THE STELLAR DISTRIBUTION ABOVE THE GALACTIC PLANE: AN INTRODUCTION

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This paper will truly be an introduction — a presentation and a discussion of the basic problems, to set the stage for detailed research results that will be reported by others.

The problem of star distribution perpendicular to the galactic plane has one central theme, which can be played in two modes. The theme is the drop-off of star-density with z, and the two modes are the general density profile and the stratification of populations. (This last phrase is a convenient one, but I should warn against taking it too literally. By "stratification" I mean a gradual change, rather than a sharp separation; and I want to emphasize even more strongly that the word "population" should be used in a general and abstract sense, rather than as a sharp separation into two, or five, or any other discrete number of components.)

The density drop-off is a simple problem in principle; it is controlled by the velocity distribution of the stars and the force field that holds them to the plane. The interrelationships are conveniently described by the familiar "hydrodynamical" equation

$$\frac{\partial(N\langle Z^2\rangle)}{\partial z} \boxed{+ \frac{\partial(N\langle \Pi Z\rangle)}{\partial\tilde{\omega}} + \frac{N}{\tilde{\omega}}\langle \Pi Z\rangle} = -N\frac{\partial V}{\partial z} \quad . \tag{1}$$

(The terms in the box have often been omitted. Although studies of the third integral have shown that these terms are not quite equal to zero, they can safely be ignored in the present discussion, because even at z = 2000 pc they produce a correction of only 20 per cent or so.)

If the $\langle \Pi Z\rangle$ terms are omitted, the hydrodynamical equation takes the much simpler form

$$\frac{\partial(N\langle Z^2\rangle)}{\partial z} = -N\frac{\partial V}{\partial z} \quad . \tag{2}$$

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Even so, this equation still has no straightforward solution, because $\langle Z^2 \rangle$ depends on z in a way that is determined by the form of the Z-velocity distribution.

In one case, however, the solution of Eq. (2) becomes very simple. If the velocity distribution is Gaussian, then it can easily be shown that $\langle Z^2 \rangle$ is independent of z, and along a line of constant $\tilde{\omega}$ the solution is then

$$N(z) = N_o \exp(-\Delta V/\langle Z^2 \rangle) , \qquad (3)$$

where ΔV is the increase of the potential over its minimum value, in the plane. Note the role of $\langle Z^2 \rangle$ in scaling ΔV; it determines how strongly the change in potential will affect the density. Note also the simple power-law relationship between the densities of different stellar groups; if group B has a value of $\langle Z^2 \rangle$ that is n times as large as that of group A, its density changes are only the $1/n$ power of those of group A. It is this sensitive dependence of N(z) on $\langle Z^2 \rangle$ that produces the stratification of stellar types.

At the same time, this very behavior of Gaussian velocity distributions should remind us how dangerous it can be to represent stellar density distributions directly by Equation (3). Few distributions in nature are truly Gaussian, and real stellar velocity distributions tend to have a tail that is much more extended than the meager tail of a Gaussian. This tail can be represented by adding a small percentage of stars that have one or more Gaussian velocity distributions with higher values of $\langle Z^2 \rangle$. Although these contribute very little to the density at $z = 0$, at higher z they become dominant, and the overall N(z) looks quite different from one simple term of the form given by Eq. (3).

This way of representing the relationship between velocities and densities was introduced by Oort (1932), in his pioneering discussion of motions perpendicular to the plane. It is an effective method of representation, because even though a family of Gaussians is not a mathematically complete basis set, a superposition of them does in fact represent the local distribution of Z velocities rather well. However, one cannot overemphasize the sensitivity of N(z) to small admixtures of high-velocity stars. A graphic example is the difference in the treatment of the same observational data by Oort (1960) and by Hill (1960). What Oort concluded was that the observed numbers of faint stars demanded the addition of a higher-velocity group that contributed only 1% of the star density at $z = 0$; at higher z, however, this group made a crucial difference.

In the face of such sensitivity to the shape of the tail, it might seem hopeless to choose a reliable velocity distribution. The task is helped very much, however, by a few observations of velocity dispersions at appreciable distances from the galactic plane, where a great deal of segregation has already taken place and the high-velocity-dispersion component shows itself much more clearly.

