PART 5.

Magnetic Fields And Viscosity

https://doi.org/10.1017/S0252921100042627 Published online by Cambridge University Press

Accretion Phenomena and Related Outflows, IAU Colloquium 163
ASP Conference Series, Vol. 121, 1997
D.T. Wickramasinghe, L. Ferrario, and G.V. Bicknell, eds.

Three-Dimensional Simulations of Accretion Disks

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Abstract. The transport of angular momentum is the central issue in accretion disk dynamics. We review recent three-dimensional simulations that investigate possible transport mechanisms. Purely hydrodynamic local instabilities and turbulence are ruled out; global spiral waves remain a possibility. MHD turbulence, arising from a local MHD instability, has been shown effective in transporting angular momentum at dynamically important rates. These results establish the basic picture of accretion disk transport.

1. Introduction

Until recently, the dynamics of accretion disks were more a matter of "conventional wisdom" than of first-principle physics. Over the years a standard picture was formulated, with individual components built upon seemingly plausible arguments. Nevertheless, key parts of this picture remained frustratingly unproven—for good reasons as it turns out.

Although there are several variations, one form of the conventional accretion disk went something like this: Accretion disks accrete because of the outward transport of angular momentum. This requires a viscosity, but the known sources, *e.g.*, molecular viscosity, are too small by a large factor to provide the necessary transport. Hence their must be an "anomalous viscosity." The origin of this anomalous viscosity is mysterious. However, because disks have such a high Reynolds number (*i.e.*, low molecular viscosity) they probably become turbulent. The resulting turbulence will produce enhanced transport. Because disks are hydrodynamically, locally, linearly stable according to the Rayleigh criterion ($dL/dR > 0$, where L is the angular momentum, usually assumed to be at or near the Keplerian value), they must be *nonlinearly* unstable in a manner analogous to simple shear flows. The source of the turbulence may be less ambiguous if a disk becomes convectively unstable in the vertical direction. Then convective instability will lead to turbulence and enhanced radial angular momentum transport. A possible alternative involves magnetic fields; they can, in principle, transport angular momentum via the Maxwell stress. The difficulty with this scenario is that magnetic fields will not be significant unless they are strong. While the presence of weak fields is almost certain, dynamically significant fields may be harder to come by. It is likely, however, that the pre-existing hydrodynamic turbulence in the disk will produce a kinematic disk dynamo that will amplify these weak fields. This follows because shear plus turbulence pro-

https://doi.org/10.1017/S0252921100042627 Published online by Cambridge University Press

duces a dynamo. So magnetic fields may well be an important addendum to disk dynamics, but, in the main, disks are basically hydrodynamic.

Remarkably, many of the statements in the preceding paragraph are wrong. The error lies with the emphasis on hydrodynamics. The realization that disks are dynamically unstable in the presence of weak magnetic fields (Balbus & Hawley 1991) means that magnetic fields cannot be relegated to a supporting role in disks; they are essential. In this and a companion paper in these proceedings, we will review the current understanding of disk dynamics. This paper will focus upon the role that multi-dimensional numerical simulations have played in distinguishing angular momentum transport mechanisms that work from those that don't.

2. Hydrodynamical Simulations

2.1. Hydrodynamical Turbulence and Transport

Hydrodynamical turbulence has long been regarded as the leading contender for the source of the anomalous viscosity in accretion disks. The argument supporting this typically involves two steps. First, it is assumed that such turbulence, once established, would naturally lead to an outward transport of angular momentum, and that such transport can be straightforwardly parameterized by a "turbulent viscosity," *e.g.* ν_t times an angular velocity gradient. Second, it is assumed that hydrodynamic turbulence can be created and sustained by some mechanism, *e.g.* an instability. The two leading candidates for this were convection, and a nonlinear hydrodynamic shear instability. However, the first assumption, that turbulent transport occurs, was often adopted without specifying the particulars of its source.

