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BIORDERED SETS OF SEMIGROUPS

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The main aim of this thesis is to describe the biordered sets of some important classes of semigroups. The body of the thesis comprises an introduction and seven chapters.

In Chapter 1 all terminology relating to biordered sets is defined, and the axioms for a biordered set are reformulated using a new arrow notation. A semigroup representation ϕ° by transformation and dual transformation semigroups is defined and an analogue ϕ constructed for an arbitrary biordered set.

In Chapter 2 it is shown that ϕ is a biordered set representation, and ϕ is used to show biordered sets abstractly characterize certain naturally occurring $\mathcal{D}$-closed partial algebras of idempotents of semigroups.

In Chapter 3 the relationship between ϕ and ϕ° is explored. It is shown that the maximum idempotent-separating H-congruence on an arbitrary semigroup exists and coincides with the kernel of ϕ° (when ϕ° is defined for all regular elements of the semigroup), which generalizes known results about maximum idempotent-separating congruences on eventually regular semigroups. It is shown that ϕ enables a reconstruction of the "fundamental" image of an arbitrary semigroup from its biordered set alone. Further a prototype theorem is given characterizing in terms of properties of ϕ the biordered sets of different classes of idempotent-surjective

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semigroups. This theorem is replaced in special instances in later chapters by characterizations in terms of sandwich sets and solid sets.

In Chapter 4 a condition on ϕ is found which determines precisely when the sandwich set of an arbitrary n-tuple of biordered set elements is non-empty. This criterion is used to show that biordered sets of eventually regular semigroups are precisely biordered sets for which all sandwich sets are "eventually" non-empty, in a sense which is made precise.

In Chapter 5 properties of solid sets are explored and used to find characterizations of biordered sets of group-bound semigroups, periodic semigroups, and a new description of biordered sets of bands.

In Chapter 6 biordered sets of arbitrary finite semigroups are studied and a condition given which enables one to determine after a finite number of steps whether a given finite biordered set comes from a finite semigroup.

In Chapter 7 examples are given to show that the classes of biordered sets of eventually regular, group-bound and periodic semigroups are all distinct. One example is the smallest biordered set which is not regular but comes from a finite semigroup.

The thesis includes material which appears in papers of the author [1], [2], [3], [4], and [5]. Chapter 2 contains some unpublished results of T.E. Hall.

References

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