

EFFECTS OF ANGULAR MOMENTUM ON SUPERNOVA EXPLOSION AND STABILITY
OF RAPIDLY ROTATING STELLAR CORES

Katsuhiko Sato and Masayoshi Kiguchi*
Department of Physics, Faculty of Science,
The University of Tokyo, Tokyo 113 Japan
*Institute of Science and Technology, Kinki University,
Higashi Osaka 577 Japan

ABSTRACT We discuss effects of angular momentum on gravitational collapse and the fate of rapidly rotating stellar cores just before the gravitational collapse. It is found that stellar cores become dynamically unstable against the modes of P_0 and/or $P_2 \pm 2$ before the central densities become higher than the neutron drip density ($\sim 4.3 \times 10^{11} \text{ g/cm}^3$). This suggests that fission occurs at pre-supernova stage or in the collapsing stage. In order to make clear whether these dynamical instabilities induce supernova explosion or not, three-dimensional hydrodynamic computation would be necessary.

1. EFFECTS OF ANGULAR MOMENTUM ON GRAVITATIONAL COLLAPSE

As is well known, massive stars on the main sequence have large angular momentum. If we define the non-dimensional angular momentum $q = J/(GM^2/c)$, the value of q is of the order of ten, where J is the total angular momentum of a star, G gravitational constant, M , the mass of the star and c , the light velocity. If the angular momentum of stellar cores are conserved during stellar evolution, it is expected that fast spinning cores are formed. At present, however, it is not well known how fast the angular momentum is transferred during the stellar evolution. If the transfer from the core to the envelope is very efficient, the effect of angular momentum is neglected. For example, if the value of q at pre-supernova stage, where the central density is higher than 10^9 g/cm^3 , is much smaller than unity, effects of angular momentum is completely neglected even in the dynamical collapsing stage. Gravitational collapse and supernova explosion in this case has been reviewed by Hillebrandt (1986) in detail. When the angular momentum q at pre-supernova stage is of the order of unity, the effects of angular momentum can be neglected at the pre-supernova stage. However it plays an essential role in the dynamical collapsing stage. Collapse in the equatorial plane is stopped by this centrifugal force. Numerical simulations of gravitational collapse of rotating cores has been carried out by LeBlanc and Wilson (1970), Müller et al. (1980), Müller and Hillebrandt (1981), and recently by Symbalisty (1984). They observed very

hot regions are formed on the rotation axis by shock wave, however explosion or mass ejection from stellar core is considerably hard. In particular, Symbalisky showed even if strong magnetic field is taken into account, explosion does not occur.

On the other hand, if $q \gg 1$ at pre-supernova stage, effects of angular momentum is important already from this stage, i. e., stellar cores rotate rapidly and are deformed from sphere greatly by the effect of centrifugal force. How these cores evolve? Do these cores evolve to neutron stars smoothly by the slow angular momentum loss without dynamical collapse? Are these cores are broken up by the dynamical growth of non-axisymmetric mode?

Glatzel et al.(1981) and Tholine(1984) discussed the fate of this rapidly rotating cores, however, their results seems to be rather qualitative because of the following two reasons: First, they assumed that matter density and angular velocity are homogeneous in a stellar core. This Maclaurin spheroid approximation seems to hold no more for stellar cores just before the gravitational collapse. Second, and this is more important, they assumed very high electron-baryon ratio Y_e in the high density region, $\rho > 10^{11}$ g/cm³. This is equivalent to assume that the neutrinos are trapped in stellar cores as in the spherical collapse (Sato 1975, Mazurek 1976). In the actual situations of rapidly rotating stars, however, neutrinos would diffuse out from stellar cores, because neutrino diffusion time is much shorter than the time for angular momentum loss.

In the following, in order to make clear the fate of rapidly rotating cores, we investigate equilibrium structure and stability of cores just before the gravitational collapse more quantitatively by computing the structure of differentially rotating cores by using adequate equation of state.

2. ASSUMPTION AND METHOD

2.1 Equation of state

When the rotation plays an important role on the evolution, the time scale of final stage of stellar evolution becomes larger. This is determined essentially by the time scales of angular momentum loss from cores, and of mass accretion onto cores due to the nuclear shell burning. During these long time intervals, the thermal energy is lost by neutrino emission sufficiently.