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Given the caution about the shape of the velocity distribution, we can proceed to examine the behavior of Gaussian groups in a more quantitative way. Since ΔV is simply $\int_0^z K_z\, dz$, where K_z is the acceleration in the z-direction, we can calculate ΔV directly from the values of K_z given by Oort (1960). Furthermore, noting that 10^{-9} cm sec^{-2} pc = 0.3084 (km/sec)^2, we can express the results in units of km/sec, in order to go quickly from velocity dispersions to density ratios. The numbers are given in Table 1. The last column can be interpreted as the velocity dispersion of a group whose density has dropped by a factor of e at the height z given in column 1. (Note, however, that the values in the last two lines of the table are at a z-level where Eq. (3) has lost some accuracy, because of the $\langle \Pi Z \rangle$ correction alluded to above, which would tend to raise the densities somewhat.)

Table 1. Force Components and Potential Differences

z (pc)	K_z (10^{-9} cm sec^{-2})	ΔV (10^{-9} cm sec^{-2})	$(\Delta V)^{1/2}$ (km sec^{-1})
0	0.00	0	0.0
100	2.50	133	6.4
200	4.28	476	12
400	6.17	1544	22
1000	8.05	5932	43
2000	8.93	14536	67
3000	9.09	23552	85

Thus a young population, with a root-mean-square Z-velocity dispersion of 10 km/sec, should have a scale height of about 150 pc, whereas an older population with $\langle Z^2 \rangle^{1/2}$ = 20 km/sec should have a scale height of nearly 400 pc. Furthermore, at that level the density of the lower-velocity population should have dropped by e^4, or a factor of 70.

These simple dynamical arguments have another interesting consequence that has not, as far as I know, been noted before: an excess hump of halo-star density in and around the disc. Since the Z-velocity dispersion of halo stars is only about 100 km/sec, the numbers in Table 1 suggest that halo stars are 2 or 3 times as numerous in the galactic plane as they would be if the gravitational force of the disc were absent. This excess should be taken into account when we use local estimates of halo density to scale the densities in an overall halo model. Thus it appears that Schmidt's (1975) recent estimate of the total mass of the halo should be lowered even further.

Since the question may occur to some of you, I should mention that "massive halos" are not an issue at all in the galactic-polar-cap problem that we are discussing here. The halo population on which Schmidt sets such a firm upper limit has no relation to the extended density distribution that has been suggested by Ostriker and Peebles

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(1973) and by others. The conventional halo, which is the one that concerns us here, is reasonably well represented by an inverse-cube density distribution, whereas the halo that has been postulated to explain the disturbingly flat rotation curves in spiral galaxies must have a much gentler drop-off — somewhere around inverse-square. Or, to look at the problem from the other direction, if our Galaxy has a "massive halo," its predominant mass must lie much father out, and we should expect vanishingly few of its objects to be found in the solar neighborhood.

This whole dynamical discussion depends on our knowing the function $K_z(z)$. But in fact, this function is of even greater interest for its own sake, and its determination is one of the prime problems of high-latitude studies. The slope of its initial rise tells us the spatial mass density in the solar neighborhood, and the value at which K_z levels off tells us the surface density. In principle the determination of $K_z(z)$ is simple. We need only compare the spatial stellar density N(z) with the local distribution of velocity components $\phi(Z)_{z=0}$. Then with the aid of Eq. (1), or with the use that has been described for Eq. (3), or with some other way of applying the Liouville equation from which these equations are derived, $K_z(z)$ can be found directly from these two functions.

In practice, however, the determination of K_z has been an unending headache. Different studies have found quite different values of the local density, ρ_o; and, worse, attempts to derive the form of $K_z(z)$ have consistently produced a dip that implies the absurdity of a negative density somewhere. Obviously the problem is a lack of adequate observational data. There are three ways in which this trouble may arise. First, the stellar group used may not be one to which the dynamical hypotheses apply. An example is the A stars, which might not be distributed as smoothly as the theory assumes. Second, different parts of the data may refer to different subgroups within the sample studied. This is a particular danger among the K giants, which occupy the most inhomogeneous region of the whole HR diagram. The relative proportion of M11-type red giants to M67-type red giants varies by a large factor in the very z-range in which we need to do our study. Furthermore, these stars have different absolute magnitudes even though they are spectroscopically very similar. The third source of possible trouble is systematic errors in the data. Photometric errors are distressingly common, and spectral classes have their problems too. Note, for example, the suggestion by Oort (1960) that the spectral classifications with which he is dealing shift systematically at fainter magnitudes.

As a result of these difficulties, we still have no reliable determination of $K_z(z)$, and even the local volume density and surface density are rather uncertain. It has been a long journey through the wilderness; Professor Oort was given a glimpse of the promised land in 1932, but we still have not yet entered it.

