

part of his treatise, he turns to detailed questions of motion and develops theorems relating to infinite and infinitesimal speeds within closed boundaries of motion.

Professor Clagett's extensive commentary on the main text (pp. 437-517) relates Oresme's principal mathematical insights to their modern notations, as well as to texts and commentaries on Euclid that were accessible to medieval writers, and to the relevant ideas of some of those writers. Their original texts are usually cited in evidence, so that this valuable commentary becomes itself a source-book of important extracts from medieval mathematics not elsewhere easily available. The appendices likewise provide texts, as well as translations, of some supplemental medieval treatises related to configuration doctrine, the exploration of convergent series of certain types, and the treatment of uniform and non-uniform change.

Oresme's mathematical genius so far transcended that of his contemporaries that despite the early and widespread study of his works, evidenced by multiple surviving manuscript copies, two or three centuries were to elapse before other mathematicians fully attained some of his conceptions and pushed them further. As an example of this, Cantor cited the development of fractional exponents.

Professor Clagett, like all modern students of medieval mathematics, believes that Galileo's work on acceleration in free fall must have been rooted in mean-speed considerations drawn from the tradition to which Oresme was the greatest contributor. The principal work he has given us in this book is tantalizingly probable as such a source for Galileo, since it was composed in Latin (rather than in French, as was Oresme's astronomical work), and since four ancient manuscript copies are preserved in Italy (where no copies of the astronomical work can be traced). Yet he candidly grants that no connection has been established between Oresme's work and Galileo's. It is striking that Oresme commented specifically on the odd-number law that links squares to uniform growth, though not specifically for distances and times in local motion. On the other hand, Oresme was concerned with motion or change to a specific terminus ad quem and with rules that would permit indefinite increase of speed only within a bounded space. Whether Galileo began with those rules and extended them, or whether he arrived at true conclusions consistent with Oresme's by a quite different route, remains a fascinating topic of further historical research.

The overwhelming impression gained from a study of Oresme's work is one of astonishment at the progress he made under the handicap of a completely inadequate analytical notation. His perception that recourse to geometric figures made possible rigorous reasoning about conceptions of motion and change, and offered a means of their classification in precise ways, greatly extended the scope of applied mathematics beyond the geometry of Euclid. If others did little to extend that perception before Descartes, that in no way detracts from the importance of Oresme to our understanding of the history of mathematics, and particularly the history of mathematical physics.

Professor Clagett's scholarly care in editing, translating, and clarifying these texts is evident on every page. The University of Wisconsin Press likewise deserves praise for the production of an attractive and accurately printed book. I have noted but a single and trivial misprint in the entire volume.

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Das parallelenproblem im corpus Aristotelicum, by Von Imre Tóth.  
Archive for History of Exact Sciences, Vol. 3, No. 4/5, 1967. 173 pages.

This very extensive and elaborate study is concerned with the problem of

parallels in the works of Aristotle. For the great philosopher geometrical theorems served mainly to illustrate propositions in logic, philosophy, and ethics; for us Aristotle's remarks can provide a source of information on the state of the parallel problem in his days. The problem is this: is it possible to prove Euclid's fifth postulate ("that, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles"), or was Euclid right in placing this statement among the basic principles of geometry which must be accepted without proof? In the first book of his 'Elements' Euclid had proved the theorems 1 - 28 without using postulate V. As Theorem 27, in particular, he established that "if a straight line falling on two straight lines make the alternate angles equal to one another, the straight lines will be parallel to one another." It therefore seemed very strange that in order to prove Theorem 29 (a straight line falling on parallel straight lines makes (1) the alternate angles equal to one another; (2) the exterior angle equal to the interior and opposite angle; and (3) the interior angles on the same side equal to two right angles") he should have to take refuge to postulate V.

Chapter I of the present study deals with attempts to give a direct proof. Point of departure is the 'Analytica Priora' (II 16, 65a 4-7) where Aristotle illustrates the fault called 'petitio principii' - the (implicit) application of the statement to be proved in the proof itself. According to the author, the attempts to give a direct proof of Euclid I, 29.1 ran through several stages: (1) at first, the existence of parallels was proved in I, 27 by means of a direct construction as given in I, 31; (2) for the geometers of the first half of the fourth century B. C. the uniqueness of the constructed parallel seemed obvious; (3) therefore I, 29.1 was derived from the assumption of the existence of a unique parallel line; (4) about the middle of the fourth century the generation influenced by Eudoxos raised the standards of exact proofs; (5) in consequence, the method of proof described in (1) seemed no longer satisfactory; (6) the necessity of Theorem I, 30 (transitivity of being parallel), the author assumes, was only recognized in connection with the attempts to give a satisfactory proof, and it was first inserted into a textbook by Euclid; (7) the discovery of a logical circle in the direct proof must have been widely known, as can be concluded from Aristotle's elliptic way of mentioning the fact.