When the central density becomes higher than $\sim 10^{9.5}$ g/cm³, electron captures of iron nuclei begin. At this point, non-rotating cores become unstable and begin to collapse. In the case of rotating cores, however, cores can be stable due to the effect of rotation. Therefore, neutrinos can diffuse out from cores and can't be trapped in cores. With increasing density, electron captures of nuclei proceed sufficiently and the electron baryon ratio Y_e becomes very small. In this situation, the structure of the rotating core is described by the equation of state of the matter with low entropy and small Y_e . This equation of state may be

well approximated by that of the cold catalyzed matter. This approximation becomes accurate for densities higher than 10^{10} g/cm^3 , because at these densities electron captures proceed very quickly. We will, therefore, use the equation of state given by Baym, Pethick and Sutherland(1970) in the present work.

2.2 Angular momentum distribution

The structure of spinning core depends critically on the angular momentum distribution, but it is not known what is the most natural distribution. Here We simply assume that the angular momentum distribution is given by

$$j(m)=J/M \cdot 2m,$$

which corresponds to that of uniformly rotating thin disk and is an extreme case of differential rotation. (We calculated various models for angular momentum, details, see the original paper, Kiguchi and Sato(1985).)

2.3 Method

The calculation is made by using the Ostriker and Mark's self-consistent method(1968). At the same time, the perturbations of the second order virial equations are calculated by giving the Lagrange displacement of the type of $L_{ij}x_j$ (Tassoul and Ostriker,1968), and the instability of the rotating cores is studied.

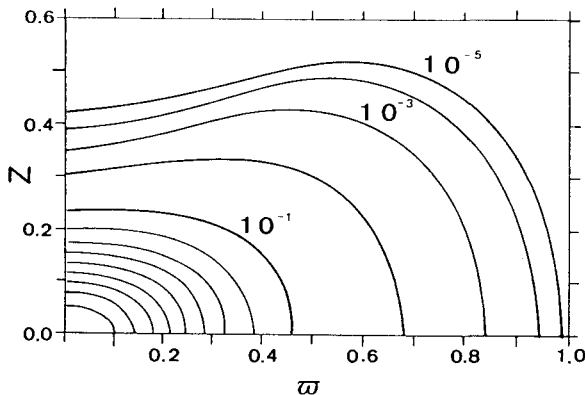


Fig.1. Density contours in a meridional plane of the core with the mass of $2.2M_{\odot}$ and the total angular momentum of $q(=GM^2/c)=4$. The contours correspond to $0.9, \dots, 0.1, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}$ times ρ_c^3 respectively. The central density ρ_c of this models is $9.84 \times 10^{10} \text{ g/cm}^3$ and the equatorial radius R_e is $7.84 \times 10^7 \text{ cm}$.

3. RESULTS

In Fig. 1, we display a view of a model of cores with the mass of $2.2 M_{\odot}$ in the meridional plane. This model is near the onset of dynamical instability for non-axial deformation. As seen from this figure, the shape is extremely flattened. This is very different from the case of Maclaurin spheroid, in which this instability occurs only when the eccentricity is small.

Summary of results of these calculations is shown in Fig 2. In this diagram, stellar cores evolve along mass constant curve increasing their central densities and T/W-ratios.

When the T/W-ratio becomes larger than about 0.14, non-axial secular instability occurs before the dynamical instability. However, since the time scale of this instability is determined by dissipations and is very long compared with the evolution time scale, this instability can be neglected in practice.

