function. In chapter III are studied further methods for the development of rational function approximations to functions. Examples are given of the numerical computation. In chapter IV are considered the generalized continued fractions proposed by Euler. Particular attention is given to methods associated with matrices, and examples demonstrate the computation of roots and the solution of polynomial equations. The book concludes with a list of 17 books in the Russian language on the general theory of continued fractions, a list of 109 books and articles used in this text, and a list of 10 supplementary references (added by the translator).

In the translation of this book, the translator states that he has allowed himself a certain degree of freedom.

This text contains a tremendous mass of valuable formulas in continued fraction theory. Due to this fact, it can be considered as a useful reference manual for such formulas as well as a text on methods for research in analysis and in computational work.

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Calculus of Variations, by I. M. Gelfand and S. V. Fomin. Revised English edition, translated and edited by R. I. Silverman. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963. 232 + vii pages. $7.95.

Of three recent volumes bearing the same title, all translated from the Russian [Elsgolc (1962), Akhiezer (1962)], the present text is the most modern and sophisticated.

After a brief introduction to the theory of functionals, necessary conditions for extreme values of simple integrals are derived from the first variation with more than the usual rigour. The first three chapters of the book are devoted to this problem and its generalizations: the fixed end problem for n unknown functions, functionals depending on higher order derivatives, variational problems with subsidiary conditions, discontinuous solutions. The sections dealing with subsidiary conditions are somewhat disappointing as they make no mention at all of the deep results of Carathéodory (Acta Math. 47(1926), 199-236; Comment. Math. Helv. 5 (1933), 1-19) which must nowadays be regarded as classical. It must also be pointed out that the authors' definition of "non-holonomic" constraints (p. 48) is incomplete and therefore misleading in that they do not distinguish at all between integrable and non-integrable conditions.

Chapter 4 introduces the canonical equations of motion and furnishes an introduction to the Hamilton-Jacobi theory and its applications to classical mechanics. A very lucid form of Noether's theorem for single integrals is given.

Sufficient conditions for weak and strong extrema are derived via the usual theory of the second variation in chapters 5 and 6 respectively, the latter depending to a large extent on the concept of fields of extremals which is introduced with great care. While the condition of Jacobi appears relatively early (p. 112), the excess function of Weierstrass is defined for the first time at an unusually late stage (p. 146).

Multiple integral problems with application to continuous mechanical systems are treated in chapter 7. A detailed calculation based on the first variation leads to Noether's theorem and its role in field theory. Again it seems a pity that the remarkable but distinct theories of multiple integral problems due on the one hand to Carathéodory (Acta Szeged Sect. Scient. Math. 4 (1929), 193-216; also reproduced in collected works, vol. 1) and on the other to H. Weyl (Ann. Math. 36 (1935), 607-629) are ignored completely, for these theories first opened the way to the rigorous study of sufficient conditions for extreme values.

Chapter 8 gives a lucid introduction to the theory of direct methods (e.g. the Ritz method). In appendix 1 the propagation of disturbances is briefly treated by Finsler space methods (although the authors do not say so). Appendix 2 furnishes an account of the application of variational methods to problems of optimal control, which is a fairly recent development not treated in standard texts. This unique feature adds lustre to what is in general an excellent book.

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Variationsrechnung, by L. Koschmieder. Sammlung Göschen, Band 1075. Walter de Gruyter and Co., Berlin, 1962.

This is a compact but fairly rigorous introduction to the basic concepts of the calculus of variations. The text is restricted largely to problems based on single integrals whose integrands involve only one or two dependent functions. The methods are classical, a clear distinction being made between the theories based on fixed and movable end points. These are described in the first two chapters, while the third and last gives an excellent description of classical isoperimetric problems.

This book, which conforms in every respect with the fine tradition of the well-known "Göschen"-series, is to be followed by a text dealing with more general problems and modern direct methods.

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