

Verbal small cancellation

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The general problem considered in this thesis is, in the context of an arbitrary free group, to find a sharp lower bound for the cyclically reduced length of an expression in two non-commuting variables, expressed in terms of the cyclically reduced lengths of these variables.

We present a method for solving such problems for two-variable expressions, and we discuss the possibility of applying a similar method to find minimum length estimates for expressions in more than two variables.

The method is applied to two classes of expressions in two variables: the class of Engel words, and the words

$$[x, y][[x^2, y^2], [x, y]]^N,$$

where N is an arbitrary positive integer. Our motive for considering the latter class of words is the possibility of constructing a group variety in which every group has a perfect first derived group, but in which not every group is abelian; whether such varieties exist is an open question.

For the Engel words we show that the cyclically reduced length of the N -th Engel word

$$E_N(x, y) = [x, y, \dots, y], \quad N \geq 2,$$

is at least $2^{N-1}(\lambda^*(y)+2)$, where $\lambda^*(y)$ denotes the cyclically reduced length of y .

As a byproduct of our proof, we show that for $N \geq 4$ the N -th Engel word $E_N(x, y)$ has, *after* all cancellations and cyclic reductions are carried out, the appearance of a cyclic conjugate of some $E_{N-3}(u, v)$

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before any cancellations are carried out. This enables one to recognise when a given element of a free group is *not* conjugate to some Engel word $E_N(x, y)$.