Consider first the issue of the hydrodynamic stability of disks, and the source of the supposed turbulence. The notion that the presence of shear alone in an accretion disk might be sufficient to guarantee hydrodynamic turbulence and significant angular momentum transport has long been a staple of accretion disk lore. Although disks are linearly stable to hydrodynamic perturbations, it was thought they might be unstable to finite-amplitude (nonlinear) perturbations. Such finite amplitude instabilities are not without a fluid mechanical precedent: they are present, in fact, for simple shear flows.

For many years that is where the matter stood. Their very nature conveniently places the proof of such instabilities beyond the realm of linear perturbation theory. Given this difficulty, numerical simulations would appear to be the only way to proceed. A practical difficulty is that nonlinear instabilities in shear flows are three-dimensional (3D) in nature; 3D simulations remain challenging, even with the latest supercomputers. A second apparent difficulty, namely the inevitable "numerical viscosity" inherent in simulations, seems more serious. In experimental fluid mechanics the presence of substantial viscosity can prevent the development of turbulence. In other words, low Reynolds number flows tend to be laminar while high Reynolds number flows exhibit turbulence. The terms "turbulence" and "high Reynolds number" became so associated with one another that some came to regard the high Reynolds numbers of accretion disks as leading inevitably to disk turbulence.

The presumed congruence of turbulence and high Reynolds number flow led to pessimism regarding the possibility of simulating disk flows, *viz.* that numerical simulation codes have too much numerical viscosity to permit the development of such instabilities. However, the numerical "viscosity" in a code that solves the Euler equations is not equivalent to a low Reynolds number Navier-Stokes problem. This point has been explored by Porter *et al.* (1990) who compared a Navier-Stokes code with an Euler code on a series of test problems. They found that the Euler code resolved much finer detailed structure and shorter wavelength instabilities than the highest Reynolds number Navier-Stokes simulation that was feasible. They concluded that a good Euler code can model quite high Reynolds number flows with even modest resolution.

The reason for this has to do with the role played by viscosity, and the distinction between true viscous instabilities and dynamical (inviscid) instabilities. In a viscous instability, or in viscous boundary layers, the viscous wavelength will be a dynamically important lengthscale. The physical viscosity wavelength would then have to be resolved by the numerical grid. Instabilities arising within Eulerian dynamics, however, do not depend on viscous lengthscales. Viscosity acts only as a sink for the resulting turbulence. The Rayleigh criterion for an unbounded, differentially rotating flow, for example, applies to all wavenumbers. In an Euler code simulation, if there are instabilities present for wavelengths that are resolved on the grid, and the growth rate of the instability is greater than the numerical diffusion rate at any of the unstable wavelengths, the instability will be seen. Thus, *if* there exists an ubiquitous, local, nonlinear instability in disks, even a fairly low resolution 3D simulation should find it. And, indeed, this expectation is borne out in simulations of simple shear systems (Balbus, Hawley, & Stone 1996), where the nonlinear instability is easily observed.

Differentially rotating systems prove to be quite different, however. The first indication of this came when Hawley, Gammie & Balbus (1995) carried out local hydrodynamic simulations of differentially rotating systems as controls for MHD simulations. They found no sign of sustained hydrodynamic turbulence in the absence of magnetic fields. The hydrodynamic stability of accretion disks was later demonstrated more systematically in Balbus, Hawley & Stone (1996) who ran a series of simulations for different background angular velocity distributions. Not surprisingly, Rayleigh-unstable background velocity distributions were found to be unstable. In sharp contrast, the perturbations in Keplerian simulations decay rapidly. If the tidal and Coriolis forces are removed, to produce a simple shear flow, the known nonlinear shear instability appears, and the perturbations again grow. The stabilizing role of the Coriolis force is further demonstrated by a simulation of a disk with constant angular momentum. In a constant angular momentum distribution, radial perturbations in a particle's orbit do not result in epicyclic motion; there is no restoring force. In the absence of this force, the constant angular momentum case proves to be as unstable as the simple shear flow (see Balbus & Hawley, these proceedings for a further discussion of these results). Thus in this series of simulations, all done with the same numerical code and the same grid resolution, situations known to be unstable became unstable, and those without known instabilities were stable. Stated this way, the results hardly seem surprising, yet the notion of "nonlinear instabilities" can now be put to rest.