Before leaving the area of dynamics I should mention one more consequence of the vertical separation of groups of different velocity dispersion. This is a progressive change of mean transverse velocity with height above the plane. It results from the correlation between mean rotational velocity and velocity dispersion, which is described by the equation (cf. Oort 1965, Eq. [38])

$$\Theta^2_{circ} - \Theta^2_m = \langle \Pi^2 \rangle \left\{ -\frac{\partial \ln \nu}{\partial \ln \tilde{\omega}} - \left[1 - \frac{\langle \Theta_{pec}^{\;2} \rangle}{\langle \Pi^2 \rangle}\right] - \left[1 - \frac{\langle Z^2 \rangle}{\langle \Pi^2 \rangle}\right] \right\} \quad (4)$$

If we note that for nearly all stellar types $\langle \Pi^2 \rangle \approx 4 \langle Z^2 \rangle$, then with reasonable values for various of its quantities Eq. (4) can be approximated as

$$\Theta \approx \Theta_{circ} - \langle Z^2 \rangle / 7^2 \quad (5)$$

Thus at large $\underline{z}$, as $\langle Z^2 \rangle$ increases the solar motion should shift systematically. Whereas this effect is unlikely to be significant for nearby stars such as those studied by Murray and Sanduleak (1972), for more distant stars it can become important.

Having talked about the behavior of stellar groups in general, I should now like to discuss a more specific question. When we look at the stars in a high-latitude field, what should we expect to see? More specifically, if we distinguish by magnitude and color, what stars dominate each part of the color-magnitude array? In answering this question I am fortunate to be able to draw on the calculations of a Berkeley student, Kate Brooks, who is modeling the star distribution in high latitudes. She does this by representing the distribution as a mixture of disc and halo populations, with a luminosity function and a density distribution for each, and by adjusting the parameters so that the computed numbers at each magnitude and color agree with the observed numbers. The model then gives a breakdown at each point in the color-magnitude array, showing which types of stars are the major contributors. The procedure is similar to that followed by Luyten (1960), but the data are accurately measured colors and magnitudes, and the stellar population is broken down in detail.

Using results kindly provided by Mrs. Brooks, I have prepared Figure 1, which is an attempt to show schematically what kinds of stars we see at the galactic pole. The figure does not extend brighter than 12th magnitude, because the small fields that we are studying have too few brighter stars to be significant. At the faint end, however, I have extended it beyond the range of our own observations, in order to show what stars are registered by the new 4-meter telescopes on fine-grain emulsions. Note the diagonality of the diagram. At the top right are some subgiants (and there would be some red giants too, if the diagram went brighter). The main-sequence stars of the disc population dominate a broad strip across the middle of the figure. Note that in general at fainter apparent magnitudes we tend to see stars that are both less luminous and farther away. Then among the faint white stars the halo

