in the tachocline would then imply current densities of order $\mathcal{J} \approx \mathcal{B}/\mu\delta \approx 0.1Am^{-2}$. If we take $\eta = \eta^{molecular} = 0.2m^2s^{-1}$, corresponding to the density and temperature in the tachocline, we obtain a Joule dissipation $\mathcal{L}^2/\sigma$ of order $3\cdot 10^{-9}Wm^{-3}$, which when integrated over a tachocline of volume $3\cdot 10^{25}m^3$ would give $P_\odot \approx 10^{17}W$, which is no embarrassment when compared with $L_\odot = 4\cdot 10^{24}W$. The large 1T zonal fields might, however, raise problems if they were accompanied by the large irregular fields generated by a fully developed turbulence. The Joule dissipation in a turbulent dynamo creating a mean zonal field of order 1T may be larger by a factor of order $R^{microscale} = vl/\eta^{molecular}$ than that in the corresponding laminar dynamo; see Krause and Rädler (1980). Since $R_\odot^{microscale} \approx 10^7$, this would give $P_\odot \approx 10^{24}W$.

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