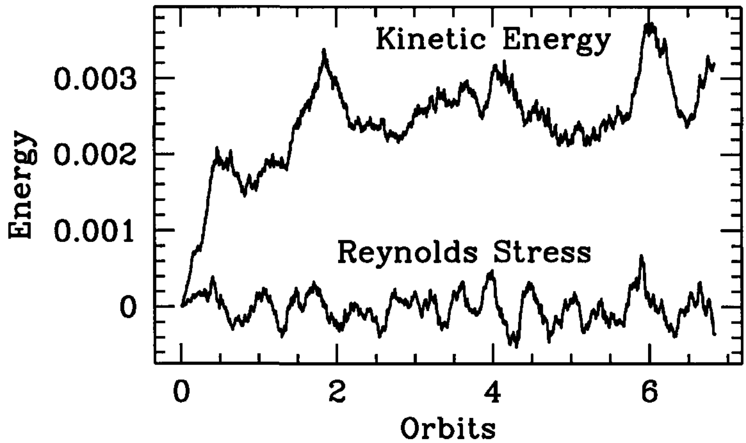

Figure 1. Evolution of the energy density as a fraction of the thermal energy for the perturbed kinetic energy and the Reynolds stress in a hydrodynamic Keplerian flow. The perturbed kinetic energy results from random forcing, but the resulting turbulence produces no net angular momentum transport.

If nonlinear instabilities do not exist, what of convection? If present, convectively unstable temperature gradients from the disk midplane to the surface can certainly lead to turbulence, but does it necessarily follow that radially outward angular momentum transport will occur? Ryu & Goodman (1992) carried out a detailed analysis of the linear stage of a convective instability and found *inward* angular momentum transport. The full 3D nonlinear simulations of Stone & Balbus (1996) found that inward transport is favored, consistent with and generalizing Ryu and Goodman's analysis. Similar numerical results have been obtained by Cabot (1996).

The failure of convection to transport angular momentum highlights the difference between turbulent mixing of scalar quantities and turbulent transport of momentum, a point emphasized by Prinn (1990). A numerical demonstration is seen in Figure 1. In this simulation, a local Keplerian flow is stirred by a random forcing function. The perturbed kinetic energy rapidly climbs to a sustained level, but the averaged Reynolds stress, $\langle \rho v_r \delta v_\phi \rangle$ oscillates around zero; there is no net transport of angular momentum. Thus, even when put in by hand, hydrodynamic turbulence need not act like an anomalous viscosity in an accretion disk.

The hydrodynamic stability of disks to nonlinear perturbations, and the failure of convectively-driven turbulence to transport angular momentum outward have a common cause in the interaction between the mean flow and the turbulent velocity fluctuations. Although the minimum energy state in a differentially

https://doi.org/10.1017/S0252921100042627 Published online by Cambridge University Press

rotating fluid is one with constant angular velocity, the angular momentum fluctuations try to reduce the angular *momentum* gradient. In a disk the angular velocity and angular momentum gradients are oppositely directed and hydrodynamic turbulence dies out. This is discussed in more detail in the companion paper (Balbus and Hawley, these proceedings).

2.2. Global Hydrodynamic Processes

Although the outlook appears grim for hydrodynamics as the source of angular momentum transport in disks, the discussion so far has been limited to local processes. Global waves remain a viable hydrodynamic angular momentum transport mechanism. There have been several numerical simulations that have investigated the efficacy of such global waves.

Again, something is needed to provide a driving mechanism for the waves. One possibility is the parametric instability in tidally distorted disks found by Goodman (1993). Ryu & Goodman (1994) simulated the action of this instability on a local section of the shearing sheet. They found that the instability could drive local hydrodynamic turbulence, but, significantly, no net angular momentum transport was produced within the disk. This is consistent with the results discussed in §2.1 above. Ryu, Goodman, & Vishniac (1996) argue that the instability will nevertheless produce global waves that will transport angular momentum, although simulations demonstrating this have not been done.