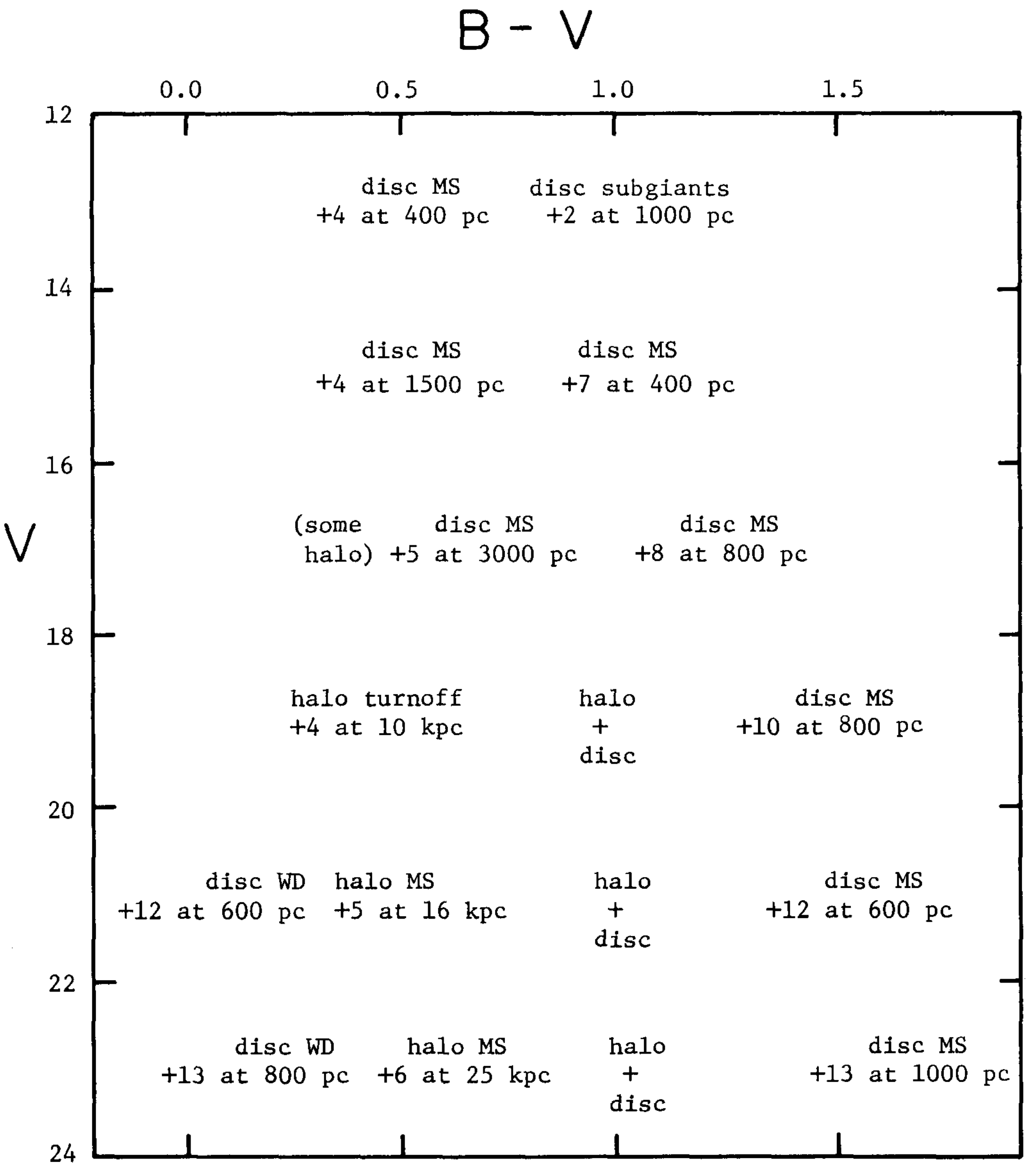

Figure 1. The dominant stars at various colors and magnitudes, at the galactic poles. The numbers are only rough approximations; at each point there is a mixture of absolute magnitudes and distances. Also, the diagram should be thought of as more continuously filled.

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makes its appearance, first at the main-sequence turnoff and then on the main-sequence itself. Finally, among the bluer colors the white dwarfs of the disc take over.

Some stellar types do not appear in the figure. The red giants of the halo are not dominant in any region; apparently they can be distinguished only by methods more sophisticated than the simple sorting of colors and magnitudes. The halo main sequence barely shows up; the number of stars on its lower part appears to be a study reserved for the days of the Space Telescope, which should reach several magnitudes beyond the lower bound of this table.

A striking characteristic of the data in the figure is how far away the stars are that we see. For most stellar types, counting and classifying such faint stars is not a way of determining densities in our immediate neighborhood; the densities refer to far-away points. Take the halo population, for instance; we find out nothing about its local density but can only count up stars that are tens of kiloparsecs away. Even for the "local" M dwarfs, a survey such as this one relates predominantly to stars that are several hundred parsecs away. To count up the M dwarfs at smaller distances we need surveys at the brighter magnitudes, over larger areas of the sky, such as the color-magnitude survey of Weistrop (1972a, 1976a) or the spectral survey of Sanduleak (1964, 1976).

Consideration of these surveys brings us right back to the problem of systematic errors, however. The surveys of both Weistrop and Sanduleak claimed to find high space densities of M-dwarf stars, and the literature was then filled with a spate of less consequential papers that echoed this claim. It now seems clear, however, that there is no appreciable excess of M dwarfs over the numbers in Luyten's (1968) luminosity function and that both Weistrop and Sanduleak had been led astray by errors in their photometric systems or scales (Faber et al. 1976; Weistrop 1976a, 1976b). With wrong colors they had deduced wrong distances, which led to large errors in density.

If there is a generalization to be drawn from this whole discussion of high-latitude problems, it is that the principles are easy but the practice is difficult. And if there is a moral in the whole story, it is that we will never build a sound understanding until we have sound and reliable observational data.

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