to include an account of metric spaces and to make brief mention of general topological spaces. Some material on conformal mapping has been added and the inclusion of the Schwarz-Christoffel transformation will add to the value of the book to students of applied mathematics. A chapter of twenty pages has been included on elliptic functions and the modular function introduced there is used later in the book along with the monodromy theorem to prove Picard's theorem. It was a defect of the first edition that hardly enough examples were provided by which the student could test his skill and discover the gaps in his understanding; this has been remedied in the present edition.

The admirable qualities of the first edition have, of course, been preserved in the revision and the book is one which every serious student of complex variable theory should possess. D. MARTIN

KLETENIK, D. V., *A Collection of Problems in Analytical Geometry* (Pergamon Press, 1966): Part I, *Analytical Geometry in the Plane*, ix+186 pp., 18s. 6d.; Part II, *Three-dimensional Analytical Geometry*, ix+137 pp., 15s.

These two little books contain 1261 problems on analytical geometry, with answers. Part II includes sections on vector algebra and determinants. The problems are mostly very elementary in nature, demanding straightforward calculations rather than theoretical proofs. The collection might prove useful for pupils needing much practice in routine coordinate geometry, but it can hardly be described as inspiring.

D. MONK

MAGNUS, WILHELM; KARRASS, ABRAHAM; AND SOLITAR, DONALD, *Combinatorial Group Theory* (Interscience Publishers, New York, 1966).

This book is concerned with the presentation of groups in terms of generators and defining relations, and, in particular, with free groups and free products. The adjective " combinatorial " arises from the frequent occurrence of combinatorial methods in this theory. A very clear and thorough treatment is given, and it seems certain that the book will become indispensable to mathematicians working in this area. There are numerous problems of considerable interest for the student to attempt and very full hints are given for their solution. Of particular interest to the reviewer were the frequent illustrations of the general theory chosen from the theory of the modular group and other groups of matrices. Burnside's problem and the fundamental problems of Max Dehn (of which the word problem is only one) are discussed in the later part of the book where a brief introduction is given to recent work on these very deep problems.

R. A. RANKIN

BRAUN, H., AND KOECHER, M., *Jordan-Algebren* (Springer-Verlag, Berlin-Heidelberg-New York, 1966), xiv+357 pp., DM 48.

Jordan algebras arise in the study of subspaces of an associative algebra which are closed under the squaring operation, or more particularly, subspaces of symmetric elements in an algebra with an involution. This was the motivation for Jordan's initial work in the 1930's and it also lies behind the recent work of Vinberg in classifying homogeneous convex domains. Between these two applications a great deal of pure theory was developed and it is this theory which the authors describe here, in the first full-length book on the subject.

The authors lay—rightly—much emphasis on the quadratic transformation $P(x)$: $u \to 2(ux)x - u \, . \, x^2$ (corresponding to $u \to x \, . \, u \, . \, x$ in an associative algebra). It is defined in terms of inverses by the equation $P(x^{-1})u = u \dfrac{\partial x^{-1}}{\partial x}$. This makes it applicable to any algebra in which there are generic units (e.g. finite-dimensional

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algebras with 1), a fact which the authors exploit by carrying out the discussion for any power-associative algebra. The reduction to simple algebras is made by means of associative linear forms and the radical is defined in terms of such forms. This leads to a speedy proof of the direct sum decomposition of semisimple algebras, valid for flexible strictly power-associative algebras of characteristic not two, although it postpones the task of actually finding associative forms. Such a form is obtained by looking at the trace of the right-multiplications, but the algebras have to be further restricted by a technical assumption, which is satisfied by Jordan algebras. This treatment enables the authors to cut down on formal calculations, which can so easily proliferate in this subject. However, it restricts the discussion to finite-dimensional algebras, and at times is rather hard to read because the definitions made are frequently not motivated until many pages later. This part, about two-fifths of the total, forms a concise introduction to finite-dimensional power-associative algebras.

The rest of the book concentrates on Jordan algebras, including a brief mention of the non-commutative and the characteristic-two case, but keeping to finite dimensionality throughout. It is shown that the theory developed so far is applicable, and in particular, that the group $\Pi(A)$ generated by the $P(x)$, for invertible x, acts transitively on the units. This emphasises the importance of isotopes, here called mutations, for in a Jordan algebra A, the elements of $\Pi(A)$ are essentially the autotopisms. Now follow some examples of special Jordan algebras; the exceptional case is dealt with in a separate chapter which also includes the basic facts on alternative algebras. There remains the classification of simple algebras. This is prefaced by a chapter on derivations and one on the Peirce decomposition relative to a complete system of idempotents. The structure theorem itself gives the classification of simple Jordan algebras over an algebraically closed field of characteristic not two. The book concludes with a brief treatment of formally-real Jordan algebras. The 15-page bibliography is fairly complete, not only on Jordan, but also on general non-associative algebras.

The authors have given an interesting and in many ways novel treatment of a difficult subject. Clearly the book is a " must " for anyone interested in Jordan algebras, although there are several topics which receive no mention: general Jordan rings, representation theory, Lie triple systems, applications to convex domains. Of these the reviewer regrets most the omission of the last one, about which the authors are particularly well qualified to write. Possibly they felt that this subject deserves a book to itself; this may well be so, and one hopes that it comes to be written soon.

P. M. COHN

GEL'FOND, A. O., AND LINNIK, YU. V., *Elementary Methods in the Analytic Theory of Numbers*, translated by D. E. Brown (Pergamon Press, 1966), xi+232 pp., 63s.

This is one of two translations into English of a book originally published in Russian in 1962, the other translation having been published in 1966 by Allen and Unwin; it seems regrettable to the reviewer that the work involved in a translation should have been duplicated in this way.

The stated and very worth-while aim of the book is to collect, systematise to some extent and simplify where possible solutions by elementary methods (that is, methods which do not use the theory of functions of a complex variable) of some of the problems of analytic number theory, and with this end in view the authors consider a wide collection of well-known problems. The twelve chapters of the book are on the following topics: 1. Additive properties of numbers, 2. Waring's problem, 3. The distribution of primes, 4. The law of distribution of Gaussian primes, 5. The sieve of Eratosthenes, 6. Atle Selberg's method, 7. The distribution of the fractional parts of numerical sequences, 8. Computation of the integral points within contours, 9. The distribution of power residues, 10. Hasse's theorem, 11. Siegel's theorem, 12. The