Another source of global angular momentum transport is the generation of spiral shocks by a tidal potential. Różyczka & Spruit (1993) carried out two-dimensional (R, ϕ) simulations of accretion disks in binary systems. They found that the accretion was quite nonsteady, with a low average accretion rate corresponding to an effective "α" value around 10^{-3}. Sawada & Matsuda (1992) have performed a 3D simulation of binary accretion and, although the challenging nature of such simulations limited the evolution time, the results are generally consistent with those of Różyczka & Spruit (1993).

The global mode of Papaloizou & Pringle (1984) is an example of a hydrodynamic instability that does not require tidal forcing. However, it does require somewhat special conditions: the largest growth rates are found only in nearly constant angular momentum accretion tori rather than Keplerian disks. Three dimensional simulations of such tori (Hawley 1991) have shown that the nonlinear amplitude of the resulting spiral wave decreases with increasing radial width in the initial torus. Further, as pointed out by Blaes (1987), the presence of accretion can eliminate the growth of these modes entirely.

To date, then, hydrodynamic simulations of global waves find some angular momentum transport, although at rather low rates even under the most favorable of conditions. While it is possible to assign these rates an average Shakura & Sunyaev (1973) α value, it is worth remembering that the local physics of the disk will not be described by the standard α formalism which was intended to describe highly dissipative turbulence. Shakura & Sunyaev's α provides a scaling for angular momentum transport by a local (turbulent) stress in terms of the pressure P. Physically, one expects some rough equipartition between the turbulent velocities (and magnetic fields) and disk pressure, hence the motivation for the parameterization. Transport of angular momentum by global waves, on the other hand, is inherently nonlocal.

3. Magnetohydrodynamical Simulations

Hydrodynamic processes have largely proved ineffective as the source of anomalous viscosity in accretion disks. We turn, then, to magnetic fields. It has been traditional to divide the torques due to magnetic fields into two categories: internal and external. Internal torques are due to the local Maxwell stress, $\langle -B_r B_\phi \rangle / 4\pi$. External torques result from large-scale magnetic fields passing through the disk and carrying off angular momentum in a rotating wind or jet. Numerical simulations are presently investigating the consequences of both of these effects.

3.1. MHD Jets and Winds

MHD jet simulations are reviewed in the article by Stone (these proceedings). Here we will briefly mention only one aspect of such simulations, the angular momentum transport in an accretion disk due to a global vertical magnetic field.

One of the most important developments for future simulations will be the investigation of fully global 3D MHD accretion disk models. The first step in that direction has been taken out by Matsumoto (these proceedings). This work is the successor to earlier 2D global simulations of accretion disks threaded by vertical magnetic fields first pioneered by Uchida & Shibata (1985; also Shibata & Uchida 1986). These authors investigated the creation of MHD jets as a sub-Keplerian accretion disk falls inward, dragging a vertical magnetic field with it. The infall produced radial fields that are then wrapped up by differential rotation into strong toroidal fields that drive dynamic outflows along the vertical field lines. Although such jets are a transient phenomenon resulting from a special choice of initial conditions, the simulations were the first time-dependent demonstration of the efficacy of magnetic fields for jet acceleration and collimation.

More recently, Stone & Norman (1994) carried out similar computations of an initial Keplerian disk with a vertical field and found that when the field is strong ($B^2/8\pi > P$) magnetic braking proceeds rapidly, collapsing the disk on an orbital timescale. When the magnetic field is weak, *i.e.* subthermal, the disk *still* collapses on an orbital timescale, in this case due to internal magnetic stresses and a weak field magnetic instability. These simulations thus illustrate both internal and external magnetic torques, each in its appropriate regime. We take up the subject of internal MHD stresses next.