Chapter II opens with a discussion of 'Analytica Priora' (II 17, 66a 11-15) which is interpreted to mean: if (i) the inner angle which a secant forms with one of two parallels is greater than the outer angle formed by the secant with the other parallel on the same side, or if (ii) the sum of the angles in a triangle is greater than two right angles, then the parallels will intersect. Now this is the famous hypothesis of the obtuse angle which was developed by G. Saccheri in his book 'Euclides ab omni naevo vindicatus' (1733). Such an attempt to prove the parallel postulate in an indirect way by exhibiting a contradiction (namely, that the parallels will intersect) must therefore have been well known at Aristotle's time. According to the author, it was necessitated by the fact that no direct proof had been found.

Chapter III deals with the relation between the sum of the angles in a triangle and the commensurability between diagonal and side of a square; it centers around Aristotle's 'De Caelo' (I 12, 281b 5-7). The author's interpretation (contrary to the traditional translation of this passage) is: "If the sum of the angles in a triangle does not equal  $2R$ , then the diagonal of the square will be commensurable with its side." This translation therefore assumes that Aristotle - and the mathematicians of his time - had possessed such a deep insight into the structure of non-euclidean geometry that they recognized the said consequence.

In Chapter IV the author pursues the relations between geometry and ethics in Aristotle's works. In nature there is no freedom and no choice, plants always generate plants of the same species, and so do animals; the essence remains the same. On the other hand, human beings in their behaviour can choose between good or bad principles. But once having done so their reactions will be determined just as from geometrical principles (axioms) the consequences follow with necessity. As example, Aristotle refers to the fact that an angle sum of  $2R$  in a triangle implies one of  $4R$  in a quadrangle, while an angle sum of  $3R$ , for instance, in a triangle yields  $6R$  in a quadrangle. In other words: the statement that the angle sum in a triangle is  $2R$  is a principle which can be replaced by a contradicting one. In the author's opinion such a claim could only be made by Aristotle if he was familiar with contemporary attempts to solve the parallel problem.

Chapter V considers the angle sum as essence, as *raison d'etre*, of the triangle. That is, here are studied the places where the philosopher touches on the question whether the concept 'triangle' is invariably connected with an angle sum of  $2R$ . This gives occasion to discuss certain aspects of Aristotelian philosophy and logic in general, and the position of principles and the role of syllogisms in the whole set-up of geometry in particular.

The subject of Chapter VI is the relation between the angle sum of the triangle and the straightness of its sides. Aristotle states (in order to illustrate the meaning of necessity) that if the sides are straight, the angle sum will be  $2R$ , and vice versa; and if the sum is not equal to  $2R$ , then the sides cannot be straight lines. He seems to have overlooked, however, that the definitions of a straight line known to him were not used when properties of geometrical figures were derived.

In footnote 282 (extending over more than two pages) the author discusses the amazing fact that none of the many scholars who studied Aristotle after non-Euclidean geometry had been developed (including such men as Heiberg and Heath) seems to have realized to what extent contra-Euclidean theorems are contained therein. It is true, Heath once wrote concerning a certain Aristotelian fragment: "It is as if he had a sort of prophetic idea of some geometry based on other than Euclidean principles, such as modern non-Euclidean geometries" - yet at once Heath added: "It is not possible that Aristotle could consciously have conceived such an idea as Riemann's." In the author's opinion, Aristotle effectively had the ideas of Saccheri who distinguished between geometries with an angle sum of less than, equal to, and greater than two right angles for a triangle. That none of these scholars should have become aware of this fact during the last century or so, the author comments with the words "eine tolle Geschichte!" Doubtless this shocking story, and the claim presented here on more than 170 pages, will give rise to further discussion.

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The art of philosophizing and other essays, by Bertrand Russell. Philosophical Library, Inc., 15 E. 40th Street, New York 10016, 1968. 119 pages. U.S. \$3.95.

This book contains three essays: The Art of Rational Conjecture; The Art of Drawing Inferences; and The Art of Reckoning. The title essay seems to be missing.

(From the publisher's preface:) "The essays in this little volume, published here for the first time in book form, were written by Bertrand Russell during the