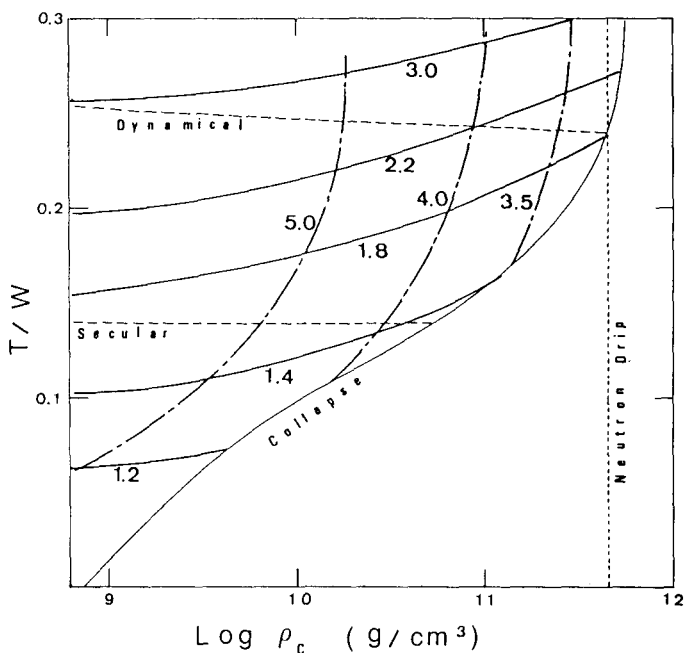


Fig.2. T/W ratios are shown as a function of the central density for various values of the core mass M in units of solar mass (— solid curves) and of the total angular momentum q (- · - · dot-dashed curves). Onset lines of the non-axial instabilities are shown by dashed curves. The collapse instability is also shown by solid line. Stellar cores evolve by the angular momentum loss along mass constant curves increasing T/W-ratio. Eventually stellar cores become unstable against P_0 and/or $P_2^{\pm 2}$ before ρ_c becomes higher than the neutron drip density.

When T/W becomes larger than about 0.25 (precisely speaking, this value shows a change, see Fig. 2), non-axial dynamical instability occurs. For example, if we take $2.2M_{\odot}$, the stellar cores become unstable when the central density becomes 10^{11}g/cm^3 . The time scale of this instability is of the order of one rotation time. It is expected that the binary is formed in the stellar core, because fission occurs usually near this point. If not, the core would evolve along this instability curve ($T/W = 0.25$) in the $T/W - \rho_c$ diagram by transferring the angular momentum to the outer envelope.

It is, anyway, impossible to avoid the dynamical instability eventually as displayed in Fig. 2. If the angular momentum is large, say $q > 4$, evolution path crosses the onset line of non-axial deformation. On the other hand, if the value is smaller than this value, evolution path runs directly into the onset line of the collapse.

Recently, Müller and Eriguchi (1985) also obtained the essentially same result, but dynamical instability was not discussed.

4. CONCLUSION

In the present talk, we showed that stellar cores become dynamically unstable against the modes p_{03} and/or $P_{2 \pm 2}$ before the central densities become higher than $4 \times 10^{11} \text{g/cm}^3$ (the neutron drip density).

In order to make clear whether these dynamical instabilities induce supernova explosion or not, three dimensional hydrodynamic computation would be necessary, because in this case no symmetric axis exist and the situation is fully three dimensional. In this computation, it is also necessary to take into account the magnetic fields, because magnetic field would be greatly strengthened by rapidly differential rotation (Bisnovatyi-Kogan et al., 1976, Meier et al., 1976, Kundt, 1976).

References

- Baym, G., Pethick, C. J., and Sutherland, P. 1970, *Ap. J.*, **170**, 299.
 Bisnovatyi-Kogan, G. S., 1976, *Astrophys. & space Sci.*, **41**, 287
 Müller, E. and Eriguchi, Y., 1985 (Submitted to *Astron. and Astrophys.*)
 Glatzel, E., Fricke, K., and El Eid, M. 1981, *Astr. and Ap.* **93**, 395.
 Hillebrandt, W. 1986 in this volume.
 Kiguchi, M. and Sato, K. Preprint of Univ. of Tokyo, 1985 UTAP-30 (MAY)
 Kundt, W., 1976, *Nature*, **261**, 673.
 LeBlanc, J. M., and Wilson, J. R. 1970, *Ap. J.*, **161**, 541.
 Mazurek, T. J. 1976, *Ap. J.*, **207**, L87.
 Meier, D. L. et al., 1976, *Ap. J.*, **204**, 864.
 Müller, E., Rozyka, M., Hillebrandt, W. 1980, *Astr. Ap.* **81**, 288
 Müller, E., and Hillebrandt, W. 1981, *Astr. and Ap.* **103**, 358.
 Ostriker, J. P., and Mark, J. W-K, 1968 *Ap. J.* **151**, 1075.
 Sato, K. 1975, *Prog. Theor. Phys.* **68**, 595 and **54**, 1325.
 Symbalisty, E. M. D. 1984, *Ap. J.*, **285**, 729.
 Tassoul, J. L., and Ostriker, J. P. 1968, *Ap. J.* **154**, 613.
 Tohline, J. E. 1984, *Ap. J.*, **285**, 1984.