3.2. Angular Momentum Transport

Hydrodynamic turbulence appears largely incapable of transporting net angular momentum outward through an accretion disk. Fortunately, magnetic fields have no such difficulty, nor do they require any particular "bootstrap" mechanism such as a kinematic dynamo to amplify them to large strengths. Subthermal magnetic fields are *unstable* in differentially rotating flows (Balbus & Hawley 1991) whenever $d\Omega^2/d\ln R < 0$, a condition universally satisfied for accretion disks.

There is now a growing body of numerical simulations of the local nonlinear behavior of this magnetorotational instability. The simulations range from nonstratified, differentially rotating domains using the shearing box (Hawley, Gammie & Balbus 1995, 1996; Matsumoto & Tajima 1995), to stratified local

https://doi.org/10.1017/S0252921100042627 Published online by Cambridge University Press

disk cross sections (Brandenburg *et al.* 1995; Stone *et al.* 1995). While the details in these simulations vary, one aspect remains consistent: magnetized disks generate self-sustaining, nonisotropic turbulence that transports angular momentum outward. This is true regardless of the initial field topology (toroidal as well as poloidal fields are unstable), or field strength (so long as the field is weak).

In the fully nonlinear state, the turbulent stresses are about 80% magnetic (Maxwell stress) and 20% kinematic (Reynolds stress). Most of the energy and angular momentum flux is concentrated at the largest scales. The results for stratified and nonstratified simulations are very similar; buoyancy does not appear to play an important role as a saturation mechanism. However, buoyancy provides a significant vertical transport mechanism for magnetic field and other quantities. The simulations suggest that a likely outcome is a disk with a weakly magnetized core surrounded by a strongly magnetized corona. To date the average angular momentum transport in these local, stratified disk simulations, as parameterized by a time and space-averaged α value, are about $\alpha \sim 0.01 - 0.001$. The exception to this has been simulations with a net poloidal field piercing the computational domain; these can produce $\alpha \sim 0.1$.

To some, a value of $\alpha \sim 0.01$ may seem disappointingly low. Since some disks (CVs in outburst, for example) apparently have $\alpha \sim 1 - 0.1$, are the simulation results a cause for concern? In general, the total stress, both Reynolds and Maxwell, is proportional to the magnetic pressure with a constant of proportionality around 0.5. Thus, the question of the angular momentum transport in the disk becomes one of the strength of the magnetic field at nonlinear saturation of the instability. What determines this level in a simulation? Larger computational domains, higher grid resolution, the presence of net magnetic flux, and larger numerical magnetic Prandtl numbers (ratio of kinematic viscosity to magnetic resistivity) have all been shown to increase the turbulent magnetic energy level, and with it the value of α. One should view the present simulation results as lower limits on α rather than the final word.

3.3. Dynamos

The issue of the amplification and maintenance of a magnetic field is the disk dynamo question. Can an accretion disk act as a dynamo, and, if so, under what circumstances will a "seed field" be amplified to equipartition strength? Traditionally a distinction has been drawn between *kinematic dynamos* that exhibit dynamo behavior for a specified velocity field $\mathbf{v}$ independent of the magnetic field, and *hydromagnetic dynamos* where the velocity is itself a function of the induced magnetic field through the influence of Lorentz forces. Although the latter is the only type of dynamo used by nature, the former has been the object of considerable theoretical interest, because the kinematic assumption renders the dynamo problem tractable. However, Balbus & Hawley (1991) and succeeding papers, have made it clear that what is meant by the term "weak magnetic field" must be revised. Since it is precisely the weak field that is linearly unstable in accretion disks, Lorentz forces can *never* be neglected, and the kinematic dynamo approximation is inappropriate for accretion disk dynamo studies. This shows that the problem of describing an accretion disk dynamo must be ap-

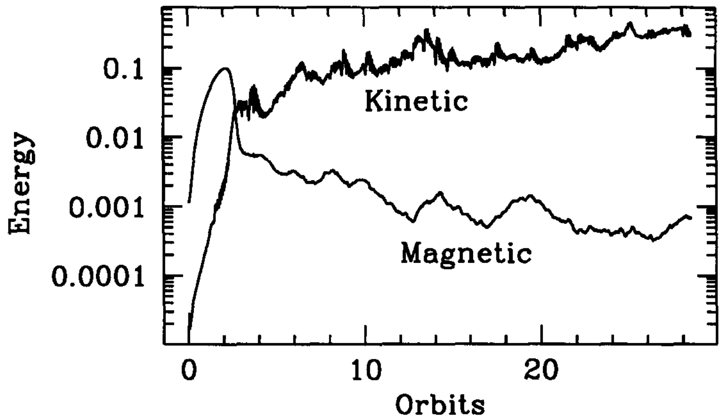

Figure 2. Evolution of the kinetic and magnetic energy densities as a fraction of the thermal energy in a simple shear flow. The perturbed kinetic energy results from the nonlinear shear instability, but the resulting turbulence does not lead to sustained magnetic fields. Shear plus turbulence are not sufficient to guarantee a dynamo.

proached with the full MHD equations intact. Generally, numerical techniques are required to investigate such hydromagnetic dynamos.

Hawley, Gammie & Balbus (1996) compared the long-term evolution of the magnetorotational instability with a series of models reproducing the 3D MHD dynamo simulations of Meneguzzi, Frisch & Pouquet (1981). These simulations found field amplification in a uniform box stirred by a random, nonhelical forced velocity field. In the case of the disk, however, no external forcing is required; field is amplified and sustained by the direct action of the instability, behaving qualitatively like a nonhelical dynamo.

The stratified disk simulations of Brandenburg *et al.* (1995) and Stone *et al.* (1996) show a similar ability to sustain indefinitely a significant magnetic pressure. Brandenburg *et al.* (1995) specifically compare their simulation results with the conventional disk "(α, Ω)" dynamo model. In this model the shear of the disk (Ω) stretches poloidal field lines in the toroidal direction while a net helicity (α) in the background turbulent velocity field converts toroidal field to poloidal. Brandenburg *et al.* (1995) conclude that the observed dynamo action of the magnetorotational instability cannot be described in terms of such a model.

The combination of turbulence and shear is often invoked as a field amplification mechanism. However, a simulation of Hawley, Gammie & Balbus (1996) calls into question even this simple assumption. In this model a nonrotating shear flow is seeded with magnetic field. The shear flow is hydrodynamically unstable to finite amplitude perturbations, and turbulence quickly develops.

However, following a brief period of amplification, the magnetic field energy decays away (Figure 2). In this case (as in all the others) the turbulence was nonhelical. The simulation demonstrates that turbulence plus shear does not necessarily equal a dynamo. In a disk, it is the action of the magnetorotational instability that sustains the magnetic field in the face of turbulent dissipation.

4. Discussion

Although most simulations to date have concentrated on the local shearing box model, they nevertheless have important implications for actual accretion disk systems, because they allow us to look at the anomalous stress from a more physical point of view. Traditionally, eddy viscosity formalisms have been adopted to describe the stress, $w_{r\phi}$, within an accretion disk. One such is the α formalism of Shakura & Sunyaev (1973) which sets $w_{r\phi} = \alpha P$. Another uses the classical ansatz (*e.g.* Lynden-Bell & Pringle 1974), $w_{r\phi} = \rho \nu_T d\Omega/dr$, where ν_T is some effective "turbulent viscosity." Although these two viewpoints are in essence the same for a Keplerian disk, differences can arise when dealing with subtle questions of viscous stability or when extending one's model beyond the simple, thin Keplerian disk. However, the present numerical simulations point to a more direct statistical formalism, as described by Balbus, Gammie & Hawley (1994). Transport in accretion disks results from correlated velocity and magnetic field fluctuations (δv, δB) in the MHD turbulence. True microscopic viscosity serves only as the ultimate sink at the large wavenumber end of a turbulent cascade. The essential point is that whereas quantities such as α are simply convenient parameterizations, physical quantities such δv are not beyond observational reach (Balbus, Gammie, & Hawley 1994; Horne 1995).

More realistic numerical simulations may soon be providing answers to more detailed questions regarding disk structure. For example, how do the vertical fluxes of magnetic field, angular momentum, and energy compare with the radial fluxes? Observations suggest that disks may often be surrounded by hot corona that both alters the disk continuum radiation and generates a significant nonthermal component. Understanding how energy and magnetic field are transported within the disk will provide clues as to how that corona is produced and what energy is transported into it. Where does magnetic field dissipation occur primarily? Does the resulting X-ray or UV emission radiation have properties consistent with cataclysmic variables and other flaring systems? How does the magnetic instability relate to the generation of winds or jets form the disk surface? Can such flows carry off angular momentum faster than turbulent viscosity can transport it outwards?

Numerical simulations have only begun to examine the physics of accretion disks. However, as we have tried to illustrate here, considerable progress has been made in the last few years. In particular, we can now rectify the conventional wisdom listed in the introduction. Accretion disks accrete because of the outward transport of angular momentum. This is due to the stresses, both Reynolds and Maxwell, present in MHD turbulence. The physical viscosity and resistivity act on the microscopic scale to dissipate the turbulence, producing the thermal heat in the disk. The origin of the anomalous turbulent viscosity is not mysterious: disks are locally, linearly unstable to the magnetorotational instability so long as there are subthermal magnetic fields present, and the criterion $d\Omega^2/d\ln R < 0$ is satisfied, as it always is in an accretion disk. The MHD disk instability acts as a hydromagnetic dynamo that amplifies initially very weak seed fields up to the levels required to produce significant angular momentum transport, thus allowing accretion disks to live up to their name and accrete.

Acknowledgments. This work is supported in part by NASA grants NAG-53058, NAGW-4431, by NSF grant AST-9423187, and an NSF International Cooperative Activity grant INT-9515471. The computations were carried out on the Cray C90 and J90 systems of the Pittsburgh Supercomputing Center.

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Discussion

D. Meier: Could you say a little bit about dissipation in you models. Is it all numerical or do you include finite diffusive terms in the simulations?

J. Hawley: In the simulations shown here we have no explicit resistivity or physical viscosity. There is an artificial viscosity for shock dissipation. Otherwise dissipation is all numerical. Reconnection of field (e.g.) lines takes place at the grid scale.

R. Narayan: You mentioned that sustained convection in an accretion disk leads to an angular momentum flux that goes inward because convection tries to set up a constant specific angular momentum configuration. I would like to understand how general this result is. Specifically, if one had a convecting rotating star with no magnetic fields, do you think the star would tend toward a constant specific angular momentum state?

J. Hawley: No. While the angular momentum equation favours driving forward constant L, the energy equation does not favour that. It only does it in the convection simulation case because there is an artificial heating set up in the disk to keep the convection going. It is not straightforward in a star to predict the outcome in general because there are large scale pressure gradients and energy transport/generation which are independent of Ω.

R. Narayan: In your simulations of the magneto-rotational instability you do not obtain values of effective α greater than about 0.01. Do you think it is possible to achieve values as large as $\alpha \sim 0.1 - 0.3$, as demanded by some observations, by changing some of the conditions in the simulations?

J. Hawley: The "α" values quoted are really measurements of P_{mag}/P_{gas} in the disk. It is better to characterise the stress as proportional to P_{mag}. Strong magnetic fields give large stress, and there are many circumstances where one can imagine generating strong fields ($\sim P_{total}$) and hence strong transport. What we see in the simulations with $\alpha = 0.01$ is from random field in a relatively small shearing box, so I would read this α as more like a lower limit than a gospel value.

M. Nowak: On what time scales are eddy's coherent (for the averages that Steve Balbus talked about)? Or, as translated by Mitch Begelman, what would the effective α be for perturbations to the disk? Is it the same for the steady-state disk?

J. Hawley: We have compared our simulations with the terms in Steve Balbus' talk (average Reynolds and Maxwell stress) averaged over the entire spatial domain. There really isn't an "eddy coherence" time or length scale (instantaneous fluid properties fluctuate very rapidly in the turbulence). This means that α is also fluctuating. You need to average over several orbits before α